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A Remark on Generalized Repunits

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Abstract

In this paper, we give some congruence for a generalization of generalized repunits with some negative numbers.

Keywords:generalized repunits, congruence

1. Introduction and Preliminaries

In 1966, Beiler [1] introduced the repunitR_n, any integerwritten in decimal form as a string of 1's, by

$$R_n := \frac{10^n - 1}{9}.$$

In 1982, Snyder [6] introduced the generalized repunits(or repunits to base b) $R_n(b)$, any integerwritten in decimal form as a string of 1's in base b, by

$$R_n(b) := \frac{b^n - 1}{b - 1}.$$

In 2013, Trojovsky[8] introduced a generalization of generalized repunits by

$$J_n(k) := \frac{(k+1)^n - nk - 1}{k^2}.$$

And then

$$J_0(k) = 0,$$

$$J_1(k) = 0,$$

$$J_n(k) = \sum_{i=0}^{n-2} \binom{n}{i} k^{n-2-i}$$

for all n ≥ 2.

Moreover, the numbers $J_n(k)$ has the recurrence forms as follows.

$$J_{n+1}(k) - (k + 1)J_n(k) = n.$$

$$J_{n+2}(k) - (k + 2)J_{n+1}(k) + (k + 1)J_n(k) = 1.$$

The generating function$j(x)$ for the numbers $J_n(k)$ can be expand by

$$j(x) = \sum_{n=0}^{\infty} \frac{(k+1)^n - nk - 1}{k^2} x^n$$

and then

$$j(x) = \frac{x^2}{(1-x-kx)(1-x)^2}.$$

The numbers $J_n(k)$ has the other formula as follows.

$$\sum_{\substack{n_1,n_2,\dots,n_m \\ n_1+n_2,\dots+n_m=n}} J_{n_1}(k)\, J_{n_2}(k) \dots J_{n_m}(k) = \sum_{i=0}^{n-2m} \binom{2m+i-1}{2m-1} \binom{n-i-m-1}{m-1} (k+1)^{n-2m-i}.$$

In this paper, we give some congruence for a generalization of generalized repunits with some negative numbers.

2. Main Results

Theorem 2.1. Let a, b, l, n be any integers such that a, $b$and n are positive. Then

$$J_n(al+b) \equiv \begin{cases} \binom{n}{2} \pmod{a} & \text{if } a \mid b \\ J_n(b) \pmod{a} & \text{if } a \nmid b. \end{cases}$$

Proof. Case $a \mid b$. Then there exists m such that

$$b = am.$$

Thus,

$$\begin{aligned}
J_n(al+b) &= J_n(a(l+m)) \\
&= \frac{(a(l+m)+1)^n - na(l+m) - 1}{(a(l+m))^2} \\
&= \frac{\sum_{i=2}^{n} \binom{n}{i} (a(l+m))^i}{(a(l+m))^2} \\
&= \sum_{i=2}^{n} \binom{n}{i} (a(l+m))^{i-2} \\
&= \binom{n}{2} + \sum_{i=3}^{n} \binom{n}{i} (a(l+m))^{i-2} \\
&\equiv \binom{n}{2} \pmod{a}.
\end{aligned}$$

Case $a \nmid b$. We note that $al+b \equiv b \pmod{a}$

$$\begin{aligned}
J_n(al+b) &= \frac{(al+b+1)^n - n(al+b) - 1}{(al+b)^2} \\
&\equiv \frac{(b+1)^n - nb - 1}{b^2} \pmod{a}
\end{aligned}$$

Therefore,

$$J_n(al+b) \equiv \begin{cases} \binom{n}{2} \pmod{a} & \text{if} \quad a \mid b \\ J_n(b) \pmod{a} & \text{if} \quad a \nmid b. \end{cases}$$

Corollary 2.2. [8] Let a, b, l, $n$be any positive integers. Then

$$J_n(al+b) \equiv \begin{cases} \binom{n}{2} \pmod{a} & \text{if} \quad a \mid b \\ J_n(b) \pmod{a} & \text{if} \quad a \nmid b. \end{cases}$$

Corollary 2.3. Let a, m, n be any integers such that $a$and$n$ are positive. Then

$$J_n(am) \equiv \binom{n}{2} \pmod{a}$$

Proof. By Theorem 2.1, we use $b = a$ and $l = m - 1$, this is in case $a \mid b$.

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