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On The Solution of Ordinary Differential Equation with Variable Coefficients using Aboodh Transform

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Abstract

In this paper, we apply a new integral transform " Aboodh transform" to solve some ordinary differential equation with variable coefficients, The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear differential equations.

Keyword: Aboodh transform- differential equations

1. INTRODUCTION

There are several integral transforms [1] like, Laplace Transform, Fourier Transform, Sumudu Transform [2-3] , Elzaki Transform [4-6], ZZ Transform [7], Natural Transform and Aboodh Transform, to crack the DEs and IEs. Of these the most widely used transform is Laplace Transform.

Recently, Khalid Aboodh, has introduced a new integral transform, named the Aboodh transform[8-11],and it has further applied to the solution of ordinary and partial differential equations. This transformation has deeper connection with the

Laplace and Elzaki Transform. The main objective is to introduce solution of Ordinary Differential Equation with Variable Coefficients by using Aboodh transform .The plane of the paper is as follows: In section 2, we introduce the basic idea of Aboodh transform, Application in 3 and conclusion in 4, respectively.

2. DEFINITIONS AND STANDARD RUSELTS

Aboodh transform :

Definition :

A new transform called the Aboodh transform defined for function of exponential order we consider functions in the set **A,** defined by:

$$\mathrm{A} = \{\mathrm{f(t)}: \exists\ \mathrm{M}\ , \mathrm{k_1}, \mathrm{k_2} > 0\ , |\mathrm{f(t)}| < M\mathrm{e}^{-\mathrm{vt}}$$

For a given function in the set M must be finite number,k_1, k_2 may be finite or infinite. Aboodh transform which is defined by the integral equation

$$A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}\,dt\ t \geq 0\ , \mathrm{k_1} \leq v \leq \mathrm{k_2} \qquad (1)$$

Aboodh transform of some functions :

$$A(1) = \frac{1}{v^2} \quad , \quad A(t^n) = \frac{n!}{v^{n+2}}\ , A(e^{at}) = \frac{1}{v^2 - av}$$

$$A(\sin(at)) = \frac{a}{v(v^2+a^2)} \quad , \quad A(\cos(at)) = \frac{1}{(v^2+a^2)}$$

Aboodh transform of derivatives :

Theorem I

If Aboodh transform of the function $f(t)$ given by $A[f(t)] = K(v)$, then:

1) $A[f\,'(t)] = vK(v) - \frac{f(0)}{v}$, $A[f\,''(t)] = v^2K(v) - \frac{f'(0)}{v} - f(0)$

$$A\left[f^{(n)}(t)\right] = v^nK(v) - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{2-n+k}}\ .$$

2) (i) $A\{tf(t)\} = -\frac{d}{dv}k(v) - \frac{1}{v}k(v)$

(ii) $A\{tf'(t)\} = -\frac{d}{dv}\left[vk(v) - \frac{f(0)}{v}\right] - \frac{1}{v}\left[vk(v) - \frac{f(0)}{v}\right]$,

(iii) $A\{tf''(t)\} = -\frac{d}{dv}\left[v^2k(v) - \frac{f'(0)}{v} - f(0)\right] - \frac{1}{v}\left[v^2k(v) - \frac{f'(0)}{v} - f(0)\right]$

(v) $A\{t^2f'(t)\} = v\frac{d^2k(v)}{dv^2} + 2\frac{d\,k(v)}{dv} - 2\frac{f(0)}{v^3}$

(iv) $A\{t^2f''(t)\} = v^2\frac{d^2k(v)}{dv^2} + 4v\frac{d\,k(v)}{dv} + 2k(v) - 2\frac{f'(0)}{v^3}$

Proof

2) (i) $A[f(t)] = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}\,dt$

$$\frac{d}{du}K(v) = K'(v) = \frac{d}{dv}\int_0^\infty \frac{1}{v}f(t)e^{-vt}\,dt = \int_0^\infty \frac{d}{dv}\left(\frac{1}{v}e^{-vt}\right)f(t)dt$$

$$= \int_0^\infty \left(-\frac{1}{v}e^{-vt}\right)\left(tf(t)\right)dt + \int_0^\infty -\frac{1}{v^2}e^{-vt}f(t)dt$$

$$= -A\left(tf(t)\right) - \frac{1}{v}K(v)$$

$$A\left(tf(t)\right) = -\frac{d}{du}K(v) - \frac{1}{v}A\left(K(v)\right).$$

(iv) $$A\{t^2f''(t)\} = -\frac{d}{dv}\left[-v^2\frac{d\,k(v)}{dv} + 2vk(v) - \frac{f'(0)}{v^2}\right]$$

$$= -\left(-v^2\frac{d^2k(v)}{dv^2} - 2v\frac{d\,k(v)}{dv} - 2v\frac{d\,k(v)}{dv} - 2k(v) + 2\frac{f'(0)}{v^3}\right)$$

$$= v^2\frac{d^2k(v)}{dv^2} + 4v\frac{d\,k(v)}{dv} + 2k(v) - 2\frac{f'(0)}{v^3}$$

3. APPLICATION

In this section: we apply the above theorem to find Aboodh transform for some differential equations

Example I

Solve the differential equation:

$$y'' + ty' - y = 0 \qquad (2)$$

With the initial condition $, y(0) = 0\ ,\ y'(0) = 1$ (3)

Solution

Using the differential property of Aboodh transform Eq.(2) can be written as

$$v^2K(v) - \frac{1}{v} - \frac{d}{dv}vk(v) - k(v) - k(v) = 0$$

$$-v\frac{d}{dv}k(v) - 3k(v) + v^2k(v) = \frac{1}{v}$$

$$\frac{d}{dv}k(v) - (v - 3v)k(v) = -\frac{1}{v^2} \qquad (4)$$

This is a linear differential equation for unknown function k, have the Solution in the form

$$k(v) = \frac{1}{v^3} + ce^{\frac{1}{2}v^2} \quad \text{and} \quad C = 0 \text{ , then: } k(v) = \frac{1}{v^3} \qquad (5)$$

By using the inverse Aboodh transform we obtain the Solution in the form of

$$y(t) = \mathrm{t} \qquad (6)$$

Example II

Solve the differential equation:

$$y'' + ty' - 4y = 6 \qquad (7)$$

With the initial condition $\ , y(0) = 0 \ , \ y'(0) = 0$ (8)

Solution

Using the differential property of Aboodh transform Eq.(7) can be written as

$$v^2K(v) - 2\frac{d}{dv}[vk(v) - 2k(v) - 4k(v)] = \frac{6}{v^2}$$

$$-2v\frac{d}{dv}k(v) - 8k(v) + v^2k(v) = \frac{6}{v^2}$$

$$\frac{d}{dv}k(v) - \left(-\frac{1}{2}v + \frac{4}{v}\right)k(v) = -\frac{3}{v^3} \qquad (9)$$

This is a linear differential equation for unknown function k, have the Solution in the form

$$k(v) = \frac{6}{v^4} + Ce^{-\frac{1}{4}v^2} \quad \text{and} \quad C = 0 \text{ , then: } k(v) = \frac{6}{v^4} \quad (10)$$

By using the inverse Aboodh transform we obtain the Solution in the form of

$$y(t) = 3\mathrm{t}^2 \quad (11)$$

Example III

Solve the differential equation:

$$t\,y'' + (1-2t)y' - 2y = 0 \quad (12)$$

With the initial condition $, y(0) = 1 \;,\; y'(0) = 2$ (13)

Solution

Using the differential property of Aboodh transform Eq.(12) can be written as

$$\frac{d}{dv}\left[\frac{2}{v} + 1 - v^2 k(v)\right] + \frac{1}{v}\left[\frac{2}{v} + 1 - v^2 k(v)\right]$$

$$+vk(v) - \frac{1}{v} - 2\left[\frac{d}{dv}\left[\frac{1}{v} - vk(v)\right]\right] + \frac{1}{v}\left[\frac{1}{v} - vk(v)\right] - 2k(v) = 0$$

$$-2v^2 \frac{d}{dv} k(v) + 2v \frac{d}{dv} k(v) - 2vk(v) + 2k(v) = 0$$

$$(v^2 - 2v)\frac{d}{dv} k(v) - (2v-2)k(v) = 0 \quad (14)$$

This is a linear differential equation for unknown function k, have the Solution in the form

$$k(v) = \frac{C}{v^2 - 2v} \quad \text{and} \quad C = 1 \text{ , then:} k(v) = \frac{1}{v^2 - 2v} \quad (15)$$

By using the inverse Aboodh transform we obtain the Solution in the form of

$$y(t) = \mathrm{e}^{2\mathrm{t}} \quad (16)$$

Example IV

Solve the differential equation:

$$t^2\, y'' + 4ty' + 2y = 12t^2 \qquad (17)$$

With the initial condition $\,, y(0) = 0\ ,\ y'(0) = 0$ (18)

Solution

Using the differential property of Aboodh transform Eq.(17) can be written as

$$v^2 \frac{d^2k(v)}{dv^2} + 4v\frac{d\,k(v)}{dv} + 2k(v) - 4v\frac{d\,k(v)}{dv} - 4k(v) + 2k(v) = \frac{24}{v^4}$$

By simplifying above equation, we have

$$\frac{d^2k(v)}{dv^2} = \frac{24}{v^6} \qquad (19)$$

The solution of this equation can be written in the form.

$$k(v) = \frac{24}{20v^4} + c_1 v + c_0 \qquad (20)$$

By substituting the initial condition (30) into equation (32) we get

$$k(v) = \frac{24}{20v^4} = \frac{3}{5}\frac{2}{v^4} \qquad (21)$$

By using the inverse Aboodh transform we obtain the Solution in the form of

$$y(t) = \frac{3}{5}t^2 \qquad (22)$$

CONCLUSION

The " Aboodh transform, whose fundamental properties are presented in this paper, is little known and not widely used . In this paper, we apply a new integral transform " Aboodh transform" to solve some ordinary differential equation with variable coefficients , The result reveals that the proposed method is very efficient, simple and can be applied to linear and nonlinear differential equations

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