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Feigenbaum Universality and Control of Chaos in a Nonlinear Chaotic Model

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Abstract

In this paper, we consider a nonlinear chaotic model, $f(x) = ax^3+bx^2 +cx +d$, where a,b,c,d are parameters to be suitably chosen ; and discuss the following results:

(i) Mitchell J Feigenbaum , an American theoretical Physicist has established a close link between the regular behaviour and chaotic behaviour by a universal route of bifurcations values in a nonlinear system . In our chaotic model, we establish this result by showing the sequence of bifurcation values approaches a strange attractor.

(ii) We have shown that there is a chaotic region at certain values of the parameter, and beyond this , there appears regular behaviour again .

(iii) We have discussed Periodic Proportional Pulse method for controlling unstable periodic orbits in strange attractors in one dimensional nonlinear cubic map. We have successfully obtained the desired results.

(iv) Some open problems are paused.

Keywords: Chaos, Strange attractors, bifurcation values, Periodic Proportional Pulse method

AMS Subject Classification: Primary 37B10, 37D45; Secondary: 37G15

1. INTRODUCTION

In the last two decades, controlling of chaos has become on focus in the nonlinear problems ranging from physics and chemistry to biology and economics. Recently, there has been increasing interest in the research area of controlling chaotic dynamical systems. There are many practical reasons for controlling chaos. First of all, chaotic system response with little meaningful information content is unlikely to be useful. Secondly, chaos can lead systems to harmful situations, therefore chaos should be reduced as much as possible. Chaos is observed as undesirable part in engineering control practice. So, controlling of chaos is an essential part of study of chaos. The idea of “controlling chaos” was first suggested in a famous paper by Ott, Grebogi and Yorke in1990 and known as OGY method . After that many techniques for controlling chaos have been proposed in these decades. The proportional pulse method was introduced by Matias and Guemez . After that N.P.Chau discussed in a similar manner but gave some restrictions on the initial conditions by which chaos can be controlled. We have taken the method of periodic proportional pulse and OGY method to control chaos .

1.1 Control of chaos in one dimensional map:

In this paper we discuss the periodic proportional pulse method to control unstable periodic orbits in strange attractors by considering a one dimensional nonlinear chaotic map. $x_{n+1} = 0.35x_n - 2.75x_n + cx_n$

where c is the control parameter whose value lies in the interval [2, 6.34] , $x \in [0,7.857]$, a=0.35, b= -2.75 and d=0. Our main aim is to study the control of chaos on the above model by using the technique of periodic proportional pulses.

1.2 Control of chaos in a cubic map.

Matias and Guemez has discussed a significant method for controlling chaos by applying instantaneous pulses on the variables x_i of a chaotic dynamical system, once every ***r*** iterations in the form

$$x_i = \eta x_i \text{ (}i\text{ is a multiple of }r\text{)} \qquad (1.1)$$

where η is a constant to be determined and ***r*** denotes the period. Chua has proposed a technique of controlling chaos in more details in discrete dynamics by considering one dimensional chaotic dynamics of the form

$$x_{n+1} = \psi(x_n) \qquad (1.2)$$

where $\psi : I \to I$, I is an interval. The critical point of (2.2) is the solution x_s of the equation

$$x_s = \phi(x_s) \qquad (1.3)$$

and the point x_s is stable if

$$\left| \frac{\mathrm{d}\psi(\mathrm{x})}{\mathrm{dx}} \right|_{x=x_s} < 1$$

Let us consider a composite function

$$h(x) = \eta \psi_c^r(x) \qquad (1.4)$$

where the dynamics of the system is kicked by multiplying its value by a factor η once after every ***r*** iteration.

As stated above a fixed point of ***h(x)*** is any solution x_s of

$$h(x) = \eta \psi_c^r(x_s) = x_s \qquad (1.5)$$

and the fixed point is locally stable if

$$\left| \eta \frac{d\psi^r(x)}{dx} \right| < 1 \qquad (1.6)$$

It is observed that a stable fixed point of ***h(x)*** can be viewed as a stable periodic point of period ***r*** of the original dynamics ψ, kicked by the control parameter. We assume that the original map ψ is chaotic and has to be controlled so as to obtain stable periodic orbits of period ***r***, by kicking once every ***r*** iteration following equation (1.1).

To obtain a suitable point x_s and a factor η satisfying equation (1.5) and (1.6) , another function $C_r(x)$ is defined as

$$C_r(x) = \frac{x}{\psi^r(x)} \frac{d\psi^r(x)}{dx} \qquad (1.7)$$

Substituting from (1.5), equation (1.6) becomes

$$|C_r(x)| < 1$$

Now if a value of the variable *x* is picked in such a way that $|C_r(x)| < 1$ and obtain a suitable value of η using equation (1.4) such that *x* becomes a stable critical point of ***h(x)***. ***h(x)*** gives a stable periodic point of period ***r*** if the value of ***h(x)*** is inserted after ***r*** iterations of ψ. It is important to observe that, if the impulse of η is too strong , it may kick the dynamics out of the basin of attraction, and in that case, the orbit will escape to infinity. While using pulse control, this precaution has to be kept in mind.

2. THE MAIN RESULTS: PERIODIC PROPORTIONAL PULSES ON OUR MODEL

We consider a nonlinear cubic map of the form

$$x_{n+1} = 0.35x_n - 2.75x_n + cx_n \qquad (2.1)$$

with ***c*** as the control parameter. It is observed that the map undergoes period doubling route to chaos, and the following results can be obtained by using MATHEMATICA software.

Table: Bifurcation values and Feigenbaum constant ‘delta’

Bifurcation Points	Feigenbaum delta
C_1=3.326	
C_2=4.086…..	
C_3=4.286…..	δ_1 =3.8
C_4=4.3257	δ_2 =5.03778
C_5=4.3367	δ_3 =3.60909
C_6=4.33875	δ_4 =5.36585
C_7=4.3391890475115	δ_5 =4.6691985399
C_8=4.3928307810543	δ_6 =4.6691985400
C_9=4.3393032165923	δ_7 =4.66919853953
C_{10}=4.339075296424	δ_8=4.669185405152
C_{11}=4.339308453366	δ_9=4.669185254980
C_{12}=4.339308651199	δ_{10}=4.66919864715
C_{13}=4.339308693569	δ_{11}=4.66918931282

The period doubling cascade accumulates at the accumulation point ***c***=4.339308….. beyond which chaos occurs. A bifurcation graph of the cubic map is given below reveals the chaotic behavior after the accumulation point. So for the parameter c=4.99 the system is chaotic.

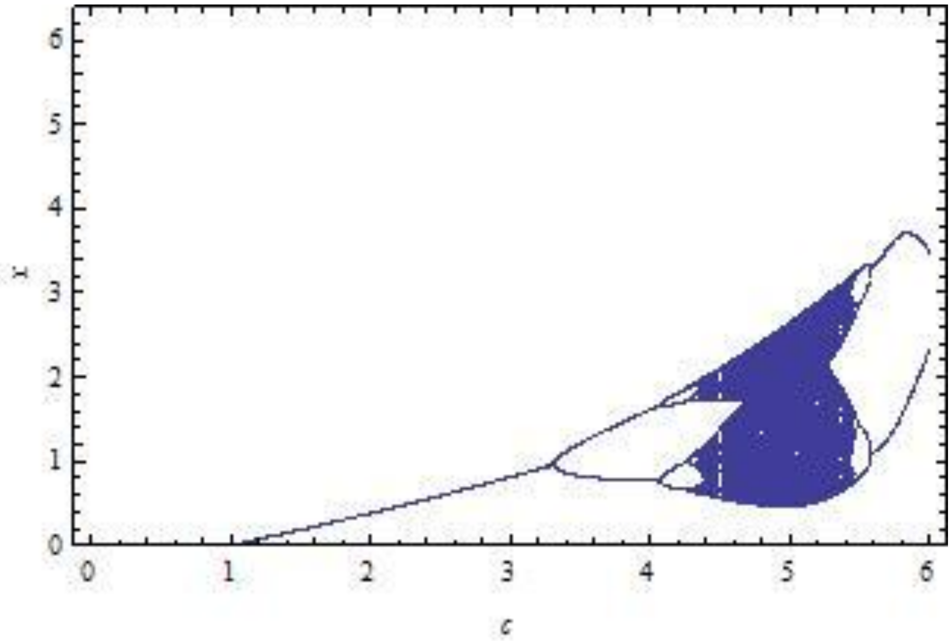

Fig. 2.1(a)- Bifurcation graph of the map, x-axis represents the value of the control parameter & y-axis represents f(x)

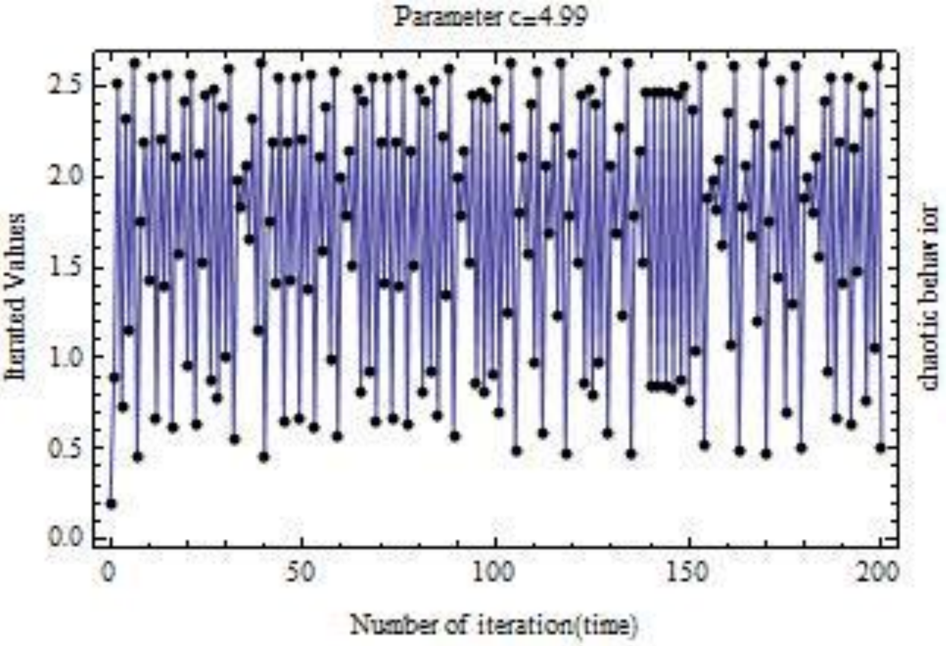

Fig:2.1(b) Time series plot showing chaotic behavior, the control parameter c=4.99, initial point x_0=0.9, x-axis shows number of iterations and y-axis shows iterated values.

Next, let us discuss the control of chaos:

For r=1 and c=4.99, the control curve $C_1(x)$ is drawn in the following figures:

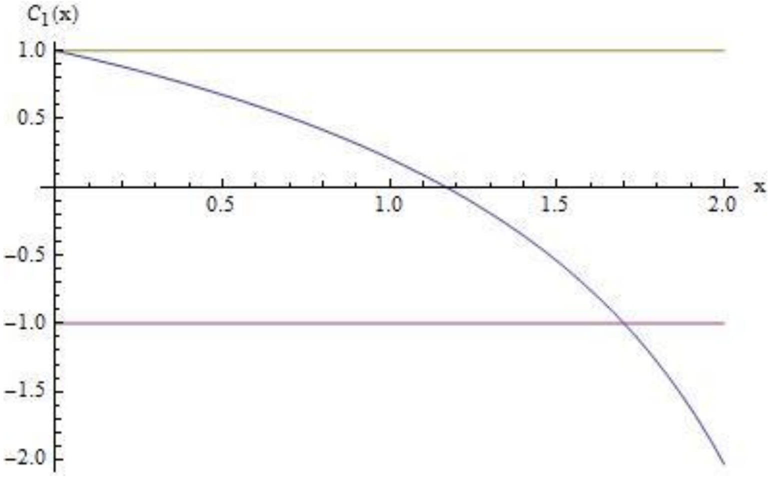

Fig:2.2(a) Control curve for parameter c=4.99, r=1 and x_0=0.9

The value of x is considered such that the range is $-1 < C_r(x) < 1$, r =1 and the value of x in this interval can stabilize orbits of period 1(critical points of period one) at every point x_s in the range about (0,1.70029). By solving the equation $C_r(x) = 1, r = 1$ and $C_r = -1, r = 1$ we obtain this range. Now taking x=1.2 in the range stated above the value of the kicking factor $\eta = 0.455789$ for which x becomes a stable fixed point of h(x). Now by applying the value of η at every iteration in our map the trajectory attains the stable fixed point.

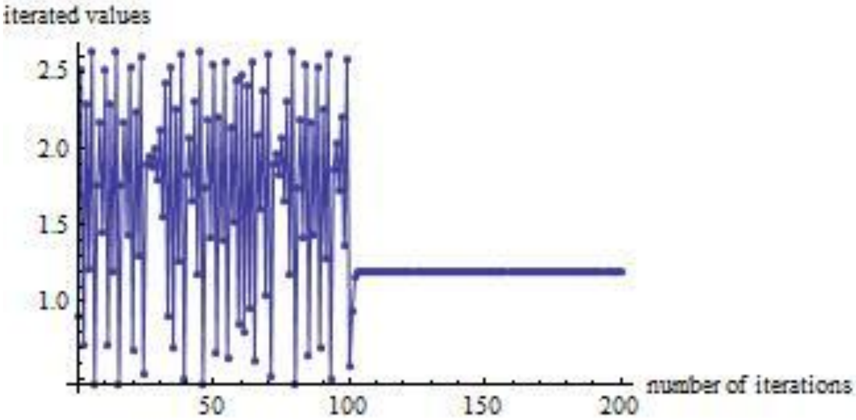

Fig 2.2(b) Time series plot showing chaotic behavior and c=4.99

Again for r=2 and c= 4.99 the control curve of $C_2(x)$ is drawn in the figure below.

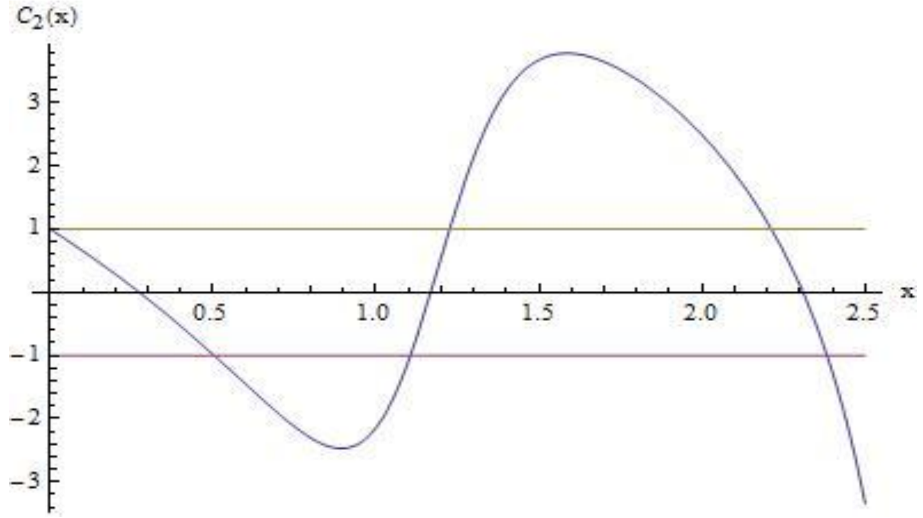

Fig:2 .3(a) Control curve for parameter c=4.99, r=2 and x_0=0.9

In this case also the range is restricted to $-1 < C_r(x) < 1$, r=2 and it is observed that we can stabilize orbits of period two at point x_s only in the range of (0, 0.504248). Now taking x=0.4811 in the range stated above the value of the kicking factor is obtained as $\eta = 0.228179$ for which x_s gives two stable fixed point of h(x)

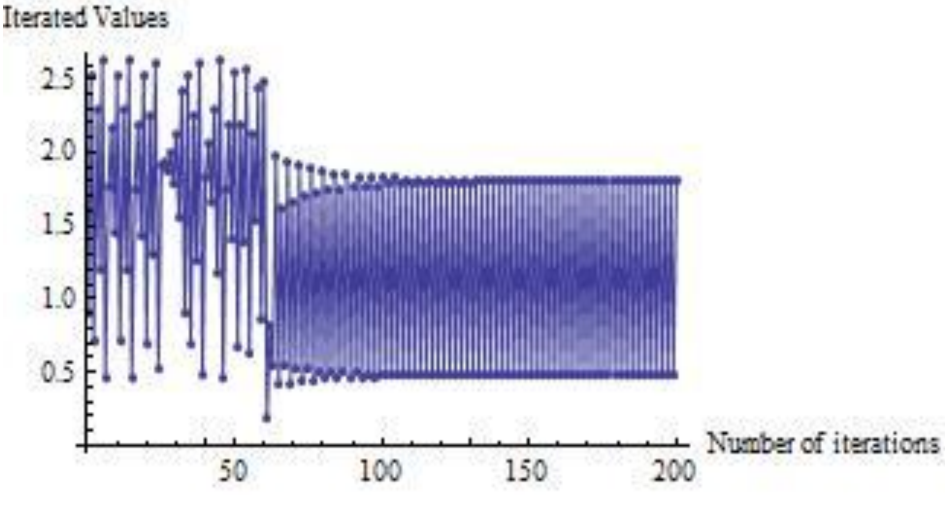

Fig 2.3(b) Time series plot showing chaotic behavior and c=4.99

Similarly for r=3,4 and c=4.99 the control curves $C_3(x)$ and $C_4(x)$ is drawn in the figure given below.

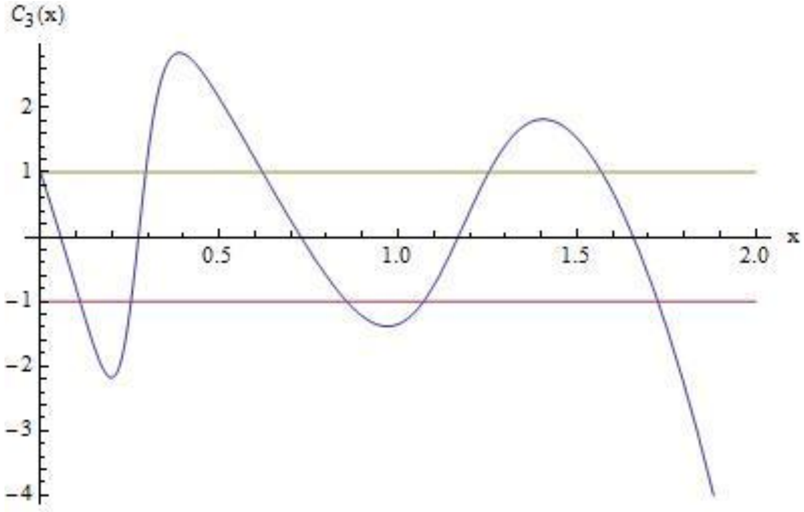

Fig:2.3(a) Control curve for parameter c=4.99, r=3 and x_0=0.9

By solving the equation $C_r(x)=1, r=3$ and $C_r=-1, r=3$ we obtain the range as [0.294089, 0.620965]. Now taking x=0.256323 in the range stated above the value of the kicking factor $\eta=0.541713$ for which *x* becomes a stable fixed point of *h(x)*. Now by applying the value of η at every iteration in our map the trajectory attains three stable fixed point.

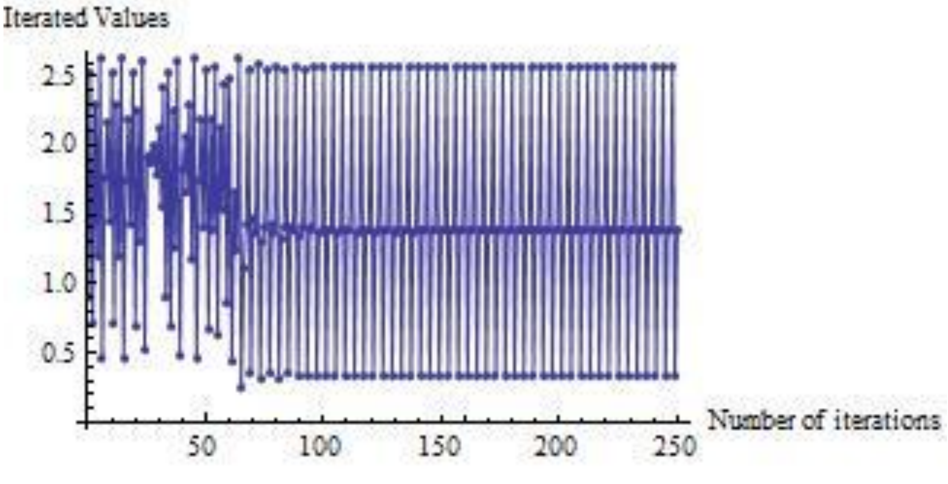

Fig-3.3(b) Time series plot showing chaotic behavior and c=4.99

For r=4 and c=4.99, the control curve $C_4(x)$ is drawn in the figure below.

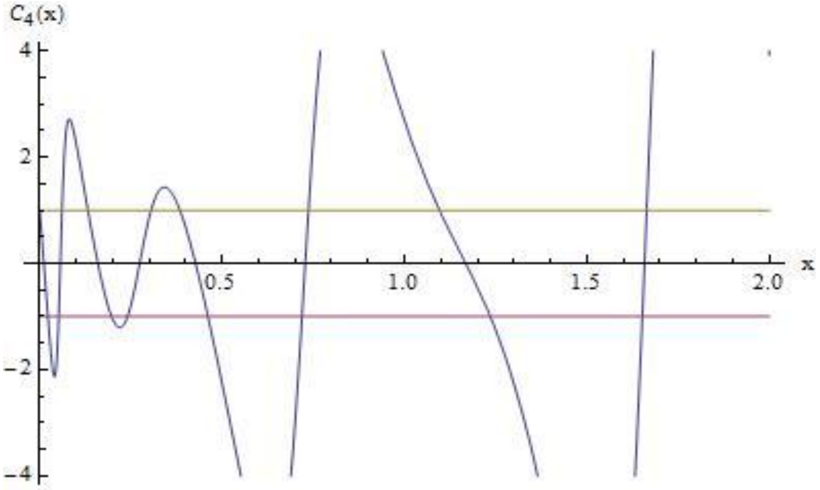

Fig:2.4(a) Control curve for parameter c=4.99, r=3 and x_0=0.9

By solving the equation $C_r(x)=1, r=4$ and $C_r=-1, r=4$ we obtain the range [0.240565,04614]. Now taking x=0.52867514 in the range stated above the value of the kicking factor $\eta=0.455789$ for which x becomes a stable fixed point of h(x). Now by applying the value of η at every iteration in our map the trajectory attains the stable fixed point.

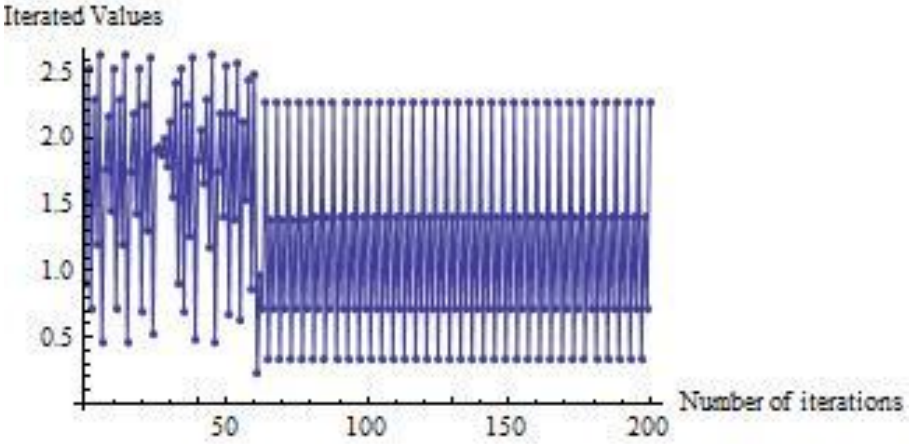

Fig 2.4(b) Time series plot showing chaotic behavior and c=4.99

In a similar way the control process stabilizes the dynamics of period 5 , 6 and so on

CONCLUSION

It is observed that an irregular orbit of any period can be controlled. It is seen that the control ranges are getting smaller as the periodicity increases. The system can be stabilized to many different points on and even off the chaotic attractors.

PROBLEMS: We can raise the following problems :

(i) Can we apply the same technique to control chaos in any other chaotic model ?
(ii) We fix the parameter values in our model as a=0.35 , b= -2.75 , and d=0.0

For any other parameter values, can we study the same results ?

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