Advances in Theoretical and Applied Mathematics
ISSN 0973-4554 Volume 12, Number 1 (2017), pp. 7-13

http://www.ripublication.com

The Use of Kamal Transform for Solving Partial Differential Equations

[1,2]Abdelilah. K. Hassan Sedeeg and [3,4] Zahra. I. Adam Mahamoud

*[1]Mathematics Department Faculty of Education,
Holy Quran and Islamic Sciences University, Sudan.*

*[2]Mathematics Department Faculty of Sciences and Arts,
Almikwah, Albaha University, Saudi Arabia.*

*[3]Mathematics Department Faculty of Sciences and Arts,
Bukirayh, Alqassim University, Saudi Arabia.*

*[4]Mathematics Department Faculty of Education,
Omdurman Islamic University, Sudan.*

Abstract

Kamal Transform of partial derivative is derived, and its applicability demonstrated using four different partial equations. In this paper we find the particular solutions of these equations.

1. INTRODUCTION

The differential equation have played a central role in every aspect of applied mathematics for every long time and with the advent of the computer, their importance has increased father. Thus investigation and analysis of differential equations cruising in applications led to many deep mathematical problems; therefore, there are so many different techniques in order to solve differential equations. The integral transform were extensively used and thus there are several words on the theory and applications of integral transforms such as the Laplace, Foureir, Mellin, Hankel and Sumudu, to name but a few. Recently, Abdelilah Kamal introduced a new integral transform, named the Kamal transform, and further applied it to the solution of ordinary and partial differential equations.

In this paper we drive the formulate for Kamal transform of partial derivatives and apply them in solving some types of initial value problems. Our purpose here is to show the applicability of this interesting new transform and its effecting in solving such problems.

2. DEFINITION AND DERIVATIONS KAMAL TRANSFORM OF DERIVATIVES:

Kamal transform of the function $f\ (t)$ is defined as

$$[f(t)] = \int_0^\infty f(t)e^{\frac{-t}{v}}dt = \ G(v)\ ,\ t > 0\ ,\ k_1 \leq v \leq k_2 \tag{1}$$

To obtain Kamal transform of partial derivatives we use integration by parts as follows:

$$K\left[\frac{\partial f(x,t)}{\partial t}\right] = \int_0^\infty \frac{\partial f(x,t)}{\partial t} e^{\frac{-t}{v}}dt = \lim_{p\to\infty} \int_0^p e^{\frac{-t}{v}} \frac{\partial f(x,t)}{\partial t} dt$$

$$= \lim_{p\to\infty} \left(\left[e^{\frac{-t}{v}} f(x,t) \right]_0^p + \frac{1}{v} \int_0^p e^{\frac{-t}{v}} f(x,t)dt \right)$$

$$= -f(x,0) + \frac{1}{v}G(x,v)$$

Thus, $$K\left[\frac{\partial f(x,t)}{\partial t}\right] = v^{-1}G(x,v) - f(x,0) \tag{2}$$

To find $K\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right]$, let $\frac{\partial f(x,t)}{\partial t} = g(x,t)$, then by using Eq. (2) we have:

$$K\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = K\left[\frac{\partial g(x,t)}{\partial t}\right] = v^{-1}K\left[\frac{\partial g(x,t)}{\partial t}\right] - g(x,0)$$

$$K\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = v^{-2}G(x,v) - v^{-1}f(x,0) - \frac{\partial f(x,0)}{\partial t} \tag{3}$$

We can easily extend this result to the nth partial derivative by using mathematical induction.

Now, we assume the $f(x,t)$ is piecewise continuous and is of exponential order. Then,

$$K\left[\frac{\partial f(x,t)}{\partial x}\right] = \int_0^\infty e^{\frac{-t}{v}} \frac{\partial f(x,t)}{\partial x} dt$$

Using the Leibniz' rule $K\left[\frac{\partial f(x,t)}{\partial x}\right] = \int_0^\infty e^{\frac{-t}{v}} \frac{\partial f(x,t)}{\partial x} dt = \frac{\partial}{\partial x}\int_0^\infty e^{\frac{-t}{v}} f(x,t) dt$

Thus, $K\left[\frac{\partial f(x,t)}{\partial x}\right] = \frac{d}{dx}\big(G(x,v)\big)$ (4)

Also we can find : $K\left[\frac{\partial^2 f(x,t)}{\partial x^2}\right] = \frac{d^2}{dx^2}\big(G(x,v)\big)$ (5)

In summary, $K\left[\frac{\partial^n f(x,t)}{\partial x^n}\right] = \frac{d^n}{dx^n}\big(G(x,v)\big)$ (6)

3. APPLICATION OF KAMAL TRANSFORM OF PARTIAL DIFFERENTIAL EQUATIONS:

In this section we solve first order Partial differential Equations and the Second order partial differential equation, wave equation, heat equation, Laplace equation and Telegraphers equation which are known as four Fundamental equations in mathematical physics and occur in many branches of physics, in applied mathematics as well as in engineering.

Example 3. 1:

Find the solution of the first - order initial value problem:

$$u_x(x,t) - 2u_t(x,t) = u(x,t)\ ,\ \ x > 0, t > 0,\ \ u(x,0) = e^{-3x} \quad (7)$$

Taking Kamal transform of Eq. (7), we have

$$G'(x,v) - 2v^{-1}G(x,v) + 2u(x,0) = G(x,v)$$

Where $G(x,v)$ is Kamal transform of $u(x,t)$.

By applying the initial condition, we get

$$G'(x,v) - \left(\frac{2}{v} + 1\right) G(x,v) = -2e^{-3x}$$

This is the linear ordinary differential equation, it has the integration factor

$$F = e^{-\int\left(\frac{2}{v}+1\right)dx} = e^{-\left(\frac{2}{v}+1\right)x}$$

Therefore, $G(x,v) = \frac{v}{1+2v} e^{-3x} + ce^{\left(\frac{2}{v}+1\right)x}$ (8)

Since $G(x,v)$ is bounded, c Should be zero, if we take the inverse Kamal transform to Eq. (8), then the solution of Eq. (7) is:

$$u\,(x,t\,) = e^{-3x-2t}$$

Example 3. 2:

Let's consider the wave equation :

$$u_{tt} - u_{xx} = 0\,,\;\; 0 \le x \le \pi\,,\;\; t \;\ge 0 \qquad (9)$$

$$u(x,0) = \sin x, u(0,t) = 0\,,\quad u_t(x,0) = 0\,,\quad u(\pi,t) = 0$$

Applying Kamal transform to both sides of this equation and using the differential property of Kamal transform, Eq. (9) can be written as:

$$v^{-2}G(x,v) - v^{-1}u(x,0) - u_t(x,0) - G''(x,v) = 0$$

And, $$G''(x,v) - \frac{1}{v^2}G(x,v) = \frac{-\sin x}{v}$$

This is the second - order ordinary differential equation have the solution in the form :

$$G(x,v) = \frac{-sinx}{v\left(D^2-\frac{1}{v^2}\right)} = \frac{-sinx}{v\left(-1-\frac{1}{v^2}\right)} = \frac{v\;sinx}{1+v^2} \qquad (10)$$

Now, if we take the inverse Kamal transform to Eq. (10), then the particular solution of Eq. (9) is:

$$u(x,t) = \; sinx\;cost$$

Example 3. 3:

Let's consider the homogeneous heat equation in one dimension in a normalized form:

$$u_t = u_{xx}\;,\;\; (x,0) = \sin\frac{\pi}{l}x\;,\;\; u(0,t) = u(l,t) = 0 \qquad (11)$$

Applying Kamal transform to both sides of this equation and using the differential property of Kamal transform, Eq. (11) can be written as:

$$v^{-1}G(x,v) - u(x,0) = G''(x,v)$$

And, $$G''(x,v) - \frac{1}{v}G(x,v) = -\sin\frac{\pi}{l}x$$

This is the second - order ordinary differential equation have the particular solution in the form

$$G(x,v) = \frac{-\sin\frac{\pi}{l}x}{D^2-\frac{1}{v}} = \frac{-\sin\frac{\pi}{l}x}{-\left(\frac{\pi}{l}\right)^2-\frac{1}{v}} = \frac{v\,\sin\frac{\pi}{l}x}{1+\left(\frac{\pi}{l}\right)^2 v} \qquad (12)$$

Now, if we take the inverse Kamal transform to Eq. (12), then the solution of Eq. (11) is:

$$u(x,t) = \sin\frac{\pi}{l}x \ e^{-\left(\frac{\pi}{l}\right)^2 t}$$

Example 3. 4:

Let's consider the homogeneous Laplace equation:

$$u_{xx} + u_{tt} = 0 \qquad u(x,0) = 0\ , u_t(x,0) = \cos x \quad x,t > 0 \qquad (13)$$

Applying Kamal transform to both sides of this equation and using the differential property of Kamal transform, Eq. (13) can be written as:

$$G''(x,v) + v^{-2}G(x,v\ ) - v^{-1}u(x,0) - u_t(x,0) = 0$$

$$G''(x,v) + v^{-2}G(x,v\ ) = \cos x$$

This is the second - order ordinary differential equation have the particular solution in the form:

$$G(x,v\ ) = \frac{cos\,x}{D^2+v^{-2}} = \frac{cos\,x}{-1+v^{-2}} = \frac{v^2 cos\,x}{1-v^2} \qquad (14)$$

Now, if we take the inverse Kamal transform to Eq. (14), then the solution of Eq. (13) is :

$$u(x,t) = \cos x \sinh t$$

Example 3. 5:

Consider the telegraphers equation:

$$u_{tt}(x,t) + 2\alpha u_t(x,t) = \alpha^2 u_{xx}(x,t)\ , 0 < x < 1\ , t > 0 \qquad (15)$$

With the initial conditions:

$$u(x,0) = cosx \ \ , u_t(x,0) = 0$$

Take Kamal transform of Eq. (15), we get:

$$v^{-2}G(x,v) - v^{-1}u(x,0) - u_t(x,0) + 2\alpha v^{-1}G(x,v) - 2\alpha u(x,0) = \alpha^2 G''(x,v)$$

Applying the initial conditions, we get

$$\alpha^2 G''(x,v) - \left(\frac{1+2\alpha v}{v^2}\right) G(x,v) = -cosx\left(\frac{1+2\alpha v}{v}\right)$$

This is the second - order ordinary differential equation have the particular solution in the form :

$$G(x,v) = \frac{-cosx\left(\frac{1+2\alpha v}{v}\right)}{\alpha^2 D^2 - \left(\frac{1+2\alpha v}{v^2}\right)} = \frac{-cosx(v+2\alpha v^2)}{-\alpha^2 v^2 - (1+2\alpha v)} = \frac{cosx(v+2\alpha v^2)}{(1+\alpha v)^2}$$

Or

$$G(x,v) = cosx\left(\frac{\alpha v^2}{(1+\alpha v)^2} + \frac{v}{1+\alpha v}\right) \quad (16)$$

Now, if we take the inverse Kamal transform to Eq. (16), then the solution of Eq. (15) is :

$$u(x,t) = cosx\,(1+\alpha t)e^{-\alpha t}$$

REFERENCES

[1] Abdelilah Kamal. H. Sedeeg, The New Integral Transform “Kamal Transform”, Advances in Theoretical and Applied Mathematics, Vol. 11, No. 4, 2016, pp. 451-458.

[2] Tarig M. Elzaki and Salih M. Elzaki, Applications of New Transform“ELzaki Transform” to Partial Differential Equations, Global Journal of Pure and Applied Mathematics,,2011,pp65-70.

[3] Abdelilah. K. Hassan Sedeeg and Mohand M. Abdelrahim Mahgob Comparison of New Integral Transform " Aboodh Transform" and Adomian Decomposition Method, International Journal of Mathematics And its Applications Volume 4, Issue 2-B (2016), p127-135. ISSN: 2347-1557.

[4] K. Sankara Rao, Introduction to Partial Differential Equations, 1995

[5] Lokenath Debnath and D. Bhatta. Integral transform and their Application second Edition, Chapman & Hall /CRC (2006).

[6] G. K. watugala, simudu transform- anew integral transform to Solve differential equation and control engineering problems. Math. Engrg Induct. 6 (1998), no 4,319-329.

[7] A. Kilicman and H. E. Gadain. An application of double Laplace transform and sumudu transform, Lobachevskii J. Math. 30 (3) (2009), pp. 214-223.

[8] J. Zhang, Asumudu based algorithm m for solving differential equations, Comp. Sci. J. Moldova 15(3) (2007), pp – 303-313.

[9] Christian Constanda, Solution Techniques for Elementary Partial differential Equations, New York, 2002.

[10] Dean G. Duffy, Transform Methods for solving partial differential Equations, 2nd Ed, Chapman & Hall / CRC, Boca Raton, FL, 2004.

[11] SunethraWeera Koon, Application of Sumudu transform to partial differential equation. INT. J. MATH. EDUC. Scl. TECHNOL, 1994, Vol. 25, No2, 277- 283.

[12] Hassan Eltayeb and Ademkilicman, A Note on the Sumudu Transforms and differential Equations, Applied Mathematical Sciences, VOL, 4,2010, no. 22,1089-1098.

[13] Kilicman A. & H. ELtayeb. A note on Integral transform and Partial Differential Equation, Applied Mathematical Sciences, 4(3) (2010), PP. 109-118.

[14] Hassan ELtayeh and Ademkilicman, on Some Applications of a new Integral Transform, Int. Journal of Math. Analysis, Vol, 4, 2010, no. 3,123-132.

[15] A. Aghili, B. Salkhordeh Moghaddam, Laplace transform Pairs of N- dimensions and second order Linear partial differential equations with constant coefficients, Analysis Mathematicae et Informaticae, 35 (2008),pp, 3-10.

[16] Hassan ELtayeb, Ademkilicman and Brian Fisher, A new integral transform and associated distributions, Integral Transform and special Functions. Vol, co, No, 0 Month 2009, 1-13.

[17] M. G. M. Hussain, F. B. M. Belgacem. Transient Solution of Maxwell's Equations Based on Sumudu Transform, Progress in Electromagnetic Research, PIER, 74. 273-289, (2007).