Sharpe’s Single Index Model and its Application
Portfolio Construction: An Empirical Study

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Abstract
There are many investors who want to invest there savings in various Investment Product to gain a return and those who are risk takers they invest there savings in the stock Market. It is an volatile market which moves towards up and down on the basis of simple economic concept demand and supply. This volatility is known as the risk of the market and to save the investors against these volatility of the market. Stock exchange introduce a new concept i.e. Portfolio where the investors get the opportunity to reduce the risk through the segmenting the total investment amount in the bunch of securities. In the different ratio the best method of constructing a Portfolio is Sharp Single Index Model which covers all the factors like portfolio return, portfolio variance, portfolio standard deviation, and whieghtage of stocks. the data should be taken from www.bseindia.com for the time period of January 2010 to December 2013 on monthly basis in bse sensex as market performer. The main objectives of the study are to get an insight into the idea embedded in Sharpe’s Single Index Model, to construct an optimal portfolio empirically using the Sharpe’s Single Index Model, to determine return and risk of the optimal portfolio constructed by using Sharpe’s Single Index Model. The method is formulated a unique cut off rate and select those securities whose excess return to beta ratio is greater than a cut off rate. then proportion of investment in each selected securities should decide on the basis of the beta value, unsystematic risk, excess return to beta ratio and cut off rate of each security is concerned.

Keywords: volatile market, portfolio, risk taker investors, sharp index model, variance, beta, standard deviation, unsystematic risk, excess return to beta ratio and cut off rate.
1. Introduction
To determine the variance of the portfolio, the covariance between each possible pair of securities must be computed, which is represented in a covariance matrix. Thus, increase in the number of securities results in a large covariance matrix, which in turn, results in a more complex computation. If there are n securities in a portfolio, the Markowitz’s model requires n average (or expected) returns, n variance terms and \( \frac{n(n-1)}{2} \) covariance terms (i.e. in total \( \frac{n(n+3)}{2} \) data-inputs). Due to these difficulties analysts did not like to perform their task and they searched a simplified model to perform their tasks which is known as Sharp single Index Model given by William F. Sharp in 1963.

2. Literature Review
(Radhika Desai, September 2013)In this research they constructed Sharpe single index optimum portfolio by using data of fifty companies CNX NSE Nifty index for period of 2010-2012. (Andrade, 2012) the aims of this paper is to developing an optimal portfolio of equity of IT sector, through Sharpe’s Single Index Model. For the study, six top performing IT companies traded in BSE were taken and the optimal portfolio was constructed with 5 companies. (Tripathy) this paper tried to give a bird’s eye view about the concept of Single Index Model given by William Sharpe for the practical application to find out the returns in public sector banks from Indian context. (Kamal, 2012)The aims of the paper is constructing an optimal portfolio by applying Sharpe’s single index model of capital asset pricing in different scenarios, one is ex ante stock price bubble scenario and stock price bubble and bubble burst is second scenario (Sarker*, November, 2013) The main purpose of this paper is to construct an optimal portfolio by using Sharpe’s single-index model on the basis of monthly returns of 164 stocks of DSE for the period of July 2007 to June 2012. (ARUN KUMAR .S.S, April, 2013)Based on these models, we compute the portfolio return and its characteristics. ‘A portfolio optimization is only possible once we have a model of the portfolio return, therefore the aim of model is to control the financial risk that an investor takes. (Amit, 2012)This study attempts 50 stocks of NSE-NIFTY to construct an optimal portfolio by using Sharpe's Single index model for the period of April 2006 to December 2011 on daily return basis, selected out of 50 short listed scripts, giving the return of 0.116%. (Mandal, Jan-Jun2013)It is found there is a significant difference between the total risk of the optimal portfolio under SIM and that of under Markowitz's model. (Sen, November 2010)This paper aims at building an optimal portfolio from all 100 scrips of S&P CNX 500, using Sharpe's Single Index Model. (Nageswari*, Selvam**, & Bhuvaneswari***, November 2013)the present study highlights the optimal portfolio selection using Sharpe's Single Index model, through which a significant reduction in the riskiness or variability of the return of securities can be obtained.

Objective: The main objectives of the study are:
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1. To get an insight into the idea embedded in Sharpe’s Single Index Model.
2. To construct an optimal portfolio empirically using the Sharpe’s Single Index Model.
3. To determine return and risk of the optimal portfolio constructed by using Sharpe’s Single Index Model.

3. Methodology
The relevant data has been collected from the secondary sources of information (www.Bseindia.com). In this, BSE Sensex taken as a Market index. The sample should be collect all 30 stocks of Bse Sensex from the duration of January 2010 to December 2013 on monthly return basis. Also computing the monthly return of market index. Taking a return of each securities and computing with Market Return to proposed method formulates a unique Cut off Rate and selects those securities whose ‘Excess Return-to-Beta Ratio’ is greater than the cut off rate. Then move towards the optimal portfolio construction, which suggest the Investment proportion of the securities in the portfolio is computed on the basis of unsystematic risk, beta value, excess return to beta ratio and the cut off rate of the security.

3.1 Analysis
From table 1, it is found that there 8 securities out of 30 samples of securities which contribute a negative return viz, Gail India ltd., Sesa Sterlite ltd., NTPC ltd., Bharat Heavy Electricals Ltd., Tata Steel Ltd., Reliance Industry Ltd., HINDALCO Industries Ltd., State Bank of India. It is also found that out of 30 samples there are 11 securities (Bharat Heavy Electricals Ltd., Tata Steel Ltd., Reliance Industry Ltd., Tata Motors Ltd., Tata Power co. Ltd., Maruti Suzuki India Ltd., Larsen toubro Ltd., AXIS Bank Ltd., HINDALCO Industries Ltd., ICICI Bank Ltd., State Bank of India) bearing Beta ratio is greater than 1, have contributed Monthly Mean Return from 0.355 to 2.16 (Tata Motors Ltd., Tata Power co. Ltd., Maruti Suzuki India Ltd., Larsen toubro Ltd., AXIS Bank Ltd., ICICI Bank Ltd., State Bank of India). While remaining securities gives negative return from -0.0375 to -1.9125 (Bharat Heavy Electricals Ltd., Tata Steel Ltd., Reliance Industry Ltd., HINDALCO Industries Ltd., State Bank of India), these 11 securities are called “aggressive securities” according to their Beta value, which is greater than 1 and the remaining securities are called defensive securities according to their Beta value, which is less than 1. All the defensive securities having positive return range from 0.135 to 2.477 except (Gail India ltd., Sesa Sterlite ltd., NTPC ltd.) having negative return from -0.275 to -1.12 and Hindustan Unilever Ltd. Which having negative Beta ratio i.e. -0.12277. The systematic risk ($\beta_i^2 \sigma_m^2$) of all the securities range from 0.031863 to 13.34784, the unsystematic risk($\sigma_i^2$) of all the securities range from 0.06397 to 4.676346, the total risk ($\sigma_i^2$) of all the securities range from 0.136219 to 16.54903, the coefficient of correlation ($r_{im}$) of all the securities range from 0.312367061 to 0.997208543, and the covariance ($\sigma_{jm}$) with the market of all the securities range from 0.3759563 to 6.4706875.
From table 2, it shows that the Tata consultancy Services Ltd. having highest excess return to beta ratio (5.016942) would be first place. HDFC Bank Ltd. having second highest excess return to beta ratio (3.079425) put on second place. ITC Ltd. Having third highest excess return to beta ratio (2.969724) would placed third and the fourth securities place fourth and so on. Bharti Airtel Ltd. Having lowest excess return to beta ratio (0.032022) placed on last 18th position within the data set. Now it is necessary determine those securities which occupies excess return to Beta ratio is greater than a particular $c^*$ value that is $C_i$.

From table 3, $\frac{R_i - R_f}{\beta_i}$ value of the first four security exceed the $C_i$ values of the respective securities. The $C_i$ value of Sun Pharma Ltd. is (2.691289) below which ‘excess return to beta ratio’ is less than the respective $C_i$ value of the security and the collection these four securities use make it a Optimal portfolio.

From table 4, it is found that, that weight $W_i$ for the selected securities in the optimal portfolio of stocks viz. Tata Consultancy Service Ltd., HDFC Bank Ltd., ITC Ltd., Sun Pharma Ltd., are 20%, 13%, 53%, 14% respectively.

On the basis of information arranged in Table-5 the following results can be extracted:

i. Portfolio Return $\sum W_i R_i = 1.90\%$ per month
ii. Portfolio beta $\sum W_i \beta_i = 0.50\%$ which indicates the defensive in nature.
iii. Systematic risk of the portfolio $\sum W_i \beta_i \sigma_i = 0.50*1.77 = 0.885$ which comes from economic factors.
iv. Unsystematic risk of the portfolio $\sum W_i \sigma_i = 1.04$ which comes from firm-specific factors i.e. the internal environmental factors.
v. Total risk of the portfolio $(\sigma_i^2) = 1.04+0.885=1.925$.

Or
Total risk of the portfolio $(\sigma_i) = \sqrt{1.925} =1.3874$.

From the above results it is found that the portfolio return is higher than the average returns of the individual stock ITC Ltd., all most similar with HDFC Bank Ltd. in the optimal portfolio with the exception of Tata Consultancy Service Ltd. and Sun Pharma Ltd. The beta value of the optimal portfolio is less than one which indicates that the returns from the portfolio fluctuates at a slower rate than that of the market index. The unsystematic risk (firm specific) of the optimal portfolio is 1.04% in terms of variance which is much higher than that of the systematic risk (0.885 %), of the portfolio. The total risk of the portfolio (1.3874%, in term of SD) is less than that of the securities in the portfolio with the exception of ITC Ltd.

According to Markowitz’s Mean-Variance Model, portfolio risk (in terms of variance) is given by:

$$\sigma_p^2 = W_{11} \sigma_{11}^2 + W_{12} \sigma_{12}^2 + 2W_{11}W_{12}(r_{im}\sigma_{i1}\sigma_{i2})$$

16)

In terms of S.D. it under as
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\[ \sigma_p = \sqrt{W_{t1}^2 \sigma_{i1}^2 + W_{t2}^2 \sigma_{i2}^2 + 2W_{t1}W_{t2}(r_{im}\sigma_{i1}\sigma_{i2})} \]  
(17)

\[ \sigma_p^2 = 0.53 + 4.3998 = 4.92\% \]

\[ \sigma_p = \sqrt{4.92} = 2.21\% . \]

4. Findings

1. Therefore, there is a significant difference between the total risk of the optimal portfolio calculated under two different mechanisms found in SIM and Markowitz’s model respectively. It is observed that the total risk is found to be 2.21% in Markowitz’s model whereas total risk of the optimal portfolio is 1.3874% (in terms of SD) under SIM taking the necessary input from Markowitz’s model.

2. It is observed that as compared to the Markowitz’s Mean-Variance Model, the Sharpe’s Single Index model gives an easy mechanism of constructing an optimal portfolio of stocks for a rational investor by analyzing the reason behind the inclusion of securities in the portfolio with their respective weights. Actually, it simplifies the portfolio problems found in the Markowitz’s model to a great extent.

3. So far as the construction of optimal portfolio is concerned, there is a considerable similarity between SIM and the Markowitz’s model though, in reality, SIM requires equal input in compare to input requirement of Markowitz’s model to arrive at the risk and return of the optimal portfolio. From the study, it is observed that only Four securities out of thirty sampled securities are allowed to be included in the optimal portfolio using the steps behind its construction under SIM. To arrive at the risk and return of this portfolio, the number of inputs required in SIM is 14 (applying 3n+2) whereas the same is 14 applying \( \frac{n(n+3)}{2} \) in Markowitz’s model.

4. There is a significant difference between the total risk of the optimal portfolio calculated under two different mechanisms found in SIM and Markowitz’s model respectively. It is observed that the total risk is found to be 2.21% in Markowitz’s model whereas total risk of the optimal portfolio is 1.3874% (in terms of SD) under SIM taking the necessary input from Markowitz’s model.

5. Concluding Remarks

From the discussion and analysis so far it is clear that the construction of optimal portfolio investment by using Sharpe’s Single Index Model is easier and more comfortable than by using Markowitz’s Mean-Variance Model. In his seminal contribution Sharpe argued that there is a considerable similarity between efficient
portfolios generated by Markowitz’s Model and SIM. This model can show how risky a security is, if the security is held in a well-diversified portfolio. This study is made on the basis of small sample (n=30) i.e. 30 sampled securities but the time duration of the study is being short i.e. monthly return of four year it gives more accurate result if time duration of the study will extended.