

Solution to the Riemann Hypothesis

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Abstract

Prime numbers are numbers that are not the product of two smaller numbers and are important in pure and applied mathematics. Their distribution amongst natural numbers does not follow any regular pattern. The German mathematician G.F.B. Riemann found that an elaborate function known as the zeta function is defined in the half plane as $\Re(s) > 1$ and demonstrates the function of the complex variables.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \dots$$

The Riemann hypothesis proclaims that solutions of the equation

$$\zeta(s) = 0$$

Lie on a certain vertical straight line.

Keywords Riemann Hypothesis, Zeta Function, Number theory,

I. INTRODUCTION

By using a system of two linear equations,

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

(Washington 2005, p.139).

Solutions to the equation,

$$\zeta(s) = 0$$

that lie on a vertical line are found (Bombieri n.d, p.1: Clay mathematics Institute 2020).

The system of two linear equations is as follows:

$$x(s^0) + y = 0 \quad (1)$$

$$x(s^\infty) + y = 1 \quad (2)$$

Where:

x represents the x axis.

y represents the y axis.

s represents the prime number.

s^0 and s^∞ represent the range of the zeta function from zero to infinity where s^∞ is represented as s^s in the series equation, which value depends on the prime value being determined.

II. METHOD

For which values of s we have $\zeta(s) = 0$

Solving using a system of two linear equations.

$$x(s^0) + y = 0 \quad (1)$$

$$x(s^\infty) + y = 1 \quad (2)$$

The value of the prime number in the critical strip is determined and added to the infinite series where the zeta function in the critical strip is defined as.

$$\zeta(s) = \sum_{s=2}^{\infty} \frac{1}{(s^s - s^0)} = \frac{1}{(s^s - s^0)} + \frac{1}{(s^s - s^0)} \dots \quad (3)$$

Let s equal the first prime number (2).

From equation (1) finding y.

$$x(s^0) + y = 0$$

$$y = 0 - x(s^0)$$

$$y = -x(s^0)$$

From equation (2) finding x.

$$x(s^\infty) + y = 1$$

$$x(s^\infty) - x(s^0) = 1$$

$$x(2^2) - x(2^0) = 1$$

$$x(4) - x(1) = 1$$

$$4x - x = 1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$x = 0.333333333$$

From the system of two linear equations the value of the primes in the critical strip are determined and added to the series (3) as follows.

$$\zeta(s) = \sum_{s=2}^{\infty} \frac{1}{(s^s - s^0)} = \frac{1}{(2^2-1)} + \frac{1}{(3^2-1)} + \frac{1}{(5^2-1)} \dots$$

Demonstrating that the x plane and the y plane are the same in relation to the zeta function

Demonstrating that the real number plane x is the same as the complex number plane y using multiplication of matrices where the zeta function is used as the identity matrix with 1's for elements of the principal diagonal with other elements zero (Washington 2005, p.442).

$$xy = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} xys &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{matrix} (1)(1) + (0)(1) & (1)(1) + (0)(1) \\ (0)(1) + (1)(1) & (0)(1) + (1)(1) \end{matrix} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} sxy &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ &= \begin{matrix} (1)(1) + (0)(1) & (1)(1) + (0)(1) \\ (0)(1) + (1)(1) & (0)(1) + (1)(1) \end{matrix} \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{aligned}$$

Therefore

$$xys = sxy = xy$$

This indicates that both the x and y planes are the same in relation to the zeta function.

III. RESULTS

As each prime numbers value is determined from the equations it is added to the series.

$$\zeta(s) = \sum_{s=2}^{\infty} \frac{1}{(s^s - s^0)} = \frac{1}{(2^2-1)} + \frac{1}{(3^2-1)} + \frac{1}{(5^2-1)} \dots$$

It can be seen that the first point lies at, $\frac{1}{(2^2-1)}$

Then the following addition the point lies at, $\frac{1}{(2^2-1)} + \frac{1}{(3^2-1)}$

With the next addition the point lies at, $\frac{1}{(2^2-1)} + \frac{1}{(3^2-1)} + \frac{1}{(5^2-1)}$

Then further additions the point lies at, $\frac{1}{(2^2-1)} + \frac{1}{(3^2-1)} + \frac{1}{(5^2-1)} + \frac{1}{(7^2-1)}$

IV. CONCLUSION

A solution to the equation

$$\zeta(s) = 0$$

That lies on a vertical line in the critical strip are found by representing the zeta function as

$$\zeta(s) = \sum_{s=2}^{\infty} \frac{1}{(s^s - s^0)}$$

The equation allows the value of the primes in the critical strip to fall at the origin of the line 0.54 except for the first two points. Therefore to determine the values in the critical strip the zeta function must begin at $s = 2$. The series formula clearly demonstrates the divergence properties of the zeta function in the critical strip.

REFERENCES

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