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Solution to the Birch Swinnerton-Dyer Conjecture

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Abstract

The size of the group of rational points is related to the behaviour of an associated L function near the point of s = 1, and affirms the Birch and Swinnerton – Dyer conjecture. The Birch and Swinnerton – Dyer Conjecture asserts that if the L function is equal to zero then there are an infinite number of rational points and equally if the L function is not equal to zero then there are a finite number of rational points.

Keywords – Birch and Swinnerton – Dyer Conjecture, L function.

I. INTRODUCTION

By using a system of two linear equations

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

(Washington 2005, p.139).

The L function equal to zero and one can be determined. The linear equations being:

$x(s^0) + y = 0$ (1)

$x(s^\infty) + y = 0$ (2)

For the L function being equal to zero.

$x(s^0) + y = 0$ (3)

$x(s^\infty) + y = 1$ (4)

For the L function being equal to 1.

Where:

x equals the x plane.

y equals the y plane.

s^0 and s^∞ represents the range of the L function between zero and infinity where s^∞ is s^s in the linear equations, which value depends on the number being determined.

II. METHOD

For rational points where the L function is equal to zero.

Let x equal the rational point.

From equation (1) finding y:

$x(s^0) + y = 0$

$y = 0 - x(s^0)$

$y = - x(s^0)$

From equation (2) finding x.

$x(s^\infty) + y = 0$

$x(s^\infty) - x(s^0) = 0$

$x = \frac{0}{(s^s - s^0)}$

Therefore there are a finite number of rational points. And conversely when the L function is equal to one but not equal to zero.

Let x equal the rational point.

From equation (3) finding y.

$x(s^0) + y = 0$

$y = 0 - x(s^0)$

$y = - x(s^0)$

From equation (4) finding x.

$x(s^\infty) + y = 1$

$x(s^\infty) - x(s^0)$

$x = \frac{1}{(s^s - s^0)}$

Therefore there are an infinite number of solutions.

When the L function is equal to one but not equal to zero, that is when there are an infinite number of solutions, each number is raised to the power of the L function and multiplied by the next. The following series is created.

$$\zeta(s) = \prod_{S=2}^{\infty} \frac{1}{s^s - s^0} = \frac{1}{(2^s - s^0)} \cdot \frac{1}{(3^s - s^0)} \cdot \frac{1}{(5^s - s^0)} \cdot \frac{1}{(7^s - s^0)} \cdot \frac{1}{(11^s - s^0)} \cdots \quad (5)$$

As each multiplication is made the value approaches zero as seen in the following series.

$$\zeta(s) = \prod_{S=2}^{\infty} \frac{1}{s^s - s^0} = \frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)} \cdot \frac{1}{(5^2 - s^0)} \cdot \frac{1}{(7^2 - s^0)} \cdot \frac{1}{(11^2 - s^0)} \cdots$$

Demonstrating that NP = P using a system of two exponential equations derived from two logarithmic equations.

$x = \log_b y$ then $y = b^x$

(Washington 2005, p.373).

Let x equal the following.

$\log P = x \qquad b^x = NP$ (6)

$\log NP = x \qquad b^x = P$ (7)

Substituting x into the exponential equations.

$b^{\log P} = NP$ (8)

$b^{\log NP} = P$ (9)

$\therefore P = NP$

From equation (8).

Let P equal 2

$b^{\log P} = NP$

$10^{\log 2} = NP$

$NP = 2$

From equation (9).

Let NP equal 2.

$b^{\log NP} = P$

$10^{\log 2} = P$

$P = 2$

$\therefore P = NP$

$$\frac{NP}{P} = 1$$

The following equation found by Birch and Swinnerton-Dyer is used to calculate the number of points on an elliptic curve for a number of primes P (Wikipedia 2019, p.1).

$$\prod_{p \le x} \frac{NP}{P} \sim c \cdot (\log x)^r \quad (10)$$

(Bhargava, 2016: Wikipedia 2019, p.1).

Exponential form of equation (10).

$$\frac{\sqrt[r]{(10^{NP}/^{P})}}{c} = x \qquad (11)$$

It can be seen from equation (11) that as r becomes larger x becomes increasingly small for some constant c.

For example:

$y = \log_b x$ (Washington 2005, p.396).

From equations 8 and 9:

$$\frac{NP}{P} = 1$$

Using equation (11).

$$\frac{\sqrt[r]{(10^{NP}/^{P})}}{c} = x \qquad (11)$$

And the series equation (5)

$$\zeta(s) = \prod_{S=2}^{\infty} \frac{1}{s^s - s^0} = \frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)} \cdot \frac{1}{(5^2 - s^0)} \cdot \frac{1}{(7^2 - s^0)} \cdot \frac{1}{(11^2 - s^0)} \cdots \qquad (5)$$

The following example demonstrates r increasing as x becomes smaller and the constant increasing after each multiplication of x in the series is made.

Let r equal 2 and x equal the first point in the series.

$$\frac{\sqrt[r]{(10^{NP}/^{P})}}{c} = x \qquad (11)$$

$$\frac{\sqrt[2]{(10^1)}}{c} = \frac{1}{(2^2 - s^0)}$$

$$\frac{\sqrt[2]{(10^1)}}{\left[\frac{1}{(2^2 - s^0)}\right]} = c$$

c = 9.486832981

Let r equal 3 and x equal the first two points in the series.

$$\frac{\sqrt[r]{(10^{NP}/^{P})}}{c} = x$$

$$\frac{\sqrt[3]{(10^1)}}{c} = \frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)}$$

$$\frac{\sqrt[3]{(10^1)}}{\left[\frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)}\right]} = c$$

c =51.70643339

Let r equal 4 and x equal the first three points in the series.

$$\frac{\sqrt[r]{(10^{NP/P})}}{c} = x$$

$$\frac{\sqrt[4]{(10^1)}}{c} = \frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)} \cdot \frac{1}{(5^2 - s^0)}$$

$$\frac{\sqrt[4]{(10^1)}}{[\frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)} \cdot \frac{1}{(5^2 - s^0)}]} = c$$

c =1024.289006

Therefore as r increases x becomes smaller and the constant changes after each multiplication at an exponential rate.

III. RESULTS

Table I: Change in value of r in equation (11) as the value of x decreases for some constant c.

Value of r	Value of x	Value of constant
2	0.333333333	9.486832981
3	0.041666666	51.70643339
4	0.001736111	1024.289006

When the L function is equal to one but not equal to zero, that is when there are an infinite number of solutions each number is raised to the power of the L function and multiplied by the next. The following series is created.

$$\zeta(s) = \prod_{S=2}^{\infty} \frac{1}{s^s - s^0} = \frac{1}{(2^s - s^0)} \cdot \frac{1}{(3^s - s^0)} \cdot \frac{1}{(5^s - s^0)} \cdot \frac{1}{(7^s - s^0)} \cdot \frac{1}{(11^s - s^0)} \cdots \quad (5)$$

As each multiplication is made the value approaches zero as seen in the series (5).

$$\zeta(s) = \prod_{S=2}^{\infty} \frac{1}{(s^s - s^0)} = \frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^2 - s^0)} \cdot \frac{1}{(5^2 - s^0)} \cdot \frac{1}{(7^2 - s^0)} \cdot \frac{1}{(11^2 - s^0)} \cdots$$

If each prime value is raised to the power of itself then the series approaches zero rapidly as seen in the following series.

$$\zeta(s) = \prod_{S=2}^{\infty} \frac{1}{(s^s - s^0)} = \frac{1}{(2^2 - s^0)} \cdot \frac{1}{(3^3 - s^0)} \cdot \frac{1}{(5^5 - s^0)} \cdot \frac{1}{(7^7 - s^0)} \cdot \frac{1}{(11^{11} - s^0)} \cdots \quad (12)$$

IV. CONCLUSION

By using a system of two linear equations the number of rational points for the L function equal to zero and the L function equal to one can be determined. Therefore the Birch Swinnerton-Dyer Conjecture is refuted as when the L function is equal to zero there are a finite number of solutions and when it is equal to one there are an infinite number of solutions. This is confirmed with the series created when the value of the L function is equal to one. As x approaches infinity hence the series approaches zero, the rank becomes increasingly larger and the constant increases exponentially after each multiplication is made.

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