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The New Integral Transform ''Kamal Transform''

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Abstract

In this paper a new integral transform namely Kamal transform was applied to solve linear ordinary differential equations with constant coefficients.

Keywords: Kamal transform, Differential Equations.

1. Introduction

Kamal Transform is derived from the classical Fourier integral. Based on the mathematical simplicity of the Kamal transform and its fundamental properties. Kamal transform was introduced by Abdelilah Kamal to facilitate the process of solving ordinary and partial differential equations in the time domain.Typically, Fourier, Laplace , Sumudu , Elzaki , Aboodh and Mahgoub transforms are the convenient mathematical tools for solving differential equations, Also Kamal transform and some of its fundamental properties are used to solve differential equations.

A new transform called the Kamal transform defined for function of exponential order we consider functions in the set A defined by:

$$A = \left\{ f(t) : \exists M , k_1, k_2 > 0 \, . |f(t)| < Me^{\frac{|t|}{k_j}} , if \;\; t \in (-1)^j \times [0, \infty) \right\} \qquad (1)$$

For a given function in the set A , the constant M must be finite number ,k_1,k_2 may be finite or infinite.

The Kamal transform denoted by the operator $K(.)$, defined by the integral equation:

$$K[f(t)] = G(v) = \int_0^{\infty} f(t) e^{\frac{-t}{v}} dt \ . \quad t \geq 0 \quad , \ k_1 \leq v \leq k_2 \qquad (2)$$

The variable v in this transform is used to factor the variable t in the argument of the function f. This transform has deeper connection with the Laplace ,Elzaki, Aboodh , Mahgoub transforms.

The purpose of this study is to show the applicability of this interesting new transform and its efficiency in solving the linear differential equations.

2. Kamal Transform of the Some Functions:

For any function $f(t)$, we assume that the integral Eq. (2) exist. The sufficient conditions for the existence of Kamal transform are that $f(t)$ for $t \geq 0$ be piecewise continuous and of exponential order, otherwise Kamal transform may or may not exist.

In this section we find Kamal transform of simple functions.

i) Let $f(t) = 1$, by the definition we have:

$$K[1] = G(v) = \int_0^{\infty} e^{\frac{-t}{v}} dt = \left[-v e^{\frac{-t}{v}}\right]_0^{\infty} = v$$

ii) Let $f(t) = t$, then:

$$K[t] = \int_0^{\infty} t e^{\frac{-t}{v}} dt$$

Integrating by parts, we get $K[t] = v^2$

In the general case if $n \geq 0$ is integer number, then.

$$K[t^n] = n!\, v^{n+1}$$

iii) Let $f(t) = e^{at}$, then, $K[e^{at}] = \int_0^{\infty} e^{at} e^{\frac{-t}{v}} dt = \frac{v}{1-av}$

This result will be useful, to find Kamal transform of:

$K[\sin at] = \frac{av^2}{1+a^2v^2}$, $\quad K[\cos at] = \frac{v}{1+a^2v^2}$

$K[\sinh at] = \frac{av^2}{1-a^2v^2}$, $\quad K[\cosh at] = \frac{v}{1-a^2v^2}$

Theorem 2.1:

Let $G(v)$ is the Kamal transform of $f(t)$ $\left(K[f(t)] = G(v)\right)$ then:

i) $K[f'(t)] = \frac{1}{v}G(v) - f(0)$

ii) $K[f''(t)] = \frac{1}{v^2}G(v) - \frac{1}{v}f(0) - f'(0)$

iii) $K\left[f^{(n)}(t)\right] = v^{(-n)}G(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^{(k)}(0)$

Proof:

i) By the definition we have:

$$K[f'(t)] = \int_0^\infty f'(t)e^{\frac{-t}{v}}\,dt \quad ,$$

Integrating by parts, we get

$$K[f'(t)] = \frac{1}{v}G(v) - f(0)$$

ii) $$K\,[f''(t)] = \int_0^\infty f''(t)e^{\frac{-t}{v}}\,dt \quad ,$$

Integrating by parts, we get

$$K[f''(t)] = v^{-2}G(v) - v^{-1}f\,(0) - f'(0)$$

iii) Can be proof by mathematical induction .

3. Application of Kamal Transform of Ordinary Differential Equations:

As stated in the introduction of this paper, the Kamal transform can be used as an effective tool. For analyzing the basic characteristics of a linear system governed by the differential equation in response to initial data. The following examples illustrate the use of the Kamal transform in solving certain initial value problems described by ordinary differential equations.

Consider the first-order ordinary differential equation:

$$\frac{dx}{dt} + px = f(t) \quad , \; t > 0 \qquad (3)$$

With the initial Condition

$$x(0) = a$$

Where p and a are constants and $f(t)$ is an external input function so that its Kamal transform exists.

Applying the Kamal transform to both sides of Eq. (3), we have:

$$K\left[\frac{dx}{dt}+px\right]=K[f(t)] \tag{4}$$

Using the differential property of Kamal transform , Eq.(4) can be written *as*

$$v^{-1}K(x)-x(0)+pK(x)=G(v)$$

$$K(x)=\frac{vG(v)}{1+pv}+\frac{av}{1+pv}$$

The inverse Kamal transform leads to the solution.

Consider the second - order linear ordinary differential equation has the general form:

$$\frac{d^2y}{dx^2}+2p\frac{dy}{dx}+qy=f(x) \qquad , \; x>0 \tag{5}$$

The initial conditions are

$$y(0)=a \;\; , \;\; y'(0)=b \tag{6}$$

Where p, q, a and b are constants . Applications of the Kamal transform to this general initial value problem gives

$$v^{-2}K(y)-v^{-1}y(0)-y'(0)+2p(v^{-1}K(y)-y(0))+qK(y)=G(v)$$

by applying the initial condition in Eq.(6) , we get

$$K(y)=\frac{v^2G(v)}{qv^2+2pv+1}+\frac{v^2b}{qv^2+2pv+1}+\frac{av(1+2vp)}{qv^2+2pv+1}$$

The inverse Kamal transform gives the solution.

Example 3.1:

Let's consider the first order differential equation :

$$\frac{dy}{dx}+y=0 \quad , \quad y(0)=1 \tag{7}$$

Applying the Kamal transform to both sides of this equation and using the differential property of Kamal transform , Eq.(7) can be written as:

$$v^{-1}K(y)-y(0)+K(y)=0$$

Where $K(y)$ is the Kamal transform of the function $y(x)$,

Applying the initial condition , we get

$$(v^{-1}+1)K(y)=1 \text{ and } \quad K(y)=\frac{v}{1+v}$$

Now applying the inverse Kamal transform , we get *:* $y(x) = K^{-1}\left[\frac{v}{1+v}\right] = e^{-x}$

Example 3.2:
Solve the differential equation :

$$\frac{dy}{dx} + 2y = x \ , \ \ y(0) = 1 \tag{8}$$

Applying the Kamal transform to both sides of Eq.(8) and using the differential property of Kamal transform , Eq.(8) can be written as :

$$v^{-1}K(y) - y(0) + 2K(y) = v^2$$

$$(v^{-1} + 2)K(y) = v^2 + 1$$

$$K(y) = \frac{v(1+v^2)}{(1+2v)} = \frac{1}{2}v^2 + \frac{5}{4}\frac{v}{(1+2v)} - \frac{1}{4}v$$

The inverse Kamal transform of this equation gives the Solution:

$$y(x) = \frac{1}{2}x + \frac{5}{4}e^{-2x} - \frac{1}{4}$$

Example 3.3*:*

Let's consider the following second - order differential equation :

$$y'' + y = 0 \ \ , \ \ \ y(0) = y'(0) = 1 \tag{9}$$

Applying the Kamal transform to both sides of Eq.(9) , and using the differential property of Kamal transform , Eq.(9) can be written as :

$$v^{-2}K(y) - v^{-1}y(0) - y'(0) + K(y) = 0$$

Applying the initial condition , we get

$$(v^{-2} + 1)K(y) = v^{-1} + 1$$

We solve this equation for $K(y)$to get

$$K(y) = \frac{v^2 + v}{1 + v^2} = \frac{v^2}{1 + v^2} + \frac{v}{1 + v^2}$$

The inverse Kamal transform of this equation is simply obtained as

$$y(t) = sin\, t + cos\, t$$

Example 3.4:

Let’s consider the following differential equation :

$$y'' - 3y' + 2y = 0 \quad , \quad y(0) = 1 \quad , \quad y'(0) = 4 \tag{10}$$

Take Kamal transform of Eq.(10) , we find that:

$$v^{-2}K(y) - v^{-1}y(0) - y'(0) - 3(v^{-1}K(y) - y(0)) + 2K(y) = 0$$

Applying the initial condition , we get

$$(v^{-2} - 3v^{-1} + 2)K(y) = \frac{1}{v} + 1$$

$$K(y) = \frac{v(1+v)}{2v^2 - 3v + 1} = \frac{v^2 + v}{(2v-1)(v-1)} = \frac{2v}{v-1} - \frac{3v}{2v-1} = \frac{-2v}{1-v} + \frac{3v}{1-2v}$$

Thus, the solution is $y(t) = -2e^{t} + 3e^{2t}$

Example 3.5:

Let’s consider the following second - order differential equation**:**

$$y'' + 9y = cos\, 2t \quad , \quad y(0) = 1 \quad , \quad y\left(\frac{\pi}{2}\right) = -1 \tag{11}$$

Since $y'(0)$ is not known, let $y'(0) = c$.

Take Kamal transform of Eq.(11) and using the conditions, we have

$$v^{-2}K(y) - v^{-1}y(0) - y'(0) + 9K(y) = \frac{v}{1+4v^2}$$

$$(v^{-2} + 9)K(y) = \frac{v}{1+4v^2} + \frac{1}{v} + c$$

$$K(y) = \frac{v^3}{(1+4v^2)(1+9v^2)} + \frac{v}{1+9v^2} + \frac{cv^2}{1+9v^2}$$

We can write this equation in the form,

$$K(y) = \frac{v}{5(1+4v^2)} - \frac{v}{5(1+9v^2)} + \frac{v}{1+9v^2} + \frac{cv^2}{1+9v^2}$$

$$= \frac{v}{5(1+4v^2)} + \frac{4v}{5(1+9v^2)} + \frac{cv^2}{1+9v^2}$$

And invert to find the solution.

$$y(t) = \frac{1}{5} cos\, 2t + \frac{4}{5} cos\, 3t + \frac{c}{3} sin\, 3t$$

To determine c , note that $y\left(\frac{\pi}{2}\right) = -1$ thin we find $c = \frac{12}{5}$, then **,**

$$y(t) = \frac{1}{5} cos\, 2t + \frac{4}{5} cos\, 3t + \frac{4}{5} sin\, 3t$$

Example 3.6:

Solve the following differential equation**:**

$$y'' - 3y' + 2y = 4e^{3t} \quad , \quad y(0) = -3 \quad , \quad y'(0) = 5 \qquad (12)$$

Taking the Kamal transforms both side of Eq.(12), and using the given conditions we have,

$$v^{-2}K(y) - v^{-1}y(0) - y'(0) - 3(v^{-1}K(y) - y(0)) + 2K(y) = \frac{4v}{1-3v}$$

$$(v^{-2} - 3v^{-1} + 2)K(y) = \frac{4v}{1-3v} - \frac{3}{v} + 14$$

$$K(y) = \frac{4v^3}{(1-3v)(v-1)(2v-1)} - \frac{3v}{(v-1)(2v-1)} + \frac{14v^2}{(v-1)(2v-1)}$$

$$= \frac{-38v^3 + 23v^2 - 3v}{(1-3v)(v-1)(2v-1)}$$

Or

$$K(y) = \frac{-4v}{2v-1} + \frac{2v}{1-3v} + \frac{9v}{v-1} = \frac{4v}{1-2v} + \frac{2v}{1-3v} - \frac{9v}{1-v}$$

Inverting to find the solution in the form.

$$y(t) = 4e^{2t} + 2e^{3t} - 9e^{t}$$

Example 3.7:

Find the solution of the following initial value problem:

$$y'' + 4y = 12t \quad , \quad y(0) = 0 \quad , \quad y'(0) = 7 \tag{13}$$

Applying Kamal transform of this problem and using the given conditions , we get,

$$v^{-2}K(y) - v^{-1}y(0) - y'(0) + 4K(y) = 12v^2$$

$$(v^{-2} + 4)K(y) = 12v^2 + 7$$

$$K(y) = \frac{12v^2 + 7}{v^{-2} + 4} = \frac{v^2(12v^2 + 7)}{1 + 4v^2} = 3v^2 + \frac{4v^2}{1 + 4v^2}$$

Inverting to find the solution in the form

$$y(t) = 3t + 2\,sin\,2t$$

4. Conclusion

The definition and application of the new transform " Kamal transform" to the solution of ordinary differential equations has been demonstrated.

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