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Fuzzy Graph Structures: A Generalised Approach

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Abstract

The present authors introduced the concept of a fuzzy graph structure in an earlier paper *On Generalised Fuzzy Graph Structures*, Appl. Math. Sc., 5(4), 2011, 173-180. Here it is further generalised by defining $\rho_{i_1 i_2 \dots i_r}$ -path, $\rho_{i_1 i_2 \dots i_r}$ -cycle, $\rho_{i_1 i_2 \dots i_r}$-tree, $\rho_{i_1 i_2 \dots i_r}$-forest, fuzzy $\rho_{i_1 i_2 \dots i_r}$-cycle, fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree and fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest. Some results are obtained.

Subject Classification: 05C72, 05C05, 05C38, 05C40

Keywords: $\rho_{i_1 i_2 \dots i_r}$- path, $\rho_{i_1 i_2 \dots i_r}$-connectedness, fuzzy $\rho_{i_1 i_2 \dots i_r}$-cycle, fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree.

1.0 Introduction

The concept of fuzzy sets introduced by L.A. Zadeh in 1965 was used by A. Rosenfeld in 1975 [1] for defining fuzzy graph. E. Sampathkumar introduced the concept of graph structure in [2]. It is in particular, a generalisation of the notions like graphs, signed graphs and edge - coloured graphs with the colourings. He defined a graph structure G as $G = (V, R_1, R_2, ..., R_k)$ where V is a non-empty set and $R_1, R_2, ...,$ R_k are relations on V which are mutually disjoint such that each R_i, $i = 1, 2, ..., k$ is symmetric and irreflexive.

Based on these concepts, we introduced the concept of fuzzy graph structure in [3]. Therein we defined ρ_i-edge, ρ_i-cycle, ρ_i-tree, ρ_i-forest, fuzzy ρ_i-cycle, fuzzy ρ_i-tree, fuzzy ρ_i-forest etc. and proved some results.

In this paper, we generalise the above notions and introduce some new concepts like $\rho_{i_1 i_2 \dots i_r}$-cycle, $\rho_{i_1 i_2 \dots i_r}$-tree, $\rho_{i_1 i_2 \dots i_r}$-forest, fuzzy $\rho_{i_1 i_2 \dots i_r}$-cycle, fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree and fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest.

Essential preliminaries are given wherever necessary. For more details on Graph Theory, reference may be made to [4], on Fuzzy Graph Theory, to [5] and for Graph Structures, to [2].

2.0 Preliminaries

We first recall the definition of fuzzy graph structure [3].

Definition 2.1 [3]

Let $G = (V, R_1, R_2, ..., R_k)$ be a graph structure and μ, $\rho_1, \rho_2, \ldots, \rho_k$ be fuzzy subsets of V, $R_1, R_2, ..., R_k$ respectively such that

$\rho_i(x, y) \leq \mu(x) \wedge \mu(y) \ \forall x, y \in V, i=1, 2, ..., k$. Then $\tilde{G}= (\mu, \rho_1, \rho_2, \ldots, \rho_k)$ is a fuzzy graph structure of G.

Convention

Throughout this paper, G and $\tilde{G}$ will have the above meaning, unless otherwise stated.

The concepts of partial fuzzy subgraph structure, ρ_i-edge, ρ_i -path, ρ_i-cycle, fuzzy ρ_i-cycle, ρ_i-forest, ρ_i-tree, fuzzy ρ_i-forest, fuzzy ρ_i-tree etc. were introduced in [3] as follows.

Definition 2.2 [3]

$\tilde{F}=(\mu, \tau_1, \tau_2, \ldots, \tau_k)$ is a partial fuzzy spanning subgraph structure of $\tilde{G}= (\mu, \rho_1, \rho_2, \ldots, \rho_k)$ if $\tau_r \subseteq \rho_r$ for $r = 1, 2, ..., k$.

Definition 2.3 [3]

Let G be a graph structure and $\tilde{G}$ be a fuzzy graph structure of G. If $(x, y) \in$ supp(ρ_i), then (x, y) is said to be a ρ_i-edge of $\tilde{G}$.

Definition 2.4 [3]

A ρ_i-path of a fuzzy graph structure $\tilde{G}$ is a sequence of vertices, $x_0, x_1, \ldots, x_n$ which are distinct (except possibly $x_0 = x_n$ such that (x_{j-1}, x_j) is a ρ_i-edge for all $j=1, 2, ..., n$.

Definition 2.5 [3]

$\tilde{G}$ is a ρ_i-cycle iff (supp(μ), supp(ρ_1), supp(ρ_2), ..., supp(ρ_k)) is an R_i-cycle (where an R_i-cycle is a sequence of vertices $x_0, x_1, \ldots, x_{n-1}, x_n = x_0$ in V such that each (x_{j-1}, x_j) is an R_i-edge for $j=1, 2, ..., n$.

Definition 2.6 [3]

$\tilde{G}$ is a fuzzy ρ_i-cycle iff (supp(μ), supp(ρ_1), supp(ρ_2), ..., supp(ρ_k)) is an R_i-cycle and there exists no unique (x, y) in supp(ρ_i) such that

$\rho_i(x, y) = \wedge\{\rho_i(u, v) | (u, v) \in \text{supp}(\rho_i)\}$.

Definition 2.7 [3]
$\tilde{G}$ is a ρ_i-forest if its ρ_i-edges form an R_i-forest(where an R_i-forest is a fuzzy graph structure which does not contain R_i-cycles).

Definition 2.8 [3]
$\tilde{G}$ is a ρ_i-tree if it is a ρ_i-connected ρ_i-forest.

Definition 2.9 [3]
$\tilde{G}$ is a fuzzy ρ_i-forest if it has a partial fuzzy spanning subgraph structure

$\widetilde{F_i}$=(μ, τ_1, τ_2, …, τ_k) which is a τ_i-forest where for all ρ_i-edges not in $\widetilde{F_i}$,

$$\rho_i(x, y) < \tau_i^{\infty}(x, y).$$

Definition 2.10 [3]
$\tilde{G}$ is a fuzzy ρ_i-tree if it has a partial fuzzy spanning subgraph structure
$\widetilde{F_i}$=(μ, τ_1, τ_2, …, τ_k) which is a τ_i-tree where for all ρ_i-edges not in
$\widetilde{F_i}$, $\rho_i(x, y) < \tau_i^{\infty}(x, y)$.

3.0 Generalisation

Now we introduce some new terms by generalising the above concepts. First we define $\rho_{i_1 i_2 \ldots i_r}$-path and $\rho_{i_1 i_2 \ldots i_r}$-connectedness.

Definition 3.1
Let $x_0, x_1, \ldots, x_n$ be a sequence of distinct vertices of $\tilde{G}$. Let $\rho_{i_P}(x_{j-1}, x_j)$>0
$\forall$ *j=1, 2, …, n* for some $p \in \{1, 2, \ldots, r\}$ where $\rho_{i_1}, \rho_{i_2}, \ldots, \rho_{i_r}$are some among ρ_1, ρ_2, …, ρ_k. Then $x_0, x_1, \ldots, x_{n-1}, x_n$ is a $\rho_{i_1 i_2 \ldots i_r}$-path.

Definition 3.2
Two vertices of a fuzzy graph structure $\tilde{G}$ joined by a $\rho_{i_1 i_2 \ldots i_r}$-path are said to be $\rho_{i_1 i_2 \ldots i_r}$--connected.

Definition 3.3
The strength of a $\rho_{i_1 i_2 \ldots i_r}$-path $x_0, x_1, \ldots, x_{n-1}, x_n$ of a fuzzy graph structure $\tilde{G}$ is $\wedge_{j=1}^{n} \vee_{q=1}^{r} \rho_{i_q}(x_{j-1}, x_j)$.

We may denote the strength of a $\rho_{i_1 i_2 \ldots i_r}$-path from x to y as $\rho_{i_1 i_2 \ldots i_r}(x, y)$ and the strength of a strongest $\rho_{i_1 i_2 \ldots i_r}$-path from x to y as $\rho_{i_\infty}(x, y)$.

We now define $\rho_{i_1 i_2 \ldots i_r}$-cycle, $\rho_{i_1 i_2 \ldots i_r}$-tree, $\rho_{i_1 i_2 \ldots i_r}$-forest etc.

Definition 3.4
A sequence of vertices $x_0, x_1, \ldots, x_{n-1}, x_n = x_0$ of V of a fuzzy graph structure $\tilde{G}$ such

that each (x_{j-1}, x_j), *j=1.2, ..., n*, is an R_i-edge for some $i \in$*{1, 2, ..., r},* is said to be an $R_{i_1 i_2 \ldots i_r}$-cycle if $R_{i_1}, R_{i_2}, \ldots, R_{i_r}$ are some among R_1, R_2, …, $R_k$which are represented in it by nonzero R_i-edges, *i=1, 2, ..., k.*

Note that R_i-cycle [2] is an $R_{i_1 i_2 \ldots i_r}$-cycle with i_1=i_2=…=i_r=i.

Definition 3.5
$\tilde{G}$ is a $\rho_{i_1 i_2 \ldots i_r}$-cycle iff (supp(μ), supp(ρ_1), supp(ρ_2), ... , supp(ρ_k)) is an $R_{i_1 i_2 \ldots i_r}$-cycle and $\rho_{i_1}, \rho_{i_2}, \ldots, \rho_{i_r}$are some among ρ_1, ρ_2, …, $\rho_k$which correspond to $R_{i_1}, R_{i_2}, \ldots, R_{i_r}$.

Note that $\rho_{i_1 i_2 \ldots i_r}$-cycle is a ρ_i-cycle [3] for i_1=i_2=…=i_r=i.

Definition 3.6
$\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-cycle iff (supp(μ), supp(ρ_1), supp(ρ_2), ... , supp(ρ_k)) is an $R_{i_1 i_2 \ldots i_r}$-cycle and there exists no unique *(x, y)* in $\bigcup_{q=1}^{r} supp(\rho_{i_q})$ such that $\bigvee_{q=1}^{r} \rho_{i_q}(x,y) = \bigwedge\{\bigvee_{q=1}^{r} \rho_{i_q}(u,v) | (u,v) \in \bigcup_{q=1}^{r} supp(\rho_{i_q})\}$

Definition 3.7
A graph structure which does not contain $R_{i_1 i_2 \ldots i_r}$-cycles
for $i_1, i_2, \ldots, i_r \in$ *{1, 2, ..., k}* which need not be distinct, is an $R_{i_1 i_2 \ldots i_r}$-forest.

Definition 3.8
An $R_{i_1 i_2 \ldots i_r}$-forest is an $R_{i_1 i_2 \ldots i_r}$-tree if it is connected by a $\rho_{i_1 i_2 \ldots i_r}$-path.

Definition 3.9
$\tilde{G}$ is a $\rho_{i_1 i_2 \ldots i_r}$-forest if its ρ_{i_p}-edges *(p ∈{1, 2, ..., r})* form an $R_{i_1 i_2 \ldots i_r}$-forest.

Definition 3.10
$\tilde{G}$ is a $\rho_{i_1 i_2 \ldots i_r}$-tree if it is connected by a $\rho_{i_1 i_2 \ldots i_r}$-path and it is a $\rho_{i_1 i_2 \ldots i_r}$-forest.

Definition 3.11
$\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-forest if it has a partial fuzzy spanning subgraph structure $\tilde{F}$=(μ, $\tau_1, \tau_2, \ldots, \tau_k$) which is a $\tau_{i_1 i_2 \ldots i_r}$-forest and $\tau_{i_1}, \tau_{i_2} \ldots, \tau_{i_r}$are some among $\tau_1, \tau_2, \ldots, \tau_k$which are represented in $\tilde{F}$ where for all ρ_{i_p}-edges (p∈{1, 2, ..., r}) not in $\tilde{F}$, ρ_{i_p}*(x, y)*<$\tau_{i\infty}$ *(x, y).*

Definition 3.12
$\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-tree if it has a partial fuzzy spanning subgraph structure $\tilde{F}$=(μ, $\tau_1, \tau_2, \ldots, \tau_k$) which is a $\tau_{i_1 i_2 \ldots i_r}$-tree where for all ρ_{i_p}-edges *(p ∈{1, 2, ..., r})* not in $\tilde{F}$, ρ_{i_p}*(x, y)*<$\tau_{i\infty}$ *(x, y).*

4.0 Some results on fuzzy $\rho_{i_1 i_2 \dots i_r}$-trees and fuzzy $\rho_{i_1 i_2 \dots i_r}$-forests

We generalise some results discussed in [3] to fuzzy $\rho_{i_1 i_2 \dots i_r}$-trees and fuzzy $\rho_{i_1 i_2 \dots i_r}$-forests.

Theorem 4.1

$\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest iff in any $\rho_{i_1 i_2 \dots i_r}$-cycle, there exists some ρ_{i_p}-edges $(p \in \{1, 2, \dots, r\})$ such that $\rho_{i_p}(x, y) < \rho'_{i_\infty}(x, y)$ where $(\mu, \rho_1', \rho_2', \dots, \rho_k')$ is the partial fuzzy spanning subgraph structure obtained by deleting (x, y) and $\rho_{i_1}', \rho_{i_2}', \dots, \rho_{i_r}'$ are defined accordingly.

Proof

Sufficiency

If $\tilde{G}$ does not contain a $\rho_{i_1 i_2 \dots i_r}$-cycle, it is a fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest. Then there is nothing to prove.

Let $\tilde{G}$ contains a $\rho_{i_1 i_2 \dots i_r}$-cycle. Consider some ρ_{i_p}-edge $(p \in \{1, 2, \dots, r\})$ (x, y) with $\rho_{i_p}(x, y) < \rho'_{i_\infty}(x, y)$.

Remove (x, y). Still there may be $\rho_{i_1 i_2 \dots i_r}$-cycles. Repeat the process with some ρ_{i_q}-edge $(q \in \{1, 2, \dots, r\})$, q need not be different from p.

Strength of the deleted ρ_{i_p}-edges $(p \in \{1, 2, \dots, r\})$ increases in each step. When the fuzzy graph structure is cleared of all $\rho_{i_1 i_2 \dots i_r}$-cycles, the resultant partial fuzzy spanning subgraph structure is a $\rho_{i_1 i_2 \dots i_r}$-forest. Let it be $\tilde{F}$.

If $(x, y) \notin \tilde{F}$, it was deleted. So there exists a $\rho_{i_1 i_2 \dots i_r}$-path from x to y in $\tilde{F}$, stronger than (x, y). There will be stronger $\rho_{i_1 i_2 \dots i_r}$-paths for diverting around deleted ρ_{i_p}-edges $(p \in \{1, 2, \dots, r\})$. Repeating the process, we get a $\rho_{i_1 i_2 \dots i_r}$-path consisting only of ρ_{i_p}-edges $(p \in \{1, 2, \dots, r\})$. Hence $\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest.

Necessity

Let $\tilde{G}$ be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest. Let C be a $\rho_{i_1 i_2 \dots i_r}$-cycle. Some ρ_{i_p}-edge $(p \in \{1, 2, \dots, r\})$ (x, y) of C is not in $\tilde{F} = (\mu, \tau_1, \tau_2, \dots, \tau_k)$, the partial fuzzy spanning subgraph structure which is a $\tau_{i_1 i_2 \dots i_r}$-forest ($\tau_{i_1}, \tau_{i_2} \dots, \tau_{i_r}$ are some among $\tau_1, \tau_2, \dots, \tau_k$ which are represented in it) and $\rho_{i_p}(x, y) < \tau_{i_\infty}(x, y)$.

But $\tau_{i_\infty}(x, y) < \rho'_{i_\infty}(x, y)$ where $(\mu, \rho_1', \rho_2', \dots, \rho_k')$ is obtained from $\tilde{G}$ by deleting (x, y) and $\rho_{i_1}', \rho_{i_2}', \dots, \rho_{i_r}'$ are defined accordingly.

Therefore $\rho_{i_p}(x, y) < \rho_{i_\infty}'(x, y)$.

Theorem 4.2

Let $\tilde{G}$ be a fuzzy graph structure. If there is at most one strongest $\rho_{i_1 i_2 \dots i_r}$-path between any two vertices, then $\tilde{G}$ must be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-forest.

Proof

Suppose there exists at most one strongest $\rho_{i_1 i_2 \ldots i_r}$-path between any two vertices of $\tilde{G}$.

If possible, let $\tilde{G}$ be not a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-forest. Then by definition, there exists a $\rho_{i_1 i_2 \ldots i_r}$-cycle C such that for every ρ_{i_p}-edge ($p \in \{1, 2, ..., r\}$) (x, y) in C, $\rho_{i_p}(x, y) \geq \rho_{i_1 i_2 \ldots i_r}'(x, y)$ where (μ, ρ_1', ρ_2', …, ρ_k') is the partial fuzzy spanning subgraph structure obtained by deleting (x, y) from $\tilde{G}$ and $\rho_{i_1 i_2 \ldots i_r}'(x, y)$ is the strength of $\rho_{i_1 i_2 \ldots i_r}$-path (there is only one such $\rho_{i_1 i_2 \ldots i_r}$-path) from x to y not involving (x, y).
ie., (x, y) is the strongest $\rho_{i_1 i_2 \ldots i_r}$-path from x to y.

If (x, y) is the weakest ρ_{i_p}-edge $(p \in \{1, 2, ..., r\})$ of C, the remaining ρ_{i_p}-edges $(p \in \{1, 2, ..., r\})$ of C form a strongest $\rho_{i_1 i_2 \ldots i_r}$-path which is a contradiction.

Therefore $\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-forest.

Theorem 4.3

Let $\tilde{G}$ be a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-tree and $\tilde{G}^*$= (supp(μ), supp(ρ_1), supp(ρ_2), …, supp(ρ_k)) be not a $\rho_{i_1 i_2 \ldots i_r}$-tree. Then there exists at least one ρ_{i_p}-edge $(p \in \{1, 2, ..., r\})$ (u, v) in $\bigcup_{q=1}^{r} supp(\rho_{i_q})$ for which $\rho_{i_p}(u, v) < \rho_{i_\infty}(u, v)$.

Proof

Let $\tilde{G}$ be a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-tree. Then there exists a partial fuzzy spanning subgraph structure $\tilde{F}$ = (μ, $\tau_1, \tau_2, \ldots, \tau_k$) which is a $\tau_{i_1 i_2 \ldots i_r}$-tree and for every ρ_{i_p}-edge $(p \in \{1, 2, ..., r\})$ $(u, v) \notin \tilde{F}$, $\rho_{i_p}(u, v) < \tau_{i_\infty}(u, v)$.

Also $\tau_{i_\infty}(u, v) \leq \rho_{i_\infty}(u, v)$.

Therefore $\rho_{i_p}(u, v) < \rho_{i_\infty}(u, v)$ $\forall$ ρ_{i_p}-edge $(u, v) \notin \tilde{F}$. $\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-tree and $\tilde{G}^*$ is not a $\rho_{i_1 i_2 \ldots i_r}$-tree.

Therefore there exists at least one ρ_{i_p}-edge $(p \in \{1, 2, ..., r\}) \notin \tilde{F}$. ie., there exists at least one ρ_{i_p}-edge $(p \in \{1, 2, ..., r\})$ (u, v) in $\bigcup_{q=1}^{r} supp(\rho_{i_q})$ with $\rho_{i_p}(u, v) < \rho_{i_\infty}(u, v)$.

Lemma 1

Let $\tilde{G}$ be a fuzzy graph structure with $\rho_{i_p}(u, v) = \mu(u) \wedge \mu(v)$ for some $p \in \{1, 2, ..., r\}$ and for every ρ_{i_p}-edge $(u, v) \in \bigcup_{q=1}^{r} supp(\rho_{i_q})$ where $\bigcup_{q=1}^{r} supp(\rho_{i_q}) \neq \phi$. Then $\rho_{i_\infty}(u, v) = \rho_{i_p}(u, v)$ for that p.

Proof

$\rho_{i_p}(u, v) = \mu(u) \wedge \mu(v)$ for every ρ_{i_p}-edge (u, v) for some $p \in \{1, 2, ..., r\}$, $(u, v) \in \bigcup_{q=1}^{r} supp(\rho_{i_q})$. $\rho_{i_\infty}(u, v)$ = strength of the strongest $\rho_{i_1 i_2 \ldots i_r}$-path from u to v.
ie., $\rho_{i_\infty}(u, v) = \mu(u) \wedge \mu(v) = \rho_{i_p}(u, v)$ for that p.

Theorem 4.4

Let $\tilde{G}$ be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree. Then $\rho_{i_p}(u, v) < \mu(u) \wedge \mu(v)$ for some ρ_{i_p}-edge (u, v) $(p \in \{1, 2, \dots, r\})$, in $\bigcup_{q=1}^{r} supp(\rho_{i_q})$.

Proof

If possible, let $\rho_{i_p}(u, v) = \mu(u) \wedge \mu(v) \forall (u, v) \in \bigcup_{q=1}^{r} supp(\rho_{i_q})$.

Then by lemma, $\rho_{i_p}(u, v) = \rho_{i_\infty}(u, v)$ for that p for which $\rho_{i_p}(u, v) = \mu(u) \wedge \mu(v)$.

Let $\tilde{G}$ be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree. Then it has a partial fuzzy spanning subgraph structure $\tilde{F} = (\mu, \tau_1, \tau_2, \dots, \tau_k)$ which is a $\tau_{i_1 i_2 \dots i_r}$-tree with $\rho_{i_p}(u, v) < \tau_{i_\infty}(u, v) \forall$ ρ_{i_p}-edge $(u, v) \notin \tilde{F}$.

Therefore $\rho_{i_\infty}(u, v) < \tau_{i_\infty}(u, v)$ which is a contradiction.

Hence $\rho_{i_p}(u, v) < \mu(u) \wedge \mu(v)$ for some ρ_{i_p}-edge $(p \in \{1, 2, \dots, r\})$ (u, v).

Theorem 4.5

Let $\tilde{G}$ be a $\rho_{i_1 i_2 \dots i_r}$-cycle. $\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \dots i_r}$-cycle iff $\tilde{G}$ is not a fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree.

Proof

Let $\tilde{G}$ be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-cycle.

If possible, let it be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree also. Then it has a partial fuzzy spanning subgraph structure $\tilde{F} = (\mu, \tau_1, \tau_2, \dots, \tau_k)$which is a $\tau_{i_1 i_2 \dots i_r}$-tree. Then

$[\bigcup_{q=1}^{r} supp(\rho_{i_q}) - \bigcup_{q=1}^{r} supp(\tau_{i_q})] = \{(u, v)\}$ for some $u, v \in V$ since $\tilde{G}$ is a $\rho_{i_1 i_2 \dots i_r}$-cycle.

There does not exist a unique (x, y) in $\bigcup_{q=1}^{r} supp(\rho_{i_q})$ such that

$$\bigvee_{q=1}^{r} \rho_{i_q}(x, y) = \bigwedge \{\bigvee_{q=1}^{r} \rho_{i_q}(u, v) : (u, v) \in \bigcup_{q=1}^{r} supp(\rho_{i_q})\}.$$

Therefore there exists no $\tau_{i_1 i_2 \dots i_r}$-path in $\tilde{F}$ from u to v having greater strength than $\rho_{i_1 i_2 \dots i_r}(u, v)$. Otherwise $\tilde{G}$ will not be a fuzzy $\rho_{i_1 i_2 \dots i_r}$-cycle.

Thus $\tilde{G}$ is not a fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree.

Conversely, let $\tilde{G}$ be not a fuzzy $\rho_{i_1 i_2 \dots i_r}$-tree. Then it has no partial fuzzy spanning subgraph structure $\tilde{F}$ which is a $\tau_{i_1 i_2 \dots i_r}$-tree.

$\tilde{G}$ is a $\rho_{i_1 i_2 \dots i_r}$-cycle. Let $(\mu, \tau_1, \tau_2, \dots, \tau_k)$be a partial fuzzy spanning subgraph structure of $\tilde{G}$, which is a $\tau_{i_1 i_2 \dots i_r}$-tree with $\rho_{i_p}(u, v) < \tau_{i_\infty}(u, v)$ for all ρ_{i_p}-edge

$(p \in \{1, 2, \dots, r\})$ $(u, v) \notin \tilde{F}$.

$\tau_{i_\infty}(u, v) \leq \rho_{i_p}(u, v)$ $\forall$ ρ_{i_p}-edge (u, v) in $\bigcup_{q=1}^{r} supp(\rho_{i_q})$, $p \in \{1, 2, \dots, r\}$, where $\tau_{i_p}(u, v) = 0$ $\forall$ $p = 1, 2, \dots, r$.

$\tau_{i_p}(x, y) = \rho_{i_p}(x, y)$ $\forall$ $(x, y) \in \bigcup_{q=1}^{r} supp(\rho_{i_q}) - \{(u, v)\}$ for $p \in \{1, 2, \dots, r\}$.

Therefore there does not exist unique *(x, y)* with

$$\vee_{q=1}^{r} \rho_{i_q} (x, y) = \wedge\{\vee_{q=1}^{r} \rho_{i_q}(u, v) : (u, v) \in \text{supp}(\rho_{i_q})\}.$$

Therefore $\tilde{G}$ is a fuzzy $\rho_{i_1 i_2 \ldots i_r}$-cycle.

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