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Fuzzy and l-Fuzzy Compactness in a Fuzzy Topological TM-System

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Abstract

Recently in 2010, Tamilarasi and Megalai introduced a new class of algebras known as TM-algebras. In this papaer, we discuss the notion of fuzzy compactness in a fuzzy topological TM-system and On L-fuzzy compactness in L-fuzzy topological TM-system.

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1. Introduction

Recently in 2010, Tamilarasi and Megalai introduced a new class of algebras, called TM-algebras [7]. In their paper they claimed that the TM-algebra is a generalization of BCH/BCI/BCK and Q algebras. In [1] the authors, while studying L-fuzzy structures on TM-algebras brought out the fact that the TM-algebra is not a generalization of BCH/BCI/BCK algebras by giving counter examples.

The notion of a fuzzy set provides a natural framework for generalizing many of the concepts of general topology. The theory of fuzzy topological spaces is developed by Chang [4], Wong [5] , Lowen [6]. and others.In our papers [2] , [3] ,we have studied the notion of Fuzzy Topological subsystem on a TM-algebra and On L- Fuzzy Topological TM-system. In this paper, we discuss the notion of fuzzy compactness in a fuzzy topological TM-system and L- fuzzy compactness in L- fuzzy topological TM-system.

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2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. A TM-Algebra $(X, *, 0)$ is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:

1. $X * 0 = X$
2. $(X * Y) * (X * Z) = Z * Y \ for\ all\ x, y, z \in X.$

Definition 2.2. For any non-empty set X, $\mu : X \to [0, 1]$ is called a fuzzy set of X.

Definition 2.3. Let X be a non-empty set. A mapping $\mu : X \to L$ is called an L- fuzzy set of X,where L is a complete lattice,with sup1 and inf 0.

Definition 2.4. Let $(X, *)$ be a TM-algebra. X is said to be an L- fuzzy topological TM-system if there is a family (X, L_τ) of L- fuzzy subsets in X which satisfies the following conditions

1. $\phi, X \in L_\tau$
2. If $A, B \in L_\tau$ then $A \cap B \in L_\tau$
3. If $A_i \in L_\tau$ for each $i \in I$ then $\cup_I A_i \in L_\tau$ where I is an indexing set.

Remark 2.5. If X is a set with a L-fuzzy topology τ then (X, L_τ) is called an L-fuzzy topological space and any element in L_τ is called an L_τ-open fuzzy set in X.

Definition 2.6. Let f be a function from X to Y. Let σ be a fuzzy set in Y. The inverse image of σ under f is defined as $\sigma_{f^{-1}}(x) = \sigma(f(x))\ \forall x \in X$. Let μ be a fuzzy set in X. The image of μ under f is defined as

$$\mu_f(y) = \begin{cases} sup_{z \in f^{-1}(x)} & \mu(z),\ f^{-1}(x) \text{ is not empty} \\ 0 & \text{otherwise} \end{cases} \quad \forall y \in Y.$$

3. Fuzzy Compactness in a Fuzzy Topological TM-System

Definition 3.1. Let $(X, *)$ be a TM-Algebra. Let (X, T) be a Fuzzy Topological TM-System. $\mathcal{F} = \{\mu_\alpha / \mu_\alpha \in T\}$ be a family of fuzzy sets.A fuzzy set $\mu^\star$ in (X, T). $\mathcal{F}$ is said to be a fuzzy open cover of a fuzzy set $\mu^\star$ if $\mu^\star \subseteq \cup\{\mu_i / \mu_i \in \mathcal{F}\}$.

Example 3.2. Consider the set $X = \{0, 1, 2, 3\}$ with the following cayley table

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X, *)$ is a TM-algebra.
Let the fuzzy subsets $\mu_i : X \to [0, 1], i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ be given by

$$\mu_1(x) = \begin{cases} .9 & \text{if } x = 0 \\ .6 & \text{if } x = 1, 2 \\ .8 & \text{if } x = 3 \end{cases} \qquad \mu_2(x) = \begin{cases} .6 & \text{if } x = 0 \\ .3 & \text{if } x = 1, 2 \\ .4 & \text{if } x = 3 \end{cases} \qquad \mu_3(x) = \begin{cases} .5 & \text{if } x = 0 \\ .2 & \text{if } x = 1, 2 \\ .3 & \text{if } x = 3 \end{cases}$$

$$\mu_4(x) = \begin{cases} .7 & \text{if } x = 0 \\ .4 & \text{if } x = 1, 2 \\ .6 & \text{if } x = 3 \end{cases} \qquad \mu_5(x) = \begin{cases} .9 & \text{if } x = 0 \\ .7 & \text{if } x = 1, 2 \\ .8 & \text{if } x = 3 \end{cases} \qquad \mu_6(x) = \begin{cases} .6 & \text{if } x = 0 \\ .2 & \text{if } x = 1, 2 \\ .4 & \text{if } x = 3 \end{cases}$$

$$\mu_7(x) = \begin{cases} .4 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2 \\ .2 & \text{if } x = 3 \end{cases} \qquad \mu_8(x) = \begin{cases} .8 & \text{if } x = 0 \\ .5 & \text{if } x = 1, 2 \\ .7 & \text{if } x = 3 \end{cases} \qquad \mu_9(x) = \begin{cases} .8 & \text{if } x = 0 \\ .4 & \text{if } x = 1, 2 \\ .7 & \text{if } x = 3 \end{cases}$$

$$\mu_{10}(x) = \begin{cases} .3 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2 \\ .1 & \text{if } x = 3 \end{cases}$$

Then the fuzzy topology on X is

$$\begin{aligned} T &= \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_8, \mu_9, \mu_{10}\} \\ \mu^\star &= \mu_{10}. \\ \mathcal{F} &= \{\mu_1, \mu_3, \mu_4, \mu_6, \mu_7\} \\ \mu^\star &\subseteq \cup\{\mu_i/\mu_i \in \mathcal{F}\} \\ \mu^\star &\subseteq Max\,\{\mu_1, \mu_3, \mu_4, \mu_6, \mu_7\} \\ &= \mu_1 \therefore \mu^\star \subseteq \mu_1 \therefore \mathcal{F} \end{aligned}$$

is a fuzzy open cover of $\mu^\star$.

Definition 3.3. Let $(X, *)$ be a TM-Algebra. Let (X, T) be a fuzzy topological TM-system.A fuzzy set $\mu^\star$ in (X, T).A fuzzy open subcover of $\mu^\star$ is a finite fuzzy subsets of fuzzy open cover of $\mu^\star$ which is also a fuzzy open cover.

Example 3.4. Consider the example 3.2, $\mathcal{F}$ is fuzzy open cover of $\mu^{\star}$. A finite fuzzy subsets of $\mathcal{F}$ is $A = \{\mu_3, \mu_4, \mu_7\}$, $\mu^{\star} \subseteq \cup\{\mu_i/\mu_i \in A\}$ $\mu^{\star} \subseteq Max\,\{\mu_3, \mu_4, \mu_7\} = \mu_4$ $\therefore$ $\mu^{\star} \subseteq \mu_4$ $\therefore$ A is fuzzy open subcover of $\mu^{\star}$ in (X, T).

Definition 3.5. Let $(X, *)$ be a TM-Algebra. Let (X, T) be a fuzzy topological TM-system.A fuzzy set $\mu^{\star}$ in (X, T) is said to be fuzzy compact if each fuzzy open cover of $\mu^{\star}$ has a finite fuzzy open subcover.

Example 3.6. Let Z be the set of all integers and Let $nZ = \{nx : x \in Z\}, n \in Z$. Then $(Z, -, 0)$ and $(nZ, -, 0)$ are TM-algebras where '-' is the usual subtraction. Let the fuzzy subsets $\mu_i : Z \to [0, 1]$, $\mu_i(n) = \dfrac{n}{10^r}$, r denotes the number of digits in n, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9$ be given by

$$\mu_1(x) = \begin{cases} \dfrac{x}{10} & \text{if } x = 1, 2, \ldots, 9 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_2(x) = \begin{cases} \dfrac{x}{10^2} & \text{if } x = 10, 11, \ldots, 99 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_3(x) = \begin{cases} \dfrac{x}{10^3} & \text{if } x = 100, 101, \ldots, 999 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_4(x) = \begin{cases} \dfrac{x}{10^4} & \text{if } x = 1000, 1001, \ldots, 9999 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_5(x) = \begin{cases} \dfrac{x}{10^5} & \text{if } x = 10000, 10001, \ldots, 99999 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_6(x) = \begin{cases} \dfrac{x}{10^6} & \text{if } x = 100000, \ldots, 999999 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_7(x) = \begin{cases} \dfrac{x}{10^7} & \text{if } x = 1000000, \ldots, 9999999 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_8(x) = \begin{cases} \dfrac{x}{10^8} & \text{if } x = 10000000, \ldots, 99999999 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_9(x) = \begin{cases} \dfrac{x}{10^9} & \text{if } x = 100000000, \ldots, 999999999 \\ 0 & \text{otherwise} \end{cases}$$

Then the fuzzy topology on X is $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_9\}$. Let $\mu^\star = \mu_9$ $\mathcal{F} = \{\mu_2, \mu_5, \mu_7, \mu_8\}$ is an fuzzy open cover of $\mu^\star$. A finite fuzzy subsets of $\mathcal{F}$ is $A = \{\mu_2, \mu_5, \mu_8\}$ is fuzzy open subcover of $\mu^\star$. Similarlly, each fuzzy open cover of $\mu^\star$ has a finite fuzzy open subcover. $\therefore$ $\mu^\star$ is fuzzy compact.

Theorem 3.7. Let $(X, T), (Y, U)$ be two fuzzy topological TM-systems such that $f : (X, T) \rightarrow (Y, U)$ is a fuzzy continuous homomorphism. If $\mu^\star$ is fuzzy compact in (X, T) then $f(\mu^\star)$ is fuzzy compact in (Y, U).

Proof. Let $\mu^\star$ be a fuzzy subset of (X, T).Let $\mathcal{F} = \{\mu_\alpha/\mu_\alpha \in T\}$ be a fuzzy open cover of $\mu^\star$. So that $\mu^\star \subseteq \cup\{\mu_i/\mu_i \in \mathcal{F}\}$. Since $\mu^\star$ is fuzzy compact, every fuzzy open cover of $\mu^\star$ has a finite fuzzy open subcover.That is $\mu^\star \subseteq \cup_{i=1}^{n}\mu_i, \mu_i \in \mathcal{F}$. Let $f(\mu^\star)$ be a fuzzy subset of (Y, U). Let $\mathcal{F}' = \{\sigma_\alpha/\sigma_\alpha \in U\}$ be a fuzzy open cover of $f(\mu^\star)$. That is $f(\mu^\star) \subseteq \cup\left\{\sigma_i/\sigma_i \in \mathcal{F}'\right\}$. Since f is fuzzy continuous homomorphism, $f^{-1}(\sigma_\alpha)$ is open in (X, T). $\mu^\star \subseteq f^{-1}(\sigma_\alpha), \sigma_\alpha \in U$. Since $\mu^\star$ is fuzzy compact,

$$\begin{aligned}
&\mu^\star \subseteq \cup_{i=1}^{n} f^{-1}(\sigma_i), \sigma_i \in \mathcal{F}' \\
&\quad \Rightarrow f(\mu^\star) \subseteq f(\cup_{i=1}^{n} f^{-1}(\sigma_i)) \\
&\quad \Rightarrow f(\mu^\star) \subseteq \cup_{i=1}^{n} f(f^{-1}(\sigma_i) \\
&\quad \Rightarrow f(\mu^\star) \subseteq \cup_{i=1}^{n}(\sigma_i), \sigma_i \in \mathcal{F}'.
\end{aligned}$$

Thus the fuzzy open cover $\mathcal{F}'$ of $f(\mu^\star)$ in (Y, U) has a finite fuzzy open subcover. Hence $f(\mu^\star)$ is fuzzy compact. ■

Theorem 3.8. Let $(X, T), (Y, U)$ be two fuzzy topological TM-systems such that $f : (X, T) \rightarrow (Y, U)$ is a fuzzy continuous homomorphism. If $\sigma^\star$ is fuzzy compact in (Y, U) then $f^{-1}(\sigma^\star)$ is fuzzy compact in (X, T).

Proof. Let $\sigma^\star$ be a fuzzy subset of (Y, U).Let $\mathcal{F} = \{\sigma_\alpha/\sigma_\alpha \in U\}$ be a fuzzy open cover of $\sigma^\star$ So that $\sigma^\star \subseteq \cup\{\sigma_i/\sigma_i \in \mathcal{F}\}$. Since $\sigma^\star$ is fuzzy compact, every fuzzy open cover of $\sigma^\star$ has a finite fuzzy open subcover. That is $\sigma^\star \subseteq \cup_{i=1}^{n}\sigma_i, \sigma_i \in \mathcal{F}$.

Let $f^{-1}(\sigma^\star)$ be a fuzzy subset of (X, T). Let $\mathcal{F}' = \{\mu_\alpha/\mu_\alpha \in T\}$ be a fuzzy open cover of $f^{-1}(\sigma^\star)$. That is $f^{-1}(\sigma^\star) \subseteq \cup\left\{\mu_i/\mu_i \in \mathcal{F}'\right\}$. Since $\sigma^\star$ is fuzzy compact, $\sigma^\star \subseteq \cup_{i=1}^{n} f(\mu_i)$. Since f is fuzzy continuous homomorphism,

$$\begin{aligned}
&\Rightarrow f^{-1}(\sigma^\star) \subseteq f^{-1}(\cup_{i=1}^{n} f(\mu_i)) \\
&\Rightarrow f^{-1}(\sigma^\star) \subseteq \cup_{i=1}^{n} f^{-1}(f(\mu_i)) \\
&\Rightarrow f^{-1}(\sigma^\star) \subseteq \cup_{i=1}^{n}(\mu_i), \mu_i \in \mathcal{F}'.
\end{aligned}$$

Thus the fuzzy open cover $\mathcal{F}'$ of $f^{-1}(\sigma^\star)$ in (X, T) has a finite fuzzy open subcover. Hence $f^{-1}(\sigma^\star)$ is fuzzy compact. ■

4. On L-Fuzzy Compactness in L-Fuzzy Topological TM-System

Definition 4.1. Let $(X, *)$ be a TM-Algebra. Let (X, L_T) be an L-Fuzzy Topological TM-System. $L_{\mathcal{F}} = \{\mu_\alpha/\mu_\alpha \in L_T\}$ be a family of L-fuzzy sets. An L-fuzzy set $\mu^\star$ in (X, L_T). $L_{\mathcal{F}}$ is said to be an L-fuzzy open cover of an L-fuzzy set $\mu^\star$ if $\mu^\star \subseteq \vee\{\mu_i/\mu_i \in L_{\mathcal{F}}\}$.

Example 4.2. Consider the set $X = \{0, 1, 2, 3, 4, 5\}$ with the following cayley table

$*$	0	1	2	3	4	5
0	0	3	4	1	2	5
1	1	0	2	3	5	4
2	2	4	0	5	1	3
3	3	1	5	0	4	2
4	4	5	3	2	0	1
5	5	2	1	4	3	0

Then $(X, *)$ is a TM-algebra. Let L be a complete lattice with sup 1 and info. Let $t_1, t_2, t_3, t_4, t_5, t_6, t_7 \in L$ such that $0 \le t_1 \le t_2 \le t_3 \le t_4 \le t_5 \le t_6 \le t_7 \le 1$. Consider the L- fuzzy subsets $\mu_i : X \to L, i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ be given by

$$\mu_1(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1, 3 \\ t_3 & \text{if } x = 2, 4, 5 \end{cases} \qquad \mu_2(x) = \begin{cases} t_7 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases}$$

$$\mu_3(x) = \begin{cases} t_3 & \text{if } x = 0, 5 \\ 0 & \text{if } x = 1, 2, 3, 4 \end{cases} \qquad \mu_4(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_4 & \text{if } x = 1, 3 \\ t_3 & \text{if } x = 2, 4, 5 \end{cases}$$

$$\mu_5(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_2 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_6(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_6 & \text{if } x = 1, 3 \\ t_4 & \text{if } x = 2, 4, 5 \end{cases}$$

$$\mu_7(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_1 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_8(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_5 & \text{if } x = 1, 3 \\ t_4 & \text{if } x = 2, 4, 5 \end{cases}$$

$$\mu_9(x) = \begin{cases} t_6 & \text{if } x = 0 \\ 0 & \text{if } x = 1, 2, 3, 4 \\ t_3 & \text{if } x = 5 \end{cases} \qquad \mu_{10}(x) = \begin{cases} 1 & \text{if } x = 0 \\ t_7 & \text{if } x = 1, 3 \\ t_4 & \text{if } x = 2, 4, 5 \end{cases}$$

Then the fuzzy topology on X is $L_T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_9, \mu_{10}\}$.
Let $\mu^\star = \mu_3$. $L_{\mathcal{F}} = \{\mu_1, \mu_2, \mu_5, \mu_6, \mu_8\}$
$\mu^\star \subseteq \vee\{\mu_i/\mu_i \in L_{\mathcal{F}}\}$ $\mu^\star \subseteq Max\{\mu_1, \mu_2, \mu_5, \mu_6, \mu_8\} = \mu_6$
$\therefore \mu^\star \subseteq \mu_6$
$\therefore L_{\mathcal{F}}$ is an L-fuzzy open cover of $\mu^\star$.

Definition 4.3. Let $(X, *)$ be a TM-Algebra. Let (X, L_T) be an L-fuzzy topological TM-system. An L-fuzzy set $\mu^\star$ in (X, L_T). An L-fuzzy open subcover of $\mu^\star$ is a finite L-fuzzy subsets of L-fuzzy open cover of $\mu^\star$ which is also an L-fuzzy open cover.

Example 4.4. Consider the example 4.2, $L_{\mathcal{F}}$ is an L-fuzzy open cover of $\mu^\star$. A finite L-fuzzy subsets of $L_{\mathcal{F}}$ is $L_A = \{\mu_2, \mu_5, \mu_8\}$, $\mu^\star \subseteq \vee \{\mu_i/\mu_i \in L_A\}$
$\mu^\star \subseteq Max\{\mu_2, \mu_5, \mu_8\} = \mu_8$
$\therefore \mu^\star \subseteq \mu_8$
$\therefore L_A$ is an L-fuzzy open subcover of $\mu^\star$ *in* (X, L_T).

Definition 4.5. Let $(X, *)$ be a TM-Algebra. Let (X, L_T) be an L-fuzzy topological TM-system. An L-fuzzy set $\mu^\star$ in (X, L_T) is said to be an L-fuzzy compact if each L-fuzzy open cover of $\mu^\star$ has a finite L-fuzzy open subcover.

Example 4.6. From the example 4.2, 4.4, $\mu^\star = \mu_3$, each L-fuzzy open cover of $\mu^\star$ has a finite L-fuzzy open subcover.
$\therefore \mu^\star$ is L-fuzzy compact.

Example 4.7. Consider the example 4.2, $\mu^\star = \mu_5$, $L_{\mathcal{F}} = \{\mu_1, \mu_2, \mu_4, \mu_6, \mu_7, \mu_8\}$
$\mu^\star \subseteq \vee \{\mu_i/\mu_i \in L_{\mathcal{F}}\}$
$\mu^\star \subseteq Max\{\mu_1, \mu_2, \mu_4, \mu_6, \mu_7, \mu_8\} = \mu_6$
$\therefore \mu^\star \subseteq \mu_6$
$\therefore$ $L_{\mathcal{F}}$ is an L-fuzzy open cover of $\mu^\star$.But this L-fuzzy open cover does not have an L-fuzzy open subcover.Because an L- fuzzy subsets of an L-fuzzy open cover, $L_A = \{\mu_2, \mu_7\}$ is not an L- fuzzy open cover.
$\therefore \mu^\star$ is not an L-fuzzy compact.

Theorem 4.8. Let (X, L_T), (Y, L_U) be two L-fuzzy topological TM-systems such that $f : (X, L_T) \to (Y, L_U)$ is an L-fuzzy continuous homomorphism. If $\mu^\star$ is an L-fuzzy compact in (X, L_T) then $f(\mu^\star)$ is an L-fuzzy compact in (Y, L_U).

Proof. Let $\mu^\star$ be an L-fuzzy subset of (X, L_T). Let $L_{\mathcal{F}} = \{\mu_\alpha/\mu_\alpha \in L_T\}$ be an L-fuzzy open cover of $\mu^\star$. So that $\mu^\star \subseteq \vee\{\mu_i/\mu_i \in L_{\mathcal{F}}\}$. Since $\mu^\star$ is an L-fuzzy compact, every L-fuzzy open cover of $\mu^\star$ has a finite L-fuzzy open subcover. That is $\mu^\star \subseteq \vee_{i=1}^{n} \mu_i$, $\mu_i \in L_{\mathcal{F}}$.

Let $f(\mu^\star)$ be a L-fuzzy subset of (Y, L_U). Let $L_{\mathcal{F}'} = \{\sigma_\alpha/\sigma_\alpha \in L_U\}$ be an L-fuzzy open cover of $f(\mu^\star)$. That is $f(\mu^\star) \subseteq \vee\{\sigma_i/\sigma_i \in L_{\mathcal{F}'}\}$. Since f is an L-fuzzy continuous homomorphism, $f^{-1}(\sigma_\alpha)$is open in (X, L_T).
$\mu^\star \subseteq f^{-1}(\sigma_\alpha), \sigma_\alpha \in L_U$. Since $\mu^\star$ is an L-fuzzy compact , $\mu^\star \subseteq \vee_{i=1}^{n} f^{-1}(\sigma_i), \sigma_i \in L_{\mathcal{F}'}$
$\Rightarrow f(\mu^\star) \subseteq f(\vee_{i=1}^{n} f^{-1}(\sigma_i))$
$\Rightarrow f(\mu^\star) \subseteq \vee_{i=1}^{n} f(f^{-1}(\sigma_i)$
$\Rightarrow f(\mu^\star) \subseteq \vee_{i=1}^{n} (\sigma_i), \sigma_i \in L_{\mathcal{F}'}$.
Thus an L-fuzzy open cover $L_{\mathcal{F}'}$ of $f(\mu^\star)$ in (Y, L_U) has a finite L-fuzzy open subcover. Hence $f(\mu^\star)$ is an L-fuzzy compact. ■

Theorem 4.9. Let (X, L_T), (Y, L_U) be two L-fuzzy topological TM-systems such that $f : (X, L_T) \to (Y, L_U)$ is an L-fuzzy continuous homomorphism. If $\sigma^\star$ is an L-fuzzy compact in (Y, L_U) then $f^{-1}(\sigma^\star)$ is an L- fuzzy compact in (X, L_T).

Proof. Let $\sigma^\star$ be an L-fuzzy subset of (Y, L_U). Let $L_{\mathcal{F}} = \{\sigma_\alpha/\sigma_\alpha \in L_U\}$ be an L- fuzzy open cover of $\sigma^\star$. So that $\sigma^\star \subseteq \vee\{\sigma_i/\sigma_i \in L_{\mathcal{F}}\}$. Since $\sigma^\star$ is an L-fuzzy compact, every L-fuzzy open cover of $\sigma^\star$ has a finite L-fuzzy open subcover. That is $\sigma^\star \subseteq \vee_{i=1}^{n}\sigma_i, \sigma_i \in L_{\mathcal{F}}$.

Let $f^{-1}(\sigma^\star)$ be an L- fuzzy subset of (X, L_T). Let $L_{\mathcal{F}'} = \{\mu_\alpha/\mu_\alpha \in L_T\}$ be an L-fuzzy open cover of $f^{-1}(\sigma^\star)$. That is $f^{-1}(\sigma^\star) \subseteq \vee\{\mu_i/\mu_i \in L_{\mathcal{F}'}\}$. Since $\sigma^\star$ is an L-fuzzy compact, $\sigma^\star \subseteq \vee_{i=1}^{n} f(\mu_i)$. Since f is an L-fuzzy continuous homomorphism,
$\Rightarrow f^{-1}(\sigma^\star) \subseteq f^{-1}(\vee_{i=1}^{n} f(\mu_i))$
$\Rightarrow f^{-1}(\sigma^\star) \subseteq \vee_{i=1}^{n} f^{-1}(f(\mu_i))$
$\Rightarrow f^{-1}(\sigma^\star) \subseteq \vee_{i=1}^{n}(\mu_i), \mu_i \in L_{\mathcal{F}'}$.
Thus an L-fuzzy open cover $L_{\mathcal{F}'}$ of $f^{-1}(\sigma^\star)$ in (X, L_T) has a finite L-fuzzy open subcover. Hence $f^{-1}(\sigma^\star)$ is L-fuzzy compact. ■

References

[1] Chandramouleeswaran. M, Anusuya. R, Muralikrishna. P., An L-fuzzy subalgebras of TM-algebras, Advances in Theorectical and Applied Mathematics, ISSN 0973-4554, 65(2011), 547–558.

[2] Chandramouleeswaran. M, Annalakshmi. M., Fuzzy Topological subsystem on a TM-algebra,Intetnational Journal of Pure and Applied Mathematics, Vol. 94, No. 3 (2014).

[3] Chandramouleeswaran. M, Annalakshmi. M., On L-Fuzzy Topological TM-system, Intetnational Journal of Mathematical Sciences and Engineering Applications, Vol. 8, No. IV (July, 2014), pp. 135–145.

[4] Chang C.L., Fuzzy Topological Spaces, Journal of Mathematical Analysis and Applications, Vol. 24, (1968), pp. 182–190.

[5] Wong. C.K., Fuzzy Topology: Product and quotient theorems, J. Math. Anal. Appl. 45(1974), 512–521.

[6] Lowen. R, Fuzzy topological spaces and fuzzy compactness, J. Math. Anal. Appl. 56(1976), 621–633.

[7] Tamilarasi. A and Megalai. K, TM-algebra an introduction, CASCT, (2010).