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On Skolem Mean Labeling of Stars $(K_{1,\ell} \cup K_{1,m} \cup K_{1,n})$

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Abstract

In this paper, we prove the conjecture that if $\ell \leq m < n$, the three Star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $n \geq \ell + m + 5$.

1. INTRODUCTION

All graphs in this chapter are finite, simple and undirected. Terms not defined here are used in the sense of Harary [4]. In [1] focused on the skolem mean labeling was focused as assignment of label to the vertices $x \in V$ with distinct elements f(x) from 1, 2, . . . , p in such a way that when the edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges get distinct labels from the set {2, 3, . . . , p} and it was proved that any path is a skolem mean graph, if $m \geq 4$, $K_{1,m}$ is not a skolem mean graph and the two stars $K_{1,m} \cup K_{1,n}$ is a skolem mean graph if and only if $|m - n| \leq 4$. In [2], it was proved that the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + \ell$, $\ell = 1, 2, 3, \ldots$; $m = 1,2,3,\ldots$; $n = \ell + m + 4$ and $\ell \leq m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + 2\ell$ for $\ell = 2, 3, 4,\ldots$; $m = 2,3,4,\ldots$; $n = 2\ell + m + 4$ and $\ell \leq m < n$; the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if |m

– n| = 7 for m = 1,2,3,…; n = m + 7 and 1 ≤ m < n. In [3], the condition for a graph to be skolem mean is that p ≥ q + 1. In this paper, we prove the existence of skolem mean graph of three star.

Definition 2.0 The three star is the disjoint union of $K_{1,\ell}$, $K_{1,m}$ and $K_{1,n}$. It is denoted by $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$.

Theorem 2.1 If $\ell \le m < n$, the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $n \ge \ell + m + 5$.

Proof: Consider the graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$.

Let $\ell \le m < n$ where $n = \ell + m + 5$ for $\ell = 1, 2, 3, \ldots$ and $m = 1, 2, 3, \ldots$. Let us take the case that $|m-n| > 4 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $n = \ell + m + 5$.

Therefore, the graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ where $n = \ell + m + 5$ for $\ell = 1, 2, 3, \ldots$ and $m = 1, 2, 3, \ldots$.

Let $\{u\} \cup \{u_i: 1 \le i \le \ell\}$, $\{v\} \cup \{v_j: 1 \le j \le m\}$ and $\{w\} \cup \{w_k: 1 \le k \le n\}$ be the vertices of G. Then G has $\ell + m + n + 3$ vertices and $\ell + m + n$ edges.

We have $V(G) = \{u, v, w\} \cup \{u_i: 1 \le i \le \ell\} \cup \{v_j: 1 \le j \le m\} \cup \{w_k: 1 \le k \le n\}$.

Case (a): The required vertex labeling $f: V(G) \to \{1, 2, 3, \ldots, \ell + m + n + 3\}$ is defined as follows:

$$f(u) = 2; f(v) = 4; f(w) = \ell + m + n + 2;$$

$$f(u_i) = 2i + 4 \text{ for } 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 4 \text{ for } 1 \le j \le m$$

$$f(w_k) = 2k - 1 \text{ for } 1 \le k \le n - 2$$

$$f(w_{n-1}) = \ell + m + n + 1 \text{ and}$$

$$f(w_n) = \ell + m + n + 3$$

The corresponding edge labels are as follows:

The edge label of uu_i is $i + 3$ for $1 \le i \le \ell$; vv_j is $\ell + j + 4$ for $1 \le j \le m$ and ww_k is $\frac{2k+\ell+m+n+1}{2}$ for $1 \le k \le n - 2$; The edge label of ww_{n-1} is $\ell + m + n + 2$ and ww_n is $\ell + m + n + 3$.

Therefore, the required edge labels of G = $\{4, \ldots, \ell + 3, \ell + 5, \ldots, \ell + m + 4, \ell + m + 4, \ldots, 2\ell + 2m + 6, 2\ell + 2m + 7, 2\ell + 2m + 8\}$. But, the edge label of vv_m is $\ell + m + 4$ and ww_1 is $\ell + m + 4$ are the same.

Hence the induced edge labels of G are not distinct. Hence the graph G is not a skolem mean graph if $|m - n| > 4 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $n = \ell + m + 5$.

Case (b): Consider the graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ for $n \ge \ell + m + 5$.

Let us assume that $G = G_1 \cup G_2 \cup G_3$ for $G_1 = K_{1,\ell}$, $G_2 = K_{1,m}$ and $G_3 = K_{1,n}$ for

$n \geq \ell + m + 5$ are the components of G.

If f(w) = t and if three of the w_k's have consecutive integers as labels then the corresponding induced edge labels are two of them will be equal. Without loss of generality, let us assume that $f(w_1) = a$, $f(w_2) = a + 1$ and $f(w_3) = a + 2$.

Case 1: If t and a are odd then the edge labels are $f(ww_1) = \frac{t+a}{2}$,

$f(ww_2) = \frac{t+a+2}{2}$ and $f(ww_3) = \frac{t+a+2}{2}$.

Case 2: If t is odd and a is even then the edge labels are $f(ww_1) = \frac{t+a+1}{2}$, $f(ww_2) = \frac{t+a+1}{2}$ and $f(ww_3) = \frac{t+a+3}{2}$. Hence, if t is odd then two of the edges have the same label.

Case 3: If t and a are even then the edge labels are $f(ww_1) = \frac{t+a}{2}$, $f(ww_2) = \frac{t+a+2}{2}$ and $f(ww_3) = \frac{t+a+2}{2}$.

Case 4: If t is even and a is odd then the edge labels are $f(ww_1) = \frac{t+a+1}{2}$, $f(ww_2) = \frac{t+a+1}{2}$ and $f(ww_3) = \frac{t+a+3}{2}$. Hence, if t is even then two of the edges have the same label. Therefore, it is enough if we show that when $n \geq \ell + m + 5$, necessarily three of the vertices in the component of G_3 will have consecutive integers as labels.

(i) Let us now consider the case when $n = \ell + m + 5$. Then the labels for the vertices are $\{1, 2, 3, \ldots, 2\ell + 2m + 8\}$. Let $f(w) = 2\ell + 2m + 8$. To get the edge label $2\ell + 2m + 8$, we must have $2\ell + 2m + 8$ and $2\ell + 2m + 7$ as the vertex labels of adjacent vertices in the component G_3. In that case, it is not possible to label the vertices without labeling three of them as consecutive integers in the component G_3. Then, the corresponding induced edge labels are two of them will be equal.

Therefore, the induced edge labels of G are not distinct.

Hence, the graph $G = K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ for $n = \ell + m + 5$ is not a skolem mean graph.

(ii) Let us now consider the case when $2\ell + 2m + 8$ is not an edge label of G.

To get the edge label $2\ell + 2m + 7$, we must have $2\ell + 2m + 7$ and $2\ell + 2m + 6$ as the vertex labels of adjacent vertices in the component G_3. Thus either $2\ell + 2m + 7$ or $2\ell + 2m + 6$ must be the label of w. In both cases, it is not possible to label the vertices without labeling three of them as consecutive integers in the component G_3. Then, the corresponding induced edge labels are two of them will be equal.

Therefore, the induced edge labels of G are not distinct.

Hence, the graph G = $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ for n = ℓ + m + 5 is not a skolem mean graph.

(iii) Let us now consider the case when n ≥ ℓ + m + 6.

Consider, the graph G = $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ for n ≥ ℓ + m + 6. Then the labels for the vertices are {1, 2, 3, . . . , 2ℓ + 2m + 9, 2ℓ + 2m + 10, 2ℓ + 2m + 11, . . . }.

Therefore, the labels for the vertices are {1, 2, 3, . . . , 2ℓ + 2m + r + 8 for r = 1, 2, 3, . . .}.

To get the edge label 2ℓ + 2m + r + 8 for r = 1, 2, 3, . . . , we must have 2ℓ + 2m + r + 7 for r = 1, 2, 3, . . . and 2ℓ + 2m + r + 8 for r = 1, 2, 3, . . . as the vertex labels of adjacent vertices in the component G_3. Thus either 2ℓ + 2m + r + 7 for r = 1, 2, 3, . . . or 2ℓ + 2m + r + 8 for r = 1, 2, 3, . . . must be the label of w. In both cases, it is not possible to label the vertices without labeling three of them as consecutive integers in the component G_3. Then, the corresponding induced edge labels are two of them will be equal.

Therefore, the induced edge labels of G are not distinct. Therefore, the graph G = $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ for n ≥ ℓ + m + 6 is not a skolem mean graph.

Hence the graph G is not a skolem mean graph if |m–n| > 4 + ℓ for ℓ = 1, 2, 3, . . . ; m = 1, 2, 3, . . . and n ≥ ℓ + m + 5.

Conclusion

We investigated the existence of skolem mean graph of three stars. That is we proved the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if |m–n| > 4 + ℓ for ℓ = 1, 2, 3, . . . ; m = 1, 2, 3, . . . and ℓ ≤ m < n.

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