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# Computation of an Accurate Implicit Block Method for Solving Third Order Ordinary Differential Equations Directly

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**Abstract**

This article discusses the derivation of block method with step-length of five for direct solution of initial value problems of third order ordinary differential equations (ODEs). This method is developed through interpolation and collocation of power series approximate solution. In addition, some basic properties of the method are also established. The developed method is then applied to solve some initial value problems of third order ODEs. The numerical results confirm the superiority of the new method over the existing methods in term of accuracy.

**Keywords**: Power Series, Interpolation, Collocation, Block method, Initial Value Problems, Direct Solution.

## 1. Introduction

This paper examines the solution of third order ordinary differential equations of the form

$$y''' = f(x, y, y', y'') \quad y(x_0) = y_0, y''(x_0) = y_2, x \in [x_0, b] \tag{1}$$

In the past, equation (1) is solved by method of reducing to its equivalent system of first order ordinary differential equations and thereafter appropriate numerical method for first order would be applied to solve the systems. However, it is shown in Awoyemi (1999, 2001) that reduction of higher order ordinary differential equations to a system of first order has serious problems which include consumption of human effort, computational burden and non-economization of computer time. This is also discussed by Fatunla (1988), Lambert (1973), Goult et al (1973) and Brugnano & Trigiante (1998).

In order to cater for the setbacks encountered in reduction method and also bring improvement on numerical method, scholars like Omar & Suleiman (1999, 2003, 2005), Badmus & Yahaya (2009) and Adesanya, et al (2012) developed block methods for solving higher order ordinary differential equations directly in which the accuracy is better than when it is reduced to system of first order ordinary differential equations.

Linear multistep methods for solving equation (1) directly have been proposed by some researchers such as Olabode (2009a) developed a block method for the solution of third order ordinary differential equation whereby the accuracy of the method is not efficient enough in terms of error. A P-stable linear multistep for direct solution of (1) was developed by Awoyemi (2003) which was implemented in predictor-corrector mode. Adesanya (2012) also proposed a new block predictor-corrector algorithm with step- length four for solving (1) directly.

This paper proposes a five-step block method for solving equation (1) without predictor. It aims to improve the accuracy of the existing methods mentioned above.

## 2. Methodology

We consider power series of the form

$$y(x) = \sum_{j=0}^{s+r-1} a_j x^j \tag{2}$$

as an approximate solution to equation (1), where $s$ is the number of collocation points for $0 \le s \le 5$ , $r$ is the number of interpolation points for $1 \le r \le 3$, and $a_j$ are the unknown parameters to be determined. The first, second and third derivatives of (2) give

$$y'(x) = \sum_{j=1}^{s+r-1} j a_j x^{j-1} \tag{3}$$

$$y''(x) = \sum_{j=2}^{s+r-1} j(j-1) a_j x^{j-2} \tag{4}$$

$$y'''(x) = \sum_{j=3}^{s+r-1} j(j-1)(j-2) a_j x^{j-3} = f(x, y, y', y'') \tag{5}$$

Interpolating equation (1) at the grid points $x = x_{n+i}, i = 1(1)3$ and collocating equation (5) at the points $x = x_{n+i}, i = 0(1)5$ produces the following equations

$$\sum_{j=0}^{s+r-1} a_j x^j = y_{n+i} \tag{6}$$

$$\sum_{j=3}^{s+r-1} j(j-1)(j-2) a_j x^{j-3} = f_{n+i} \tag{7}$$

Solving equations (6) and (7) by using Gaussian elimination method to determine the values of the unknown parameters $a_j$ which are then substituted into equation (2). This gives a continuous implicit scheme of the form

$$y(z)=\sum_{j=1}^{k=2}\alpha_j(z)y_{n+j}+h^3\sum_{j=0}^{k}\beta_j(z)f_{n+j} \tag{8}$$

where $k=5$, $\alpha_j(z)$ *and* $\beta_j(z)$ are given as

$$\begin{pmatrix}\alpha_1(z)\\ \alpha_2(z)\\ \alpha_3(z)\end{pmatrix}=\begin{pmatrix}1 & \frac{1}{2} & \frac{3}{2}\\ -3 & -4 & -1\\ 3 & \frac{5}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}z^0\\ z^1\\ z^2\end{pmatrix}$$

$$\begin{pmatrix}\beta_0(z)\\ \beta_1(z)\\ \beta_2(z)\\ \beta_3(z)\\ \beta_4(z)\\ \beta_5(z)\end{pmatrix}=\begin{pmatrix}
\frac{84}{40320} & \frac{64}{40320} & \frac{-69}{40320} & 0 & \frac{84}{40320} & \frac{28}{40320} & \frac{-14}{40320} & \frac{-8}{40320} & \frac{-1}{40320}\\
\frac{-252}{40320} & \frac{36}{40320} & \frac{625}{40320} & 0 & \frac{-560}{40320} & \frac{-168}{40320} & \frac{98}{40320} & \frac{48}{40320} & \frac{5}{40320}\\
\frac{10164}{20160} & \frac{14728}{20160} & \frac{4023}{20160} & 0 & \frac{840}{20160} & \frac{196}{20160} & \frac{-154}{20160} & \frac{-56}{20160} & \frac{-5}{20160}\\
\frac{10164}{20160} & \frac{20740}{20160} & \frac{12021}{20160} & 0 & \frac{-1680}{20160} & \frac{-56}{20160} & \frac{238}{20160} & \frac{64}{20160} & \frac{5}{20160}\\
\frac{-252}{40320} & \frac{2928}{40320} & \frac{7943}{40320} & \frac{6720}{40320} & \frac{1820}{40320} & \frac{-420}{40320} & \frac{-350}{40320} & \frac{-72}{40320} & \frac{-5}{40320}\\
\frac{84}{40320} & \frac{-44}{40320} & \frac{-267}{40320} & 0 & \frac{336}{40320} & \frac{280}{40320} & \frac{98}{40320} & \frac{16}{40320} & \frac{1}{40320}
\end{pmatrix}\begin{pmatrix}z^0\\ z^1\\ z^2\\ z^3\\ z^4\\ z^5\\ z^6\\ z^7\\ z^8\end{pmatrix} \tag{9}$$

where $z=\dfrac{x-x_{n+k-1}}{h}$.

The first and second derivatives of (9) are given below

$$\begin{pmatrix}\alpha_1'(z)\\ \alpha_2'(z)\\ \alpha_3'(z)\end{pmatrix}=\begin{pmatrix}\frac{3}{2} & 1\\ -4 & -2\\ \frac{5}{2} & 1\end{pmatrix}\begin{pmatrix}z^0\\ z^1\end{pmatrix}$$

$$\begin{pmatrix}\beta_0'(z)\\ \beta_1'(z)\\ \beta_2'(z)\\ \beta_3'(z)\\ \beta_4'(z)\\ \beta_5'(z)\end{pmatrix}=\begin{pmatrix}
\frac{64}{40320} & \frac{-138}{40320} & 0 & \frac{336}{40320} & \frac{140}{40320} & \frac{-84}{40320} & \frac{-56}{40320} & \frac{-8}{40320}\\
\frac{36}{40320} & \frac{1250}{40320} & 0 & \frac{-2240}{40320} & \frac{-840}{40320} & \frac{588}{40320} & \frac{336}{40320} & \frac{40}{40320}\\
\frac{14728}{20160} & \frac{8046}{20160} & 0 & \frac{3360}{20160} & \frac{980}{20160} & \frac{924}{20160} & \frac{-392}{20160} & \frac{-40}{20160}\\
\frac{29740}{20160} & \frac{24042}{20160} & 0 & \frac{-6720}{20160} & \frac{-280}{20160} & \frac{1428}{20160} & \frac{448}{20160} & \frac{40}{20160}\\
\frac{2928}{40320} & \frac{15886}{40320} & \frac{20160}{40320} & \frac{7280}{40320} & \frac{-2100}{40320} & \frac{-2100}{40320} & \frac{-504}{40320} & \frac{-40}{40320}\\
\frac{-44}{40320} & \frac{-534}{40320} & 0 & \frac{1344}{40320} & \frac{1400}{40320} & \frac{588}{40320} & \frac{112}{40320} & \frac{8}{40320}
\end{pmatrix}\begin{pmatrix}z^0\\ z^1\\ z^2\\ z^3\\ z^4\\ z^5\\ z^6\\ z^7\end{pmatrix} \tag{10}$$

$$\begin{pmatrix} \alpha''(z)_1 \\ \alpha_2''(z) \\ \alpha_3''(z) \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \beta_0''(z) \\ \beta_1''(z) \\ \beta_2''(z) \\ \beta_3''(z) \\ \beta_4''(z) \\ \beta_5''(z) \end{pmatrix} = \begin{pmatrix} \frac{-138}{40320} & 0 & \frac{10080}{40320} & \frac{560}{40320} & \frac{-420}{40320} & \frac{-336}{40320} & \frac{-56}{40320} \\ \frac{1250}{40320} & 0 & \frac{-6720}{40320} & \frac{-3360}{40320} & \frac{2940}{40320} & \frac{2016}{40320} & \frac{280}{40320} \\ \frac{8046}{20160} & 0 & \frac{10080}{20160} & \frac{3920}{20160} & \frac{-4620}{20160} & \frac{-2352}{20160} & \frac{-280}{20160} \\ \frac{24042}{20160} & 0 & \frac{-20160}{20160} & \frac{-1120}{20160} & \frac{7140}{20160} & \frac{2688}{20160} & \frac{280}{20160} \\ \frac{15886}{40320} & 1 & \frac{21840}{40320} & \frac{-8400}{40320} & \frac{-10500}{40320} & \frac{-3024}{40320} & \frac{-280}{40320} \\ \frac{-534}{40320} & 0 & \frac{4032}{40320} & \frac{5600}{40320} & \frac{2940}{40320} & \frac{672}{40320} & \frac{56}{40320} \end{pmatrix} \begin{pmatrix} z^0 \\ z^1 \\ z^2 \\ z^3 \\ z^4 \\ z^5 \\ z^6 \end{pmatrix} \quad (11)$$

Equation (9) is evaluated at $x = x_{n+i}, i = 0,1,3$ while equations (10) - (11) are evaluated at $x = x_{n+i}, i = 0(1)5$ to give the discrete methods and its derivatives. These schemes are combined together in matrix form and by using the matrix inversion, a block method of the following form is produced:

$$A^{(0)}Y_{N+1} = A^{(1)}Y_N + hB^{(1)}Y_N' + h^2C^{(1)}Y_N'' + h^3(D^{(0)}F_{N+1} + D^{(1)}F_N) \quad (12)$$

where

$$Y_{N+1} = [y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4}, y_{n+5}]^T, Y_N = [y_{n-4}, y_{n-3}, y_{n-2}, y_{n-1}, y_n]^T$$

$$Y_N' = [y_{n-4}', y_{n-3}', y_{n-2}', y_{n-1}', y_n']^T, Y_N'' = [y_{n-4}'', y_{n-3}'', y_{n-2}'', y_{n-1}'', y_n'']^T$$

$$F_{N+1} = [f_{n+1}, f_{n+2}, f_{n+3}, f_{n+4}, f_{n+5}]^T, F_N = [f_{n-4}, f_{n-3}, f_{n-2}, f_{n-1}, f_n]^T$$

$$A^{(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, A^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, B^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

$$C^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & \frac{9}{2} \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & \frac{25}{2} \end{pmatrix}, D^{(0)} = \begin{pmatrix} \frac{9950}{80640} & \frac{-7724}{80640} & \frac{4844}{80640} & \frac{-1766}{80640} & \frac{279}{80640} \\ \frac{1468}{1260} & \frac{-760}{1260} & \frac{488}{1260} & \frac{178}{1260} & \frac{28}{1260} \\ \frac{16119}{4480} & \frac{-4374}{4480} & \frac{4230}{4480} & \frac{-1539}{4480} & \frac{243}{4480} \\ \frac{4672}{630} & \frac{-448}{630} & \frac{1408}{630} & \frac{-400}{630} & \frac{64}{630} \\ \frac{305625}{24192} & \frac{3750}{24192} & \frac{116250}{24192} & \frac{-13125}{24192} & \frac{4125}{24192} \end{pmatrix}$$

$$D^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{7858}{80640} \\ 0 & 0 & 0 & 0 & \frac{634}{1260} \\ 0 & 0 & 0 & 0 & \frac{5481}{4480} \\ 0 & 0 & 0 & 0 & \frac{1424}{630} \\ 0 & 0 & 0 & 0 & \frac{87375}{24192} \end{pmatrix}$$

The first and second derivatives of (12) are given by

$$\begin{pmatrix} y'_{n+1} \\ y'_{n+2} \\ y'_{n+3} \\ y'_{n+4} \\ y'_{n+5} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} y'_n \\ hy''_n \end{pmatrix} + h^2 \begin{pmatrix} \frac{214}{20160} & \frac{-1364}{20160} & \frac{3764}{20160} & \frac{-6088}{20160} & \frac{8630}{20160} & \frac{4924}{20160} \\ \frac{16}{630} & \frac{-101}{630} & \frac{272}{630} & \frac{-370}{630} & \frac{1088}{630} & \frac{355}{630} \\ \frac{405}{10080} & \frac{-2592}{10080} & \frac{7830}{10080} & \frac{-648}{10080} & \frac{31509}{10080} & \frac{8856}{10080} \\ \frac{32}{630} & \frac{-160}{630} & \frac{1216}{630} & \frac{352}{630} & \frac{2848}{630} & \frac{752}{630} \\ \frac{1375}{10080} & \frac{6250}{10080} & \frac{31250}{10080} & \frac{12500}{10080} & \frac{59375}{10080} & \frac{1525}{10080} \end{pmatrix} \begin{pmatrix} f_{n+5} \\ f_{n+4} \\ f_{n+3} \\ f_{n+2} \\ f_{n+1} \\ f_n \end{pmatrix}$$

$$\begin{pmatrix} y''_{n+1} \\ y''_{n+2} \\ y''_{n+3} \\ y''_{n+4} \\ y''_{n+5} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} y''_n + h^2 \begin{pmatrix} \frac{27}{1440} & \frac{-173}{1440} & \frac{482}{1440} & \frac{-798}{1440} & \frac{1427}{1440} & \frac{475}{1440} \\ \frac{1}{90} & \frac{-6}{90} & \frac{14}{90} & \frac{14}{90} & \frac{129}{90} & \frac{28}{90} \\ \frac{3}{160} & \frac{-21}{160} & \frac{114}{160} & \frac{114}{160} & \frac{219}{160} & \frac{51}{160} \\ 0 & \frac{14}{45} & \frac{64}{45} & \frac{24}{45} & \frac{64}{45} & \frac{14}{45} \\ \frac{190}{576} & \frac{750}{576} & \frac{500}{576} & \frac{500}{576} & \frac{750}{576} & \frac{190}{576} \end{pmatrix} \begin{pmatrix} f_{n+5} \\ f_{n+4} \\ f_{n+3} \\ f_{n+2} \\ f_{n+1} \\ f_n \end{pmatrix}$$

# 3. Analysis of the Properties of the Block

## 3.1Order of the Method

Taking the Taylor series (12) at point $x$ gives

$$\begin{pmatrix} \sum_{i=0}^{\infty}\frac{(h)^i}{i!}y_n^i - y_n - hy'_n + \frac{1}{2}h^2y''_n - \frac{7858}{80640}h^3y_n^{(3)} - \sum_{i=0}^{\infty}\frac{h^{(i+3)}}{80640}y_n^{(i+3)}[278(5) - 1766(4)^i +^i 4844(3)^i \\ -7724(2)^i + 9950(1)^i] \\ \sum_{i=0}^{\infty}\frac{(2h)^i}{i!}y_n^i - y_n - 2hy'_n + 2h^2y''_n - \frac{634}{1260}h^3y_n^{(3)} - \sum_{i=0}^{\infty}\frac{h^{(i+3)}}{1260}y_n^{(i+3)}[28(5)^i - 178(4) + 488(3)^i \\ -760(2)^i + 1468(1)^i] \\ \sum_{i=0}^{\infty}\frac{(3h)^i}{i!}y_n^i - y_n - 3hy'_n + \frac{9}{2}h^2y''_n - \frac{5481}{44800}h^3y_n^{(3)} - \sum_{i=0}^{\infty}\frac{h^{(i+3)}}{44800}y_n^{(i+3)}[243(5)^i - 1539(4) + 4230)^i \\ -4374(2)^i + 16119(1)^i] \\ \sum_{i=0}^{\infty}\frac{(4h)^i}{i!}y_n^i - y_n - 4hy'_n + 8h^2y''_n - \frac{1424}{630}h^3y_n^{(3)} - \sum_{i=0}^{\infty}\frac{h^{(i+3)}}{630}y_n^{(i+3)}[64(5)^i - 400(4) + 1408(3)^i \\ -448(2)^i + 4672(1)^i] \\ \sum_{i=0}^{\infty}\frac{(5h)^i}{i!}y_n^i - y_n - 5hy'_n + \frac{25}{2}h^2y''_n - \frac{87375}{24192}h^3y_n^{(3)} - \sum_{i=0}^{\infty}\frac{h^{(i+3)}}{24192}y_n^{(i+3)}[4125(5)^i - 13125(4) + 116250(3)^i \\ +3750(2)^i + 305625(1)^i] \end{pmatrix} = 0$$

Collecting coefficients in powers of $h$ yields the order of the method to be six with the error constant $[\frac{-37}{13688}, \frac{-491}{28350}, \frac{-1917}{44800}, \frac{-1136}{14175}, \frac{-121}{943}]^T$

### 3.2. Zero Stability

**Definition 1**: The block (12) is said to be *zero-stable* if no root of the first characteristic polynomial $\rho(r)$ is having a modulus greater than one and every root of modulus one is simple.

In our method, $\rho(r) = \det[rA^{(0)} - A^{(1)}] = 0,$ where $A^{(0)}$ and $A^{(1)}$ are the coefficients of $y_{n+i}, i = 0(1)5$ (12) which is illustrated below

$$\det[rA^{(0)} - A^{(1)}] = \left| r\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right| = 0,$$

whose solution is $r = 0,0,0,01$. This confirms the zero-stability of our method. The method is consistent because the order of the method is greater than one. Since the method is zero-stable and consistent, it implies the method is convergent (Henrici, 1962).

## 4. Numerical Problems

We consider the following differential problems in order to test the accuracy of our method with the existing methods.

Problem 1: $y''' + 4y' = x, \qquad y(0) = 0, y'(0) = 0, y''(0) = 1$

Exact solution: $y(x) = \frac{3}{16}(1 - \cos 2x)$

Adesanya et. al (2012) and Awoyemi et. al. (2014) applied their methods to solve the differential equation displayed above. The same problem is solved by our method to generate numerical results and comparison is made with their result. This is shown in Table 1 below:

**Table I**: Comparison of the new method with Adesanya (2011) and Awoyemi (2014) for solving problem 1

| X | Theoretical Solution | Numerical Solution | Error in our new method K=5 | Error in Adesanya et. al (2011), K=4 | Awoyemi et.al (2014) K=4 |
|---|---|---|---|---|---|
| 0.1 | 0.004987516654767195 | 0.004987516654761906 | 5.289172E15 | 1.189947E-11 | 1.1899E-11 |
| 0.2 | 0.019801063624459044 | 0.019801063619047623 | 5.411421E-12 | 3.042207E-09 | 3.0422E-09 |
| 0.3 | 0.043999572204435337 | 0.043999571892857157 | 3.115782E-10 | 7.779556E-08 | 7.7796E-08 |
| 0.4 | 0.076867491997406501 | 0.076867486476190489 | 5.521216E-09 | 7.746692E-07 | 1.5559E-07 |
| 0.5 | 0.117443317649723790 | 0.117443303933309230 | 1.371641E-08 | 4.59901E-06 | 3.0541E-07 |
| 0.6 | 0.164557921035623750 | 0.164557934914669020 | 1.387905E-08 | 6.478349E-06 | 4.6102E-07 |
| 0.7 | 0.216881160706204830 | 0.216881253199369660 | 9.249316E-08 | 5.783963E-06 | 3.138E-07 |
| 0.8 | 0.272974910431491690 | 0.272974772791875490 | 1.376396E-07 | 2.354715E-06 | 7.0374E-07 |
| 0.9 | 0.331350392754953820 | 0.331350854513212890 | 4.617583E-07 | 3.766592E-06 | 1.0177E-06 |
| 1.0 | 0.390527531852589200 | 0.390527776051773630 | 2.441992E-07 | 1.23312E-05 | 1.6528E-06 |

Problem 2: $y''' = 3\sin x, \qquad y(0) = 1,\ y'(0) = 0, \qquad y''(0) = -2$

Exact solution: $y(x) = 3\cos x + \left(\frac{x^2}{2}\right) - 2$

The above differential problem was solved by Adesanya et. al. (2012) and Olabode & Yusuf (2009) using their developed method. The same problem is considered by our method and we compare the result generated with their result. This is demonstrated in Table 2

**Table 2**: Comparison of the new method with Adesanya (2012) and Awoyemi (2009) for solving problem 2

| X | Theoretical Solution | Numerical Solution | Error in our new method K=5 | Error in Adesanya et. al (2012), K=4 | Olabode & Yusuf (2009) K=3 |
|---|---|---|---|---|---|
| 0.01 | 0.999900001249995900 | 0.999900001249995900 | 0.000000E+00 | 0.0000E+00 | 9.992007E-16 |
| 0.02 | 0.999600019999733020 | 0.999600019999733360 | 3.330669E-16 | 9.99200E-16 | 7.660538E-15 |
| 0.03 | 0.999100101246962250 | 0.999100101246962580 | 3.330669E-16 | 1.55431E-15 | 2.287059E-14 |
| 0.04 | 0.998400319982933660 | 0.998400319982933770 | 1.110223E-16 | 3.10862E-15 | 5.906386E-14 |
| 0.05 | 0.997500781184899040 | 0.997500781184898820 | 2.220446E-16 | 4.66293E-15 | 1.153521E-13 |
| 0.06 | 0.996401619805612260 | 0.996401619805612590 | 3.330669E-16 | 6.88338E-15 | 1.982858E-13 |
| 0.07 | 0.995103000759838710 | 0.995103000759838820 | 1.110223E-16 | 9.10382E-15 | 3.127498E-13 |
| 0.08 | 0.993605118907858300 | 0.993605118907858300 | 0.000000E+00 | 1.14908E-14 | 4.635736E-13 |
| 0.09 | 0.991908199035982820 | 0.991908199035982820 | 0.000000E+00 | 1.42108E-14 | 6.542544E-13 |
| 0.1 | 0.990012495834077020 | 0.990012495834077360 | 3.330669E-16 | 1.74582E-14 | 8.885253E-13 |

Problem 3: $y''' = -y \qquad y(0) = 1,\ y'(0) = -1, \quad y''(0) = 1$

Exact solution: $y(x) = e^{-x}$

Adesanya et. al (2012) and Olabode (2007) solved the above differential problem with their method, the same problem is also solved by our method and the result generated is compared with their method. This is shown in Table 3 below.

**Table 3**: Comparison of the new method with Adesanya (2012) and Awoyemi (2007) for solving problem 3

| X | Theoretical Solution | Numerical Solution | Error in our new method K=5 | Error in Adesanya et. al (2012), K=4 | Olabode (2007) K=4 |
|---|---|---|---|---|---|
| 0.1 | 0.904837418035959520 | 0.904837418033777150 | 2.182365E-12 | 2.4525E-13 | 1.36929E-09 |
| 0.2 | 0.818730753077981820 | 0.818730753073947600 | 4.034217E-12 | 6.2109E-11 | 3.12272E-08 |
| 0.3 | 0.740818220681717770 | 0.740818220676328960 | 5.388801E-12 | 1.5746E-10 | 1.27694E-07 |
| 0.4 | 0.670320046035639330 | 0.670320045970895120 | 6.474421E-11 | 3.1477E-09 | 3.25196E-07 |
| 0.5 | 0.606530659712633420 | 0.606530659653555240 | 5.907819E-11 | 6.1617E-09 | 6.54297E-07 |
| 0.6 | 0.548811636094026390 | 0.548811636082354610 | 1.167177E-11 | 9.1732E-09 | 1.14406E-06 |
| 0.7 | 0.496585303791409470 | 0.496585303768886100 | 2.252337E-11 | 1.3329E-08 | 1.81784E-06 |
| 0.8 | 0.449328964117221560 | 0.449328964078100250 | 3.912132E-11 | 1.6378E-08 | 2.69774E-06 |
| 0.9 | 0.406569659740599110 | 0.406569659679422270 | 6.117684E-11 | 1.7134E-08 | 3.80241E-06 |
| 1.0 | 0.367879441171442330 | 0.367879441122222760 | 4.921957E-11 | 7.4405E-09 | 5.14755E-06 |

## 5. Conclusion

The new block method for solving third order ordinary differential equations is developed in this paper. The method is of step-length five and the result generated is compared with the existing methods having a step-length four and three. The numerical results obtained by using the new method are found better when comparison is made with the existing methods. Furthermore, the accuracy of the method can be improved if the step-length *k* increases.

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