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A Geometric Expression of Taylor's Law

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Abstract

Taylor's Law is a widely observed empirical pattern that relates variance to the mean in sets of non-negative measurements via an approximate power function. This note provides a geometric expression of Taylor's Law.

Taylor's law (TL) is named after Lionel R. Taylor, a British ecologist who published "aggregation, variance, and the mean" in the March 4th, 1961 issue of *Nature* (Taylor, 1961). TL is a widely observed empirical pattern (Clark and Perry 1994, Cohen 2016, Cohen 2017, Cohen, Bohk-Ewald and Rau 2018, Demers 2018, Eisler, Bartos, and Kertész 2006, Kilpatrick and Ives 2003, Swanson and Tayman, 2022, Swanson and Tedrow 2022, Tokeshi 1995) that relates the variances to the means of sets of non-negative measurements via an approximate power function: $variance_g \approx a \times (mean_g)^b$, where g indexes the group of measurements (Reuman et al. 2017: 6788).

Cohen and Courgeau (2017) found that TL applies to the distances between two randomly chosen points in various geometric shapes, subject to a few conditions. In this note, I observe that TL itself can be expressed geometrically, a perspective overlooked until now, even in kindred publications that preceded Taylor's seminal 1961 paper (e.g., Barnes 1952, Bartlett 1936, Bliss, 1941, Neyman 1926, Tweedie, 1946).

Following Swanson (1977, 2023), let the sum of the N values of random variable X equal Σn_i. These same N values form the N elements of vector V_1, There are exactly N - 1 additional distinguishable vectors, $V_2, V_3, \ldots, V_N$, that can be formed by permuting the N elements of V_1 such that $V_1 + V_2 + V_3 + ,..., + V_N = (\Sigma ni_1, , \Sigma ni_2, \Sigma ni_3, \ldots, \Sigma ni_N) = V_s$. Multiplying V_s by the scalar (l/N) gives V_m, the point in N–Space that is $(l/N)^{th}$ the distance from the origin to Vs, which can be interpreted as the mean of random variable X. Now let *D* equal the Pythagorean distance between V_1 and V_m. By multiplying *D* by the scalar $(1/N)^{½}$, the standard deviation of random variable X can be expressed as $\sigma = (1/N)^{½}(D)$, while the variance of X can be expressed as $\sigma^2 = (1/N)(D^2)$

$\approx aV_m^b$, where $aV_m^b = a \times (mean_g)^b$.

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