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The New Integral Transform ''Sawi Transform''

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Abstract

In this paper a new integral transform namely Sawi transform was applied to solve linear ordinary differential equations with constant coefficients.

Keywords: Sawi transform- Differential Equations.

1. INTRODUCTION

Sawi Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the Sawi transform and its fundamental properties. Sawi transform was introduced by Mohand Mahgoub to facilitate the process of solving ordinary and partial differential equations in the time domain. Typically, Fourier, Laplace , Sumudu, Elzaki , Aboodh , kamal , and Mohand transforms are the convenient mathematical tools for solving differential equations,

Also Sawi transform and some of its fundamental properties are used to solve differential equations.

A new transform called the Sawi transform defined for function of exponential order we consider functions in the set A defined by:

$$A = \left\{ f(t) \colon \exists M, k_1, k_2 > 0 \,.\, |f(t)| < Me^{\frac{|t|}{k_j}} \,, if \;\; t \in (-1)^j \times [0, \infty) \right\} \tag{1}$$

For a given function in the set A, the constant M must be finite number, k_1 , k_2 may be finite or infinite.

Sawi transform denoted by the operator S (.) defined by the integral equations

$$S[f(t)] = R(v) = \frac{1}{v^2} \int_0^\infty f(t) e^{-\frac{t}{v}} dt \;. \quad t \geq 0 \quad , \;\; k_1 \leq v \leq k_2 \tag{2}$$

The variable v in this transform is used to factor the variable t in the argument of the function f this transform has deeper Connection with the Laplace ,Elzaki , sumudu , kamal, and Mohand transform..The purpose of this study is to show the applicability of this interesting new transform and its efficiency in solving the linear differential equations.

2. SAWI TRANSFORM OF THE SOME FUNCTIONS

For any function $f(t)$, we assume that the integral Eq. (2) exist. The sufficient conditions for the existence of Sawi transform are that $f(t)$ for $t \geq 0$ be piecewise continuous and of exponential order, otherwise Sawi transform may or may not exist.

In this section we find Sawi transform of simple functions.

(i) let $f(t) = 1$, then , by the definition we have:

$$S[1] = R(v) = \frac{1}{v^2}\int_0^\infty 1.e^{-\frac{t}{v}}dt = \frac{1}{v^2}\int_0^\infty e^{-\frac{t}{v}}dt = \frac{1}{v^2}[-ve^{-t}]_0^\infty = \frac{1}{v}$$

(ii) let $f(t) = t$, then:

$$S[t] = \frac{1}{v^2}\int_0^\infty te^{-\frac{t}{v}}dt$$

Integrating by parts, we get $.S\,[t] = 1$

In the general case if $n = 0,1,2,\ldots$ is integer number, then.

$$S[t^n] = v^{n-1}n!$$

(iii) $S[e^{at}] = \frac{1}{v^2}\int_0^\infty e^{at}e^{-\frac{t}{v}}dt = \frac{1}{v(1-av)}$

This result will be useful, to find Sawi transform of:

$$S[\sin at] = \frac{a}{1+a^2v^2} \quad , \quad S[\cos at] = \frac{1}{v(1+a^2v^2)}$$

$$S[\sinh at] = \frac{a}{1-a^2v^2} \quad , \quad S[\cosh at] = \frac{1}{v(1-a^2v^2)}$$

Theorem 2.1:

Let $R[v]$ Sawi transform of $[S[f(t)] = R[v]]$ then:

(i) $S[f'(t)] = \frac{R(v)}{v} - \frac{f(0)}{v^2}$

(ii) $S[f''(t)] = \frac{1}{v^2}R(v) - \frac{1}{v^2}f'(0) - \frac{1}{v^3}f(0)$

(iii) $S\left[f^{(n)}(t)\right] = \frac{R(v)}{v^{(n)}} - \sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{v^{n-k+1}}$

Proof:

(i) By the definition we have:

$$S[f'(t)] = \frac{1}{v^2}\int_0^\infty f'(t)e^{-\frac{t}{v}}dt \quad ,$$

Integrating by parts, we get

$$S[f'(t)] = \frac{R(v)}{v} - \frac{f(0)}{v^2}$$

(ii) let g(t) = f′(t) , then

$$S[g'(t)] = \frac{R(v)}{v} - \frac{f(0)}{v^2}$$

we find that by using (i),

$$S[f''(t)] = \frac{1}{v^2}R(v) - \frac{1}{v^2}f'(0) - \frac{1}{v^3}f(0)$$

(iii) Can be proof by mathematical induction.

3. APPLICATION OF SAWI TRANSFORM OF ORDINARY DIFFERENTIAL EQUATIONS.

As stated in the introduction of this paper, Sawi transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of Sawi transform in solving certain initial value problems described by ordinary differential equations.

Consider the first-order ordinary differential equation:

$$\frac{dx}{dt} + p(x) = f(t) \quad , \quad t > 0 \tag{3}$$

With the initial condition

$$x(0) = a \tag{4}$$

Where *p* and *a* are constants and f(t)is an external input function so that its Sawi transform exists.

Applying Sawi transform of the Eq. (3) we have:

$$\frac{R(v)}{v} - \frac{x(0)}{v^2} + pR(v) = \bar{f}(v)$$

$$R(v) = \frac{v^2\bar{f}(v) + \mathrm{a}}{v(1+pv)}$$

The inverse Sawi transform leads to the solution.

Consider the second order linear ordinary differential equation has the general form:

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x) \quad , \; x > 0 \tag{5}$$

The initial conditions are

$$y(0) = a \quad , \quad y'(0) = b \tag{6}$$

where p, q, a and b are constants. Application of Sawi transforms to this general initial value problem gives

$$\frac{1}{v^2}R(v) - \frac{1}{v^2}f'(0) - \frac{1}{v^3}f(0) + 2p\left[\frac{R(v)}{v} - \frac{f(0)}{v}\right] + qR(v) = \bar{f}(v)$$

The use of Eq.(6) leads to the Solution for $R(v)$ as

$$R(v) = \frac{v^2\bar{\mathrm{f}}(\mathrm{v})}{1+2pv+qv^2} + \frac{a(1+2pv^2)}{v(1+2pv+qv^2)} + \frac{\mathrm{b}}{1+2pv+qv^2}$$

The inverse Sawi transform gives the solution.

Example 3.1:

Consider the first order differential equation

$$\frac{dy}{dx} + y = 0 \quad , \quad y(0) = 1 \tag{7}$$

Applying the Sawi transform to both sides of this equation and using the differential property of Sawi transform , Eq.(7) can be written as:

$$\frac{R(v)}{v} - \frac{y(0)}{v^2} + R(v) = 0$$

Where $R(v)$ is the Sawi transform of the function $y(x)$

Applying the initial condition , we get

$$\left(\frac{1}{v} + 1\right)R(v) = \frac{1}{v^2} \text{ and } R(v) = \frac{1}{v(1+v)}$$

Now applying the inverse Sawi transform , we get *:* $y(x) = e^{-x}$

Example 3.2:

Solve the differential equation

$$\frac{dy}{dx} + 2y = x \quad , y(0) = 1 \tag{8}$$

Applying Sawi transform to both sides of Eq.(8) and using the differential property of Sawi transform , Eq.(8) can be written as :

$$\frac{R(v)}{v} - \frac{y(0)}{v^2} + 2R(v) = 1$$

$$\left(\frac{1}{v} + 2\right)R(v) = 1 + \frac{1}{v^2}$$

$$R(v) = \frac{1+v^2}{v+v^2} = \frac{1}{2} + \frac{5}{4}\frac{1}{v+2v^2} - \frac{1}{4v}$$

The inverse Sawi transform of this equation gives the solution:

$$y(x) = \frac{1}{2}x + \frac{5}{4}e^{-2x} - \frac{1}{4}$$

Example 3.3:

Let us consider the second–order differential equation

$$y'' + y = 0 \quad , \quad y(0) = y'(0) = 1 \qquad (9)$$

Applying Sawi transform to both sides of Eq.(9) and using the differential property of Sawi transform , Eq.(9) can be written as :

$$\frac{1}{v^2}R(v) - \frac{1}{v^2}y'(0) - \frac{1}{v^3}y(0) + R(v) = 0$$

Applying the initial condition , we get

$$\left(\frac{1}{v^2} + 1\right)R(v) = \frac{1}{v^3} + \frac{1}{v^2}$$

$$\Rightarrow \quad R(v) = \frac{v+1}{v(1+v^2)} = \frac{1}{1+v^2} + \frac{1}{v(1+v^2)}$$

The inverse Sawi transform of this equation is simply obtained as

$$y(x) = \sin x + \cos x$$

Example 3.4:

Consider the following equation

$$y'' - 3y' + 2y = 0 \quad , \quad y(0) = 1 \quad , \quad y'(0) = 4 \qquad (10)$$

Take Sawi transform of Eq.(10), we find that:

$$\left[\frac{1}{v^2}R(v) - \frac{1}{v^2}y'(0) - \frac{1}{v^3}y(0)\right] - 3\left[\frac{R(v)}{v} - \frac{y(0)}{v^2}\right] + 2R(v) = 0$$

Applying the initial condition , we get

$$\left(\frac{1}{v^2} - \frac{3}{v} + 2\right)R(v) - \frac{4}{v^2} - \frac{1}{v^3} + \frac{3}{v^2} = 0$$

$$\left(\frac{1}{v^2} - \frac{3}{v} + 2\right)R(v) = \frac{1}{v^3} + \frac{1}{v^2}$$

$$R(v) = \frac{v+1}{v(1-3v+2v^2)} = \frac{v+1}{v(1-2v)(1-v)} = \frac{3}{v(1-2v)} - \frac{2}{v(1-v)}$$

Then the solution is $y(x) = 3e^{2x} - 2e^{x}$

Example 3.5:

Let the second order differential equation**:**

$$\mathrm{y}'' + 9\mathrm{y} = \cos 2t \quad , \quad \mathrm{y}(0) = 1 \quad , \quad \mathrm{y}\left(\frac{\pi}{2}\right) = -1 \qquad (11)$$

Since $\mathrm{y}'(0)$ is not known, let $\mathrm{y}'(0) = \mathrm{c}$.

Take Sawi transform of this equation and using the conditions, we have

$$\frac{1}{v^2}R(v) - \frac{1}{v^2}y'(0) - \frac{1}{v^3}y(0) + 9R(v) = \frac{1}{v(1+4v^2)}$$

$$\left(\frac{1}{v^2}+9\right)R(v)-c\frac{1}{v^2}-\frac{1}{v^3}=\frac{1}{v(1+4v^2)}$$

$$R(v)=\frac{v}{(1+4v^2)(1+9v^2)}+\frac{c}{1+9v^2}+\frac{1}{v(1+9v^2)}$$

We can write this equation in the form,

$$R(v)=\frac{4}{5v(1+9v^2)}+\frac{c}{(1+9v^2)}+\frac{1}{5v(1+4v^2)}$$

And invert to find the solution.

$$y(t)=\frac{4}{5}\cos 3t+\frac{c}{3}\sin 3t+\frac{1}{5}\cos 2t$$

To determine c note that $y\left(\frac{\pi}{2}\right)=-1$ thin we find $c=\frac{12}{5}$ then**,**

$$y(t)=\frac{4}{5}\cos 3t+\frac{4}{5}\sin 3t+\frac{1}{5}\cos 2t$$

Example 3.6:

Solve the differential equation**:**

$$y''-3y'+2y=4e^{3t}\quad,\quad y(0)=-3\quad,\quad y'(0)=5 \tag{12}$$

Taking Sawi transform both side of the differential Eq.(12) and using the given conditions we have,

$$\left[\frac{1}{v^2}R(v)-\frac{1}{v^2}y'(0)-\frac{1}{v^3}y(0)\right]-3\left[\frac{R(v)}{v}-\frac{y(0)}{v^2}\right]+2R(v)=\frac{4}{v(1-3v)}$$

$$\left(\frac{1}{v^2}-3\frac{1}{v}+2\right)R(v)-5\frac{1}{v^2}+3\frac{1}{v^3}-9\frac{1}{v^2}=\frac{4}{v(1-3v)}$$

$$R(v)=\frac{4v}{(1-3v)(1-v)(1-2v)}+\frac{14}{(1-v)(1-2v)}-\frac{3}{v(1-v)(1-2v)}$$

Or

$$R(v)=\frac{4}{v(1-2v)}+\frac{2}{v(1-3v)}-\frac{9}{v(1-v)}$$

Inverting to find the solution in the form.

$$y(t)=4e^{2t}+2e^{3t}-9e^{t}$$

Example 3.7:

Find the solution of the following initial value problem:

$$y''+4y=12t\quad,\quad y(0)=0\quad,\quad y'(0)=7 \tag{13}$$

Applying Sawi transform of this problem and using the given constants we get,

$$\frac{1}{v^2}R(v)-\frac{1}{v^2}y'(0)-\frac{1}{v^3}y(0)+4R(v)=12$$

$$\left(\frac{1}{v^2}+4\right)R(v)-\frac{7}{v^2}=12$$

$$\left(\frac{1}{v^2} + 4\right) R(v) = 12 + \frac{7}{v^2}$$

$$R(v) = \frac{12v^2 + 7}{(1 + 4v^2)} = 3 + \frac{4}{1 + 4v^2}$$

Inverting to find the solution in the form

$$y(t) = 3t + 2\,sin\,2t$$

4. CONCLUSION

The definition and application of the new transform " Sawi transform" to the solution of ordinary differential equations has been demonstrated.

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