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Vague Γ-Semirings

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Abstract

In this paper, the notion of vague Γ-semiring of a Γ-semiring is introduced and studied various properties. The operations and relations on vague Γ-semirings are defined and established various relations on them. Further it is shown that the class of all vague Γ-semirings is a De-Morgan algebra.

Key Words: Vague set, Vague cut, Vague characteristic set, Vague Γ-Semiring.

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1. Introduction

W.L.Gau and D.J.Buehrer[12] introduced the theory of vague sets as an improvement of the theory of fuzzy sets in approximating the real life situation. Vague sets are higher order fuzzy sets. According to them a vague set A in the universe of discourse U is a pair (t_A, f_A), where t_A and f_A are fuzzy subsets of U satisfying $t_A(u) + f_A(u) \leq 1, \forall u \in U$. Ranjit Biswas[10] initiated the study of vague algebra by introducing the concepts of vague groups and vague normal groups. H. Khan, M. Ahmad and Ranjit Biswas[1] introduced the notion of vague relations and studied some properties of them. N. Ramakrishna[7,8] continued this study by studying vague cosets, vague products and several properties related to them. Y.B.Jun and C.H.Park[13] introduced the concept of vague ideals in subtraction algebra. T.Eswarlal[11] introduced the concepts of vague ideals and normal vague ideals in semirings.

In the theory of vague sets studied so far the membership function and non-membership function assume the values in [0, 1] of real numbers. Though K.T.Attanassov indicated the development of theory of intuitionistic fuzzy sets with the membership and non-membership function taking values in [0, 1] of real numbers and an arbitrary lattice L, so far, it seems no progress is made in that direction. However the theory of fuzzy Γ-semirings has been developed extensively by many authors like Jayanta Ghosh, T.K.Samanta, T.K.Dutta, Sujit Kumar Sardar etc. The vague sets of W.L.Gau and D.J.Buehrer[12] and K.T.Attanasov[3]'s intuitionistic fuzzy sets are mathematically equivalent objects. There is a controversy about the name intuitionistic fuzzy sets. Without entering into that controversy we prefer the terminology of vague sets.

The concept of gamma in algebra was introduced and studied first by N.Nobusawa[6] in 1964 and further established Γ-ring. In fact, there have been a few slightly different definition on a Γ-ring. In 1995, M.K.Rao[5] introduced the notion of Γ-semiring as a generalization of Γ-ring as well as semiring and studied the concepts of Γ-semirings and its sub Γ-semirings with a left(right) unity. After that Γ-semirings have been analyzed by lot of mathematicians. Further on Γ-semirings, the study properties of fuzzy ideals, fuzzy prime ideals, fuzzy semiprime ideals and their generalizations play an important role in their structure theory. In this paper, the authors proposed to study vague Γ-semiring of a Γ-semiring R, as a pair of mappings (t_A, f_A), where t_A and f_A are mappings on [0, 1] satisfying the condition $t_A(x) + f_A(x) \leq 1, \forall x \in R$. we established a one-one correspondence between vague Γ-semiring A and its vague cut, $A_{(\alpha,\beta)}$, where $\alpha, \beta \in [0, 1]$ with $\alpha \leq \beta$. Also we observed that the characteristic function of a non-empty subset of a Γ-semiring R is a vague Γ-semiring of R. Further we investigated that the collection of vague Γ-semirings of a Γ-semiring R forms a De-Morgan algebra.

2.Preliminaries:

Definition 2.1: Let R and Γ be two additive commutative semigroups. Then R is called Γ-semiring if there exists a mapping $R \times \Gamma \times R \rightarrow R$ image to be denoted by $a\alpha b$ for $a, b \in R$ and $\alpha \in \Gamma$ satisfying the following conditions.

1. $a\alpha(b + c) = a\alpha b + a\alpha c$
2. $(a + b)\alpha c = a\alpha c + b\alpha c$
3. $a(\alpha + \beta)c = a\alpha c + a\beta c$
4. $a\alpha (b\beta c) = (a\alpha b)\beta c, \forall a, b, c \in R; \alpha, \beta \in \Gamma$.

Definition 2.2: A nonempty subset S of a Γ-semiring R is said to be a sub Γ-semiring of R if (S, +) is a sub semigroup of (R, +) and $a\alpha b \in S, \forall a, b \in S$ and $\alpha \in \Gamma$.

Definition 2.3: Let X be any non-empty set. A mapping $\mu : X \rightarrow [0,1]$ is called a fuzzy subset of R.

Definition 2.4: A vague set A in the universe of discourse U is a pair (t_A, f_A), where $t_A : U \rightarrow [0, 1]$, $f_A : U \rightarrow [0, 1]$ are mappings such that $t_A(u) + f_A(u) \leq 1, \forall u \in U$. The functions t_A and f_A are called true membership function and false membership function respectively.

Definition 2.5: The interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A and it is denoted by $V_A(u)$ i.e., $V_A(u) = [t_A(u), 1 - f_A(u)]$.

Definition 2.6: A vague set A is contained in the other vague set B, $A \subseteq B$ if and only if $V_A(u) \leq V_B(u)$ i.e., $t_A(u) \leq t_B(u)$ and $1 - f_A(u) \leq 1 - f_B(u), \forall u \in U$.

Definition 2.7: Two vague sets A and B are equal written as A = B, if and only if $A \subseteq B$ and $B \subseteq A$ i.e., $V_A(u) \leq V_B(u)$ and $V_B(u) \leq V_A(u), \forall u \in U$.

Definition 2.8: The union of two vague sets A and B with respective truth membership and membership functions t_A, f_A : t_B, f_B is a vague set C, written as $C = A \cup B$, whose truth membership and false membership functions are related to those of A and B by $t_C = \max\{t_A, t_B\}$ and $1 - f_C = \max\{1 - f_A, 1 - f_B\} = 1 - \min\{f_A, f_B\}$.

Definition 2.9: The intersection of two vague sets A and B with respective truth membership and membership functions t_A, f_A : t_B, f_B is a vague set C, written as $C = A \cap B$, whose truth membership and false membership functions are related to those of A and B by $t_C = \min\{t_A, t_B\}$ and $1 - f_C = \min\{1 - f_A, 1 - f_B\} = 1 - \max\{f_A, f_B\}$.

Definition 2.10: A vague set A of a set U with $t_A(u) = 0$ and $f_A(u) = 1, \forall u \in U$ is called zero vague set of U.

Definition 2.11: A vague set A of a set U with $t_A(u) = 1$ and $f_A(u) = 0, \forall u \in U$ is called unit vague set of U.

Definition 2.12: Let A be a vague set of a universe U with true membership function t_A and false membership function f_A. For $\alpha, \beta \in [0,1]$ with $\alpha \leq \beta$, the (α, β)- cut or vague cut of a vague set A is the crisp subset of U is given by
$A_{(\alpha,\beta)} = \{x \in U / V_A(x) \geq [\alpha, \beta]\}$ i.e., $A_{(\alpha,\beta)} = \{x \in U / t_A(x) \geq \alpha \text{ and } 1 - f_A(x) \geq \beta\}$.

Definition 2.13: The α-cut, A_α of the vague set A is the (α, α)-cut of A and hence given by $A_\alpha = \{x \in U / t_A(x) \geq \alpha\}$.

3. Vague Γ-semirings:

We begin with the following.

Definition 3.1: Let R be a Γ-semiring. A vague set $A = (t_A, f_A)$ on R is said to be vague Γ-semiring if the following conditions are true:
For all $x, y \in R$; $\gamma \in \Gamma$, $V_A(x + y) \geq \min\{V_A(x), V_A(y)\}$ and
$V_A(x\gamma y) \geq \min\{V_A(x), V_A(y)\}$

i.e., 1. $t_A(x + y) \geq \min\{t_A(x), t_A(y)\}$,
$1 - f_A(x + y) \geq \min\{1 - f_A(x), 1 - f_A(y)\}$ and

2. $t_A(x\gamma y) \geq \min\{t_A(x), t_A(y)\}$,
   $1 - f_A(x\gamma y) \geq \min\{1 - f_A(x), 1 - f_A(y)\}$.

Example 3.2: Let R be the set of negative integers and Γ be the set of negative even integers. Then R, Γ are additive commutative semigroups.
Define the mapping $R \times \Gamma \times R \rightarrow R$ by $a\alpha b$ usual product of a, α, b, $\forall$ a, b $\in$ R; $\alpha \in \Gamma$. Then R is a Γ-semiring.
Let $A = (t_A, f_A)$, where $t_A : R \rightarrow [0, 1]$ and $f_A : R \rightarrow [0, 1]$ defined by

$$t_A(x) = \begin{cases} 0.5 \text{ if } x = -1 \\ 0.7 \text{ if } x = -2 \\ 0.9 \text{ if } x < -2 \end{cases} \text{ and } f_A(x) = \begin{cases} 0.5 \text{ if } x = -1 \\ 0.2 \text{ if } x = -2 \\ 0.1 \text{ if } x < -2 \end{cases}$$

Then A is a vague Γ-semiring of R.

Example 3.3: Let R be the set of real numbers and Γ be the set of positive numbers. Then R, Γ are additive commutative semigroups.
Define the mapping $R \times \Gamma \times R \rightarrow R$ by $a\alpha b$ usual product of a, α, b, $\forall$ a, b $\in$ R; $\alpha \in \Gamma$. Then R is a Γ-semiring.
Let $A = (t_A, f_A)$, where $t_A : R \rightarrow [0, 1]$ and $f_A : R \rightarrow [0, 1]$ defined by

$$t_A(x) = \begin{cases} 0.8 \text{ if } x = 0 \\ 0.6 \text{ if } x \text{ is positive} \\ 0.5 \text{ if } x \text{ is negative} \end{cases} \text{ and } f_A(x) = \begin{cases} 0.2 \text{ if } x = 0 \\ 0.3 \text{ if } x \text{ is positive} \\ 0.5 \text{ if } x \text{ is negative} \end{cases}$$

Then A is a vague Γ-semiring of R.

Example 3.4: Let R be the set of all positive integers of the form 4n and Γ be the set of all positive integers of the form 3n, where n is a positive integer. Then R, Γ are additive commutative semigroups.
Define the mapping $R \times \Gamma \times R \rightarrow R$ by $a\alpha b$ usual product of a, α, b, $\forall$ a, b $\in$ R; $\alpha \in \Gamma$. Then R is a Γ-semiring.
Let $A = (t_A, f_A)$, where $t_A : R \rightarrow [0, 1]$ and $f_A : R \rightarrow [0, 1]$ defined by

$$t_A(x) = \begin{cases} 0.8 \text{ if } x > 20 \\ 0.6 \text{ if } x \leq 20 \end{cases} \text{ and } f_A(x) = \begin{cases} 0.2 \text{ if } x > 20 \\ 0.3 \text{ if } x \leq 20 \end{cases}$$

Then A is a vague Γ-semiring of R.

Theorem 3.5: A necessary and sufficient condition for a vague set $A = (t_A, f_A)$ of a Γ-semiring R to be a vague Γ-semiring of R is that t_A and $1 - f_A$ are fuzzy Γ-semirings of R.

Proof: Suppose $A = (t_A, f_A)$ is a vague Γ-semiring of R.
Let x, y $\in$ R; $\gamma \in \Gamma$.
We have $V_A(x + y) \geq \min\{V_A(x), V_A(y)\}$ and
$V_A(x\gamma y) \geq \min\{V_A(x), V_A(y)\}$
i.e., 1. $t_A(x + y) \geq \min\{t_A(x), t_A(y)\}$

$1 - f_A(x + y) \geq \min\{1 - f_A(x), 1 - f_A(y)\}$ and

2. $t_A(x\gamma y) \geq \min\{t_A(x), t_A(y)\}$

$1 - f_A(x\gamma y)$ _ $\min\{1 - f_A(x), 1 - f_A(y)\}$

Hence t_A and $1 - f_A$ are fuzzy Γ-semirings of R.
The converse part is obvious from the definition.

Theorem 3.6: Let R be a Γ-semiring. A vague set A of R is a vague Γ-semiring of R if and only if for all $\alpha, \beta \in [0, 1]$, the (α, β) - cut, $A_{(\alpha, \beta)}$ is a sub Γ-semiring of R.

Proof: Suppose A is a vague Γ-semiring of R.
Let $x, y \in A_{(\alpha, \beta)}$ and $\gamma \in \Gamma$.
$\Rightarrow V_A(x) \geq [\alpha, \beta]$ and $V_A(y) \geq [\alpha, \beta]$
We have 1.$V_A(x + y) \geq \min\{V_A(x), V_A(y)\} \geq [\alpha, \beta]$
$\Rightarrow x + y \in A_{(\alpha, \beta)}$.
2.$V_A(x\gamma y) \geq \min\{V_A(x), V_A(y)\} \geq [\alpha, \beta]$
$\Rightarrow x\gamma y \in A_{(\alpha, \beta)}$.
Hence $A_{(\alpha, \beta)}$ is a sub Γ-semiring of R.
Conversely suppose that $A_{(\alpha, \beta)}$ is a sub Γ-semiring of R.
Let $x, y \in R$ and $\gamma \in \Gamma$.
Let $V_A(x) = [\alpha_1, \beta_1]$ and $V_A(y) = [\alpha_2, \beta_2]$.
put $[\alpha, \beta] = \min\{[\alpha_1, \beta_1], [\alpha_2, \beta_2]\}$.
Then $x, y \in A_{(\alpha, \beta)}$
$\Rightarrow x + y \in A_{(\alpha, \beta)}$ and $x\gamma y \in A_{(\alpha, \beta)}$
$\Rightarrow V_A(x + y) \geq [\alpha, \beta] = \min\{V_A(x), V_A(y)\}$ and
$V_A(x\gamma y) \geq [\alpha, \beta] = \min\{V_A(x), V_A(y)\}$
Hence A is a vague Γ-semiring of R.

Corollary 3.7: Let A be a vague Γ-semiring of R. Then for $\alpha \in [0, 1]$, the α-cut A_α is a sub Γ-semiring of R.

Definition 3.8: Let $\delta = (t_\delta, f_\delta)$ be a vague set of a Γ-semiring R. For any subset S of R, the characteristic function of S taking values in [0, 1] of a vague set $\delta_S = (t_{\delta_S}, f_{\delta_S})$ by

$$V_{\delta_S}(x) = \begin{cases} [1,1] & if\ x \in S \\ [0,0] & if\ x \notin S \end{cases}$$

i.e., $t_{\delta_S}(x) = \begin{cases} 1 & if\ x \in S \\ 0 & if\ x \notin S \end{cases}$ and $f_{\delta_S}(x) = \begin{cases} 0 & if\ x \in S \\ 1 & if\ x \notin S \end{cases}$

Then δ_S is called the vague characteristic set of S in [0, 1].

Theorem 3.9: Let S be a non-empty subset of a Γ-semiring R. Then δ_S is a vague Γ-semiring of R if and only if S is a sub Γ-semiring of R.

Proof: Suppose δ_S is a vague Γ-semiring of R.
Let x, y ∈ S and γ ∈ Γ.
We have 1. $V_{\delta_S}(x + y) \geq \min\{V_{\delta_S}(x), V_{\delta_S}(y)\} = [1, 1]$ and

2. $V_{\delta_S}(x\gamma y) \geq \min\{V_{\delta_S}(x), V_{\delta_S}(y)\} = [1, 1]$.

which implies that x + y ∈ S and xγy ∈ S.
Hence S is a sub Γ-semiring of R.

Conversely assume that S is a sub Γ-semiring of R.
Let x, y ∈ R and γ ∈ Γ.
If x, y ∈ S, then x + y ∈ S and xγy ∈ S.
So, 1. $V_{\delta_S}(x + y) = [1,1] = \min\{V_{\delta_S}(x), V_{\delta_S}(y)\}$ and

2. $V_{\delta_S}(x\gamma y) = [1,1] = \min\{V_{\delta_S}(x), V_{\delta_S}(y)\}$.

If x, y ∉ S, then x + y ∉ S and xγy ∉ S.
So, 1. $V_{\delta_S}(x + y) = [0,0] = \min\{V_{\delta_S}(x), V_{\delta_S}(y)\}$ and

2. $V_{\delta_S}(x\gamma y) = [0,0] = \min\{V_{\delta_S}(x), V_{\delta_S}(y)\}$.

If x ∉ S and y ∈ S, then x + y ∉ S and xγy ∉ S.
So, 1. $V_{\delta_S}(x + y) = [0,0] = \min\{V_{\delta_S}(x), V_{\delta_S}(y)\}$ and

2. $V_{\delta_S}(x\gamma y) = [0,0] = \min\{V_{\delta_S}(x), V_{\delta_S}(y)\}$.

A Similar argument for x ∈ S and y ∉ S.
Hence δ_S is a vague Γ-semiring of R.

Theorem 3.10: Let A be a vague set of a Γ-semiring R. Then the two vague cuts $A_{(\alpha_1,\beta_1)}$ and $A_{(\alpha_2,\beta_2)}$, where $\alpha_1, \alpha_2, \beta_1, \beta_2 \in [0,1]$ with $[\alpha_1, \beta_1] < [\alpha_2, \beta_2]$ are equal if and only if there is no x ∈ R such that $[\alpha_1, \beta_1] \leq V_A(x) \leq [\alpha_2, \beta_2]$.

Proof: Suppose $A_{(\alpha_1,\beta_1)}$ and $A_{(\alpha_2,\beta_2)}$ are equal.
Suppose if possible there exists x ∈ R such that $[\alpha_1, \beta_1] \leq V_A(x) < [\alpha_2,\beta_2]$.
i.e., $V_A(x) \geq [\alpha_1,\beta_1]$
$\Rightarrow x \in A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$
$\Rightarrow V_A(x) \geq A_{(\alpha_2,\beta_2)}$.
Which is a contradiction.
Hence there exists no x ∈ R such that $[\alpha_1, \beta_1] \leq V_A(x) < [\alpha_2, \beta_2]$.

Conversely suppose that there exists no x ∈ R such that $[\alpha_1, \beta_1] \leq V_A(x) < [\alpha_2, \beta_2]$.
Suppose if possible $A_{(\alpha_1,\beta_1)} \neq A_{(\alpha_2,\beta_2)}$.
$\Rightarrow$ there exists $x \in [\alpha_1, \beta_1]$ and $x \notin A_{(\alpha_2,\beta_2)}$.
i.e., $V_A(x) \geq [\alpha_1, \beta_1]$ and $V_A(x) < [\alpha_2, \beta_2]$.
So, there exists x ∈ R such that $[\alpha_1, \beta_1] \leq V_A(x) < [\alpha_2, \beta_2]$.

Which is a contradiction.
Hence $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.

Theorem 3.11: Let S be a sub Γ-semiring of a Γ-semiring R. Then for any $0 < \alpha < \beta < 1$, there exists a vague Γ-semiring A of R such that $A_{(\alpha, \beta)} = S$.

Proof: Let S be a sub Γ-semiring of a Γ-semiring R.
Define a vague set A on R by

$$V_A(x) = \begin{cases} [\alpha,\beta] \text{ if } x \in S \\ [0,0] \text{ if } x \notin S \end{cases}$$

Clearly $A_{(\alpha, \beta)} = S$.
Let $x, y \in R$ and $\gamma \in \Gamma$.
If $x, y \in S$, then $x + y \in S$ and $x\gamma y \in S$.
So, 1. $V_A(x + y) = [\alpha, \beta] = \min\{V_A(x), V_A(y)\}$ and
2. $V_A(x\gamma y) = [\alpha, \beta] = \min\{V_A(x),V_A(y)\}$.
If $x, y \notin S$, then $x + y \notin S$ and $x\gamma y \notin S$.
So, 1. $V_A(x + y) = [0, 0] = \min\{V_A(x), V_A(y)\}$ and
2. $V_A(x\gamma y) = [0, 0] = \min\{V_A(x),V_A(y)\}$.
If $x \notin S$ and $y \in S$, then $x + y \notin S$ and $x\gamma y \notin S$.
So, 1. $V_A(x + y) = [0, 0] = \min\{V_A(x), V_A(y)\}$ and
2. $V_A(x\gamma y) = [0, 0] = \min\{V_A(x),V_A(y)\}$.
Finally a similar argument for $x \in S$ and $y \notin S$.
Hence A is a vague Γ-semiring of R such that $A_{(\alpha, \beta)} = S$.

Theorem 3.12: Let A be a Γ-semiring of a Γ-semiring of R. Then $R_A = \{x \in R / V_A(x) = V_A(0)\}$ is a sub Γ-semiring of R.

Proof: Let $x, y \in R_A$ and $\gamma \in \Gamma$.
So, $V_A(x) = V_A(0)$ and $V_A(y) = V_A(0)$.
Now, 1. $V_A(x + y) \geq \min\{V_A(x), V_A(y)\} = V_A(0)$ and
2. $V_A(x\gamma y) \geq \min\{V_A(x), V_A(y)\} = V_A(0)$.
So that $x + y \in R_A$ and $x\gamma y \in R_A$.
Thus R_A is a sub Γ-semiring of R.

Theorem 3.13: Let A, B be two vague Γ-semirings of a Γ-semiring R. Then $A \cap B$ is a vague Γ-semiring of R.

Proof: Let $A = (t_A, f_A)$ and $B = (t_B, f_B)$ be two vague Γ-semirings of a Γ-semiring R.
Let $x, y \in R$ and $\gamma \in \Gamma$.

$$\begin{aligned} t_{A\cap B}(x + y) &= \min\{t_A(x + y), t_B(x + y)\} \\ &\geq \min\{\min\{t_A(x), t_A(y)\}, \min\{t_B(x), t_B(y)\}\} \\ &= \min\{\min\{t_A(x), t_B(x)\}, \min\{t_A(y), t_B(y)\}\} \\ &= \min\{t_{A\cap B}(x), t_{A\cap B}(y)\}. \end{aligned}$$

and

$1-f_{A\cap B}(x+y) = \min\{1-f_A(x+y), 1-f_B(x+y)\}$
$\geq \min\{\min\{1-f_A(x), 1-f_A(y)\}, \min\{1-f_B(x), 1-f_B(y)\}\}$
$= \min\{\min\{1-f_A(x), 1-f_B(x)\}, \min\{1-f_A(y), 1-f_B(y)\}\}$
$= \min\{1-f_{A\cap B}(x), 1-f_{A\cap B}(y)\}$.

Similarly we can prove that $t_{A\cap B}(x+y) \geq \min\{t_{A\cap B}(x), t_{A\cap B}(y)\}$ and $1-f_{A\cap B}(x+y) \geq \min\{1-f_{A\cap B}(x), 1-f_{A\cap B}(y)\}$.
Hence $A\cap B$ is a vague Γ-semiring of R.

Theorem 3.13: Let A(R) be the set of all vague Γ-semirings of a Γ-semiring R. Then $(A(R), \subset)$ is a poset.

Proof : Let $A, B, C \in A(R)$.
1. Always $A \subset A$, for all $A \in A(R)$.
So, $\subset$ is reflexive.
2. Let $A \subset B$ and $B \subset A$
$\Rightarrow A = B$.
So, $\subset$ is anti symmetric.
3.Let $A \subset B$ and $B \subset C$
$\Rightarrow A \subset C$.
So, $\subset$ is transitive.
Thus $\subset$ is partial ordering and hence $(A(R),\subset)$ is a poset.

Theorem 3.14: $(A(R), \cup, \cap, ' ,0, 1)$ is a De-Morgan Algebra.

Proof: We will show that
1. $(A(R), \cup, \cap, ' ,0, 1)$ is a bounded distributive lattice
2.$(A')' = A$, $(A\cup B)' = A'\cap B'$ and $(A\cap B)' = A'\cup B'$, for any $A,B \in A(R)$.
Let $A = (t_A, f_A)$, $B = (t_B, f_B)$, $C = (t_C, f_C) \in A(R)$.
1.Since $0 \leq t_A(x) \leq 1$ and $1 \geq f_A(x) \geq 0$, $\forall\, x \in R$.
So A(R) is bounded.

Idempotency:
$A\cap A = (t_A, f_A) \cap (t_A, f_A) = (t_A, f_A) = A$
$A\cup A = (t_A, f_A) \cup (t_A, f_A) = (t_A, f_A) = A$

Commutativity:
$A\cap B = (t_A, f_A) \cap (t_B, f_B)$
$= (\min\{t_A, t_B\}, \max\{f_A, f_B\})$
$= (\min\{t_B, t_A\}, \max\{f_B, f_A\})$
$= (t_B, f_B) \cap (t_A, f_A)$
$= B\cap A$.
$A\cup B = (t_A, f_A) \cup (t_B, f_B)$
$= (\max\{t_A, t_B\}, \min\{f_A, f_B\})$

$= (\max\{t_B, t_A\}, \min\{f_B, f_A\})$
$= (t_B, f_B) \cup (t_A, f_A)$
$= B \cup A.$

Associativity:

$A \cap (B \cap C) = (t_A, f_A) \cap ((t_B, f_B) \cap (t_C, f_C))$
$= (\min\{t_A, \min\{t_B, t_C\}\}, \max\{f_A, \max\{f_B, f_C\}\})$
$= \min(\{\min\{t_A, t_B\}, t_C\}, \max\{\max\{f_A, f_B\}, f_C\})$
$= (A \cap B) \cap C.$

$A \cup (B \cup C) = (t_A, f_A) \cup ((t_B, f_B) \cup (t_C, f_C)$
$= (\max\{t_A, \max\{t_B, t_C\}\}, \min\{f_A, \min\{f_B, f_C\}\})$
$= \max(\{\max\{t_A, t_B\}, t_C\}, \min\{\min\{f_A, f_B\}, f_C\})$
$= (A \cup B) \cup C.$

Absorption:

$A \cap (A \cup B) = (\min\{t_A, \max\{t_A, t_B\}\}, \max\{f_A, \min\{f_A, f_B\}\})$
$= (t_A, f_A)$
$= A$

$A \cup (A \cap B) = (\max\{t_A, \min\{t_A, t_B\}\}, \min\{f_A, \max\{f_A, f_B\}\})$
$= (t_A, f_A)$
$= A$

Distributivity:

$A \cap (B \cup C) = (\min\{t_A, \max\{t_B, t_C\}, \max\{f_A, \min\{f_B, f_C\}\})$
$= (\max\{\min\{t_A, t_B\}, \min\{t_A, t_C\}\}, \min\{\max\{f_A, f_B\}, \max\{f_A, f_C\}\})$
$= (A \cap B) \cup (A \cap C).$

$A \cup (B \cap C) = (\max\{t_A, \min\{t_B, t_C\}\}, \min\{f_A, \max\{f_B, f_C\}\})$
$= (\min\{\max\{t_A, t_B\}, \max\{t_A, t_C\}\}, \max\{\min\{f_A, f_B\}, \min\{f_A, f_C\}\})$
$= (A \cup B) \cap (A \cup C).$

Therefore (A(R), ∪, ∩, ', 0, 1) is a bounded distributive lattice

2. $(A \cup B)' = (\max\{t_A, t_B\}, \min\{f_A, f_B\})'$
$= (\min\{f_A, f_B\}, \max\{t_A, t_B\})$
$= A' \cap B'.$

Therefore $(A \cup B)' = A' \cap B'$.
Similarly we can show that $(A \cap B)' = A' \cup B'$.
Also we have $A' = (f_A, t_A)$
Therefore $(A')' = A$.
Hence (A(R), ∪, ∩, ' ,0, 1) is a De-Morgan algebra.

In particular, it can be observed that (A(R), ∪, ∩, ' ,0, 1) is complete De-Morgan algebra if we consider the members in A(R) as a chain.

Consider T = { t_{α_i} /(t_{α_i} , f_{α_i}) ∈A(R) and each $t_{\alpha_i} \subseteq t_{\alpha_j}$,∀ $\alpha_i \leq \alpha_j$} and

F = { f_{α_i} /(t_{α_i} , f_{α_i}) ∈A(R) and each $f_{\alpha_i} \supseteq f_{\alpha_j}$,∀ $\alpha_i \leq \alpha_j$}

Let $t_0 = \bigcap_{\alpha_i \in \Delta} t_{\alpha_i}$ (inf T) and $f_0 = \bigcup_{\alpha_i \in \Delta} f_{\alpha_i}$ (sup S)

we show that (t_0, f_0) is a vague Γ-semiring of R.

Let x, y ∈ R; γ ∈ Γ.

$$
\begin{aligned}
t_0(x + y) &= \bigcap_{\alpha_i \in \Delta} t_{\alpha_i}(x + y) \\
&= \inf t_{\alpha_i}(x + y),\ \forall \alpha_i \in \Delta \\
&\geq \inf(\min\{ t_{\alpha_i}(x),\ t_{\alpha_i}(y)\}),\ \forall \alpha_i \in \Delta \\
&= \min\{ \inf t_{\alpha_i}(x), \inf t_{\alpha_i}(y)\},\ \forall \alpha_i \in \Delta \\
&= \min\{ \bigcap_{\alpha_i \in \Delta} t_{\alpha_i}(x),\ \bigcap_{\alpha_i \in \Delta} t_{\alpha_i}(y)\} \\
&= \min\{t_0(x), t_0(y)\}
\end{aligned}
$$

Similarly $t_0(x\gamma y) = \min\{t_0(x), t_0(y)\}$.

Also for any α_p, α_q ∈ Δ, there exists α_r ∈ Δ such that $f_{\alpha_p}(x) \leq f_{\alpha_r}(x)$, $f_{\alpha_q}(y) \leq f_{\alpha_r}(y)$

$$
\begin{aligned}
\text{So, } \min\{ f_{\alpha_p}(x),\ f_{\alpha_q}(y)\} &\leq \min\{ f_{\alpha_r}(x),\ f_{\alpha_r}(y)\} \\
&\leq f_{\alpha_r}(x + y).
\end{aligned}
$$

$$
\begin{aligned}
\text{Now, } \min\{f_0(x), f_0(y)\} &= \min\{ \bigcup_{\alpha_i \in \Delta} f_{\alpha_i}(x),\ \bigcup_{\alpha_i \in \Delta} f_{\alpha_i}(y)\} \\
&= \min\{ \sup f_{\alpha_j}(x), \sup f_{\alpha_j}(y)\},\ \alpha_i \in \Delta \\
&= \sup(\min\{ f_{\alpha_p}(x),\ f_{\alpha_q}(y)\}),\ \alpha_p, \alpha_q \in \Delta \\
&\leq \sup f_{\alpha_r}(x + y),\ \alpha_r \in \Delta \\
&= \bigcup_{\alpha_i \in \Delta} f_{\alpha_i}(x + y) \\
&= f_0(x + y)
\end{aligned}
$$

Similarly $f_0(x\gamma y) = \min\{f_0(x), f_0(y)\}$.

Thus (t_0, f_0) is a vague Γ-semiring of R.

By definition $t_0 \subseteq t_{\alpha_i}$ and $f_0 \supseteq f_{\alpha_i}$, ∀ α_i ∈ Δ.

Hence (t_0, f_0) ⊆ (t_{α_i} , f_{α_i}), ∀ α_i ∈ Δ.

This implies that (t_0, f_0) is a lower bound of A(R).

Let (t_k, f_k) be another lower bound of A(R).

Then (t_k, f_k) ⊆ (t_{α_i} , f_{α_i}), ∀ αi ∈ Δ.

i.e., $t_k \subseteq t_{\alpha_i}$ and $f_k \supseteq f_{\alpha_i}$, ∀$\alpha_i$ ∈ Δ.

This gives t_k is a lower bound of T and f_k is an upper bound of F.

But t_0 is a infimum of T and f_0 is a supremum of F.

So, $t_k \subseteq t_0$ and $f_k \supseteq f_0$.

This gives $(t_k, f_k) \subseteq (t_0, f_0)$.
Hence (t_0, f_0) is a infimum of A(R).
Thus (A(R), $\cup$, $\cap$, ' ,0, 1) is complete De-Morgan algebra.

Conclusion: In this paper, the concept of vague Γ-semirings has been introduced and we established that the class of all vague Γ-semirings of a Γ-semiring forms a De-Morgan algebra. In future it is expected that these structures are useful in developing vague ideals, normal vague ideals, vague prime ideals, vague maximal ideals and vague semiprime ideals of a Γ-semiring.

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