Soft $\partial$-Closed Sets In Soft Čech Closure Space

R. Gowri
Department of Mathematics,
Govt. College for Women’s (A),
Kumbakonam-612 001, Tamilnadu, India.
E-mail: gowrigck@rediffmail.com

G. Jegadeesan
Department of Mathematics,
Anjalai Ammal Mahalingam College,
Kovilvenni-614 403, Tamilnadu, India.
E-mail: jega0548@yahoo.co.in

Abstract
In this paper, we introduce soft $\partial$-closed sets in soft Čech closure spaces which are defined over an initial universe with a fixed set of parameters. Also we investigate the behavior relative to union, intersection and soft subspaces of soft $\partial$-closed sets as well as soft $\partial$-open sets. In the study of soft $\partial$-closed sets, we introduce and discuss the relationship between separation axioms, namely $T_{\frac{1}{2}}'$-space and $T_{\frac{1}{2}}''$-space.

AMS subject classification: 54A05, 54B05.
Keywords: Soft set, Soft $\partial$-closed set, Soft $\partial$-open set, $T_{\frac{1}{2}}'$-space, $T_{\frac{1}{2}}''$-space.

1. Introduction

Closed sets are fundamental objects in a topological space. For example, one can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. In 1970, N.Levine [9] initiated the study of generalized closed sets in topological space in order to extend some important properties of closed sets to a larger family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets.

E. Čech [2] introduced the concept of closure spaces. In Čech’s approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold
for every set $A$ of $X$. When this condition is also true, the operator becomes topological closure operator. Thus, the concept of closure space is the generalisation of a topological space.


In this paper, we introduce soft $\partial$-closed sets in soft Čech closure spaces which are defined over an initial universe with a fixed set of parameters. Also we investigate the behavior relative to union, intersection and soft subspaces of soft $\partial$-closed sets as well as soft $\partial$-open sets. In the study of soft $\partial$-closed sets, we introduce new separation axioms, namely $T'_{\frac{1}{2}}$-space and $T''_{\frac{1}{2}}$-space.

2. Preliminaries

In this section, we recall the basic definitions of soft Čech closure spaces.

**Definition 2.1.** Let $X$ be an initial universe set, $A$ be a set of parameters. Then the function $k : P(X_F A) \rightarrow P(X_F A)$ defined from a soft power set $P(X_F A)$ to itself over $X$ is called Čech closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$.

(C2) $F_A \subseteq k(F_A)$.

(C3) $k(F_A \cup G_A) = k(F_A) \cup k(G_A)$.

Then $(X, k, A)$ or $(F_A, k)$ is called a soft Čech closure space.

**Definition 2.2.** A soft subset $U_A$ of a soft Čech closure space $(F_A, k)$ is said to be soft k-closed (soft closed) if $k(U_A) = U_A$.

**Definition 2.3.** A soft subset $U_A$ of a soft Čech closure space $(F_A, k)$ is said to be soft k-open (soft open) if $k(U^C_A) = U^C_A$.

**Definition 2.4.** A soft set $Int(U_A)$ with respect to the closure operator $k$ is defined as $Int(U_A) = F_A - k(F_A - U_A) = [k(U^C_A)]^C$. Here $U^C_A = F_A - U_A$.

**Definition 2.5.** A soft subset $U_A$ in a soft Čech closure space $(F_A, k)$ is called soft neighbourhood of $e_F$ if $e_F \in Int(U_A)$. 
Definition 2.6. If \((F_A, k)\) be a soft Čech closure space, then the associate soft topology on \(F_A\) is \(\tau = \{U^C_A : k(U_A) = U_A\}\).

Definition 2.7. Let \((F_A, k)\) be a soft Čech closure space. A soft Čech closure space \((G_A, k^*)\) is called a soft subspace of \((F_A, k)\) if \(G_A \subseteq F_A\) and \(k^*(U_A) = k(U_A) \cap G_A\), for each soft subset \(U_A \subseteq G_A\).

3. Soft \(\partial\)-closed sets

In this section, we introduce and characterize a new class of soft closed sets lying between the soft closed sets and the class of soft generalized closed sets in soft Čech closure spaces.

Definition 3.1. Let \((F_A, k)\) be a soft Čech closure space. A soft subset \(U_A \subseteq F_A\) is called a soft \(\partial\)-closed set, if \(k(U_A) \subseteq G_A\), whenever \(G_A\) is an soft g-open subset of \((F_A, k)\) with \(U_A \subseteq G_A\). A soft subset \(U_A \subseteq F_A\) is called a soft \(\partial\)-open set, if its complement is soft \(\partial\)-closed.

Example 3.2. Let the initial universe set \(X = \{u_1, u_2\}\) and \(E = \{x_1, x_2, x_3\}\) be the parameters. Let \(A = \{x_1, x_2\} \subseteq E\) and \(F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}\). Then \(P(X_{F_A})\) are

\[
\begin{align*}
F_{1A} &= \{(x_1, \{u_1\})\}, & F_{2A} &= \{(x_1, \{u_2\})\}, \\
F_{3A} &= \{(x_1, \{u_1, u_2\})\}, & F_{4A} &= \{(x_2, \{u_1\})\}, \\
F_{5A} &= \{(x_2, \{u_2\})\}, & F_{6A} &= \{(x_2, \{u_1, u_2\})\}, \\
F_{7A} &= \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, & F_{8A} &= \{(x_1, \{u_1\}), (x_2, \{u_2\})\}, \\
F_{9A} &= \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, & F_{10A} &= \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, \\
F_{11A} &= \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}, & F_{12A} &= \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, \\
F_{13A} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}, & F_{14A} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, \\
F_{15A} &= F_A, & F_{16A} &= \emptyset_A.
\end{align*}
\]

An operator \(k : P(X_{F_A}) \rightarrow P(X_{F_A})\) is defined from soft power set \(P(X_{F_A})\) to itself over \(X\) as follows.

\[
k(F_{1A}) = F_{1A}, k(F_{2A}) = k(F_{3A}) = F_{3A}, k(F_{4A}) = F_{4A}, k(F_{5A}) = k(F_{6A}) = F_{6A}, k(F_{7A}) = F_{7A},
\]

\[
k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{9A}) = k(F_{13A}) = F_{13A},
\]

\[
k(F_{10A}) = k(F_{12A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.
\]

Here, the soft \(\partial\)-closed sets are \(\emptyset_A, F_{1A}, F_{3A}, F_{4A}, F_{6A}, F_{7A}, F_{11A}, F_{13A}, F_A\).

Remark 3.3. In a soft subset \(U_A\) of a soft Čech closure space \((F_A, k)\), the following implications hold.
$U_A$ is soft closed $\Rightarrow U_A$ is soft $\partial$-closed $\Rightarrow U_A$ is soft $g$-closed.

The converse of the above implications is not true from the following example.

**Example 3.4.** Let us consider the soft subsets of $F_A$ that are given in example 3.2. An operator $k : P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over $X$ as follows.

\[
k(F_1A) = F_1A, k(F_2A) = k(F_0A) = F_{12}A, \\
k(F_4A) = F_4A, k(F_5A) = k(F_8A) = F_{14}A, \\
k(F_7A) = F_7A, k(F_3A) = k(F_6A) = k(F_{10}A) = k(F_{11}A) \\
= k(F_{12}A) = k(F_{13}A) = k(F_{14}A) = k(F_A) \\
k(F_8A) = F_A, k(\emptyset_A) = \emptyset_A.
\]

Here,

$F_{11}A = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$,

is soft $\partial$-closed set but not soft closed. Also,

$F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$

is soft $g$-closed set but not soft $\partial$-closed.

**Theorem 3.5.** Let $(F_A, k)$ be a soft Čech closure space. If $U_A$ and $V_A$ are soft $\partial$-closed subsets of $(F_A, k)$, then $U_A \cup V_A$ is soft $\partial$-closed.

**Proof.** Let $G_A$ be a soft $g$-open subset of $(F_A, k)$ such that $U_A \cup V_A \subseteq G_A$. Then $U_A \subseteq G_A$ and $V_A \subseteq G_A$. Since, $U_A$ and $V_A$ are soft $\partial$-closed, $k[U_A] \subseteq G_A$ and $k[V_A] \subseteq G_A$. Then, $k[U_A \cup V_A] = k[U_A] \cup k[V_A] \subseteq G_A$. Therefore, $U_A \cup V_A$ is soft $\partial$-closed.

**Theorem 3.6.** Let $(F_A, k)$ be a soft Čech closure space. If $U_A$ is soft $\partial$-closed and $V_A$ is soft closed in $F_A$, then $U_A \cap V_A$ is soft $\partial$-closed.

**Proof.** Let $G_A$ be a soft $g$-open subset of $(F_A, k)$ such that $U_A \cap V_A \subseteq G_A$. Then $U_A \subseteq G_A \cup V_A$ and $k[U_A] \subseteq G_A \cup V_A$. That is, $U_A \subseteq G_A \cup (F_A - V_A)$ and $k[U_A] \subseteq G_A \cup (F_A - V_A)$. Then, $k[U_A] \cap V_A \subseteq G_A$. Since, $V_A$ is soft closed. Therefore, $k[U_A \cap V_A] \subseteq G_A$. Hence, $U_A \cap V_A$ is soft $\partial$-closed.

**Theorem 3.7.** Let $(H_A, k^*)$ be a soft closed subspace of $(F_A, k)$. If $V_A$ is a soft $\partial$-closed subset of $(H_A, k^*)$, then $V_A$ is a soft $\partial$-closed subset of $(F_A, k)$.

**Proof.** Let $G_A$ be a soft $g$-open subset of $(F_A, k)$ such that $V_A \subseteq G_A$. Then, $V_A \subseteq G_A \cap H_A$. Since, $V_A$ is soft $\partial$-closed and $G_A \cap H_A$ is soft $g$-open in $(H_A, k^*)$. Therefore, $k[V_A] \cap H_A = k^*[V_A] \subseteq G_A$. But $H_A$ is a soft closed subset of $(F_A, k)$ and $k[V_A] \subseteq G_A$. Hence, $V_A$ is a soft $\partial$-closed subset of $(F_A, k)$. 

**Theorem 3.8.** Let \((F_A, k)\) be a soft Čech closure space and let \(U_A \subseteq F_A\). If \(U_A\) is both soft \(g\)-open and soft \(\partial\)-closed, then \(U_A\) is soft closed.

*Proof.* The proof is obvious. 

**Theorem 3.9.** Let \((F_A, k)\) be a soft Čech closure space and let \(U_A \subseteq F_A\). If \(U_A\) is soft \(\partial\)-closed, then \(k[U_A] - U_A\) has no non-empty soft \(g\)-closed subset.

*Proof.* Suppose that \(U_A\) is soft \(\partial\)-closed. Let \(V_A\) be a soft \(g\)-closed subset of \(k[U_A] - U_A\). Then \(V_A \subseteq k[U_A] \cap (F_A - U_A)\) and so \(U_A \subseteq F_A - V_A\). Consequently, \(V_A \subseteq F_A - k[U_A]\). Since, \(V_A \subseteq k[U_A]\), \(V_A \subseteq k[U_A] \cap (F_A - k[U_A]) = \emptyset\). Thus \(V_A = \emptyset\). Therefore, \(k[U_A] - U_A\) contains no nonempty soft \(g\)-closed subset. 

**Remark 3.10.** The converse of the above theorem 3.9 is not true as shown in the following example.

**Example 3.11.** In example 3.4, take \(U_A = F_{9A}\). Then, \(k[U_A] - U_A = k[F_{9A}] - F_{9A} = F_{5A}\), which does not contain non-empty soft \(g\)-closed subset. But, \(F_{9A}\) is not soft \(\partial\)-closed.

**Theorem 3.12.** Let \((F_A, k)\) be a soft Čech closure space. A soft set \(U_A \subseteq F_A\) is soft \(\partial\)-open if and only if \(V_A \subseteq F_A - k[F_A - U_A]\), whenever \(V_A\) is soft \(g\)-closed subset of \((F_A, k)\) and \(V_A \subseteq U_A\).

*Proof.* Suppose that \(U_A\) is soft \(\partial\)-open and \(V_A\) be a soft \(g\)-closed subset of \((F_A, k)\) such that \(V_A \subseteq U_A\). Then \(F_A - U_A \subseteq F_A - V_A\). But, \(F_A - U_A\) is soft \(\partial\)-closed and \(F_A - V_A\) is soft \(g\)-open. This implies that, \(k[F_A - U_A] \subseteq F_A - V_A\). Therefore, \(V_A \subseteq F_A - k[F_A - U_A]\). Conversely, Let \(G_A\) be a soft \(g\)-open subset of \((F_A, k)\) such that \(F_A - U_A \subseteq G_A\). Then \(F_A - G_A \subseteq U_A\). Since, \(F_A - G_A\) is soft \(g\)-closed, \(F_A - G_A \subseteq F_A - k[F_A - U_A]\). This implies, \(k[F_A - U_A] \subseteq G_A\). Therefore, \(F_A - U_A\) is soft \(\partial\)-closed. Hence, \(U_A\) is soft \(\partial\)-open.

**Theorem 3.13.** Let \((F_A, k)\) be a soft Čech closure space. If \(U_A\) is soft \(\partial\)-open and \(V_A\) is soft open in \(F_A\), then \(U_A \cup V_A\) is soft \(\partial\)-open.

*Proof.* Let \(G_A\) be a soft \(g\)-closed subset of \((F_A, k)\) such that \(G_A \subseteq U_A \cup V_A\). Then \(F_A - (U_A \cup V_A) \subseteq F_A - G_A\). Hence, \((F_A - U_A) \cap (F_A - V_A) \subseteq F_A - G_A\). By theorem 3.6, \((F_A - U_A) \cap (F_A - V_A)\) is soft \(\partial\)-closed. Therefore, \(k[(F_A - U_A) \cap (F_A - V_A)] \subseteq F_A - G_A\). Consequently,

\[
G_A \subseteq F_A - k[(F_A - U_A) \cap (F_A - V_A)] = F_A - k[F_A - (U_A \cup V_A)].
\]

By theorem 3.12, \(U_A \cup V_A\) is soft \(\partial\)-open.

**Theorem 3.14.** Let \((F_A, k)\) be a soft Čech closure space. If \(U_A\) and \(V_A\) are soft \(\partial\)-open subsets of \((F_A, k)\), then \(U_A \cap V_A\) is also soft \(\partial\)-open.
Theorem 3.15. Let $G_A$ be a soft $g$-closed subset of $(F_A, k)$ such that $G_A \subseteq U_A \cap V_A$. Then, $F_A - (U_A \cap V_A) \subseteq F_A - G_A$. Consequently, $(F_A - U_A) \cup (F_A - V_A) \subseteq F_A - G_A$. By theorem 3.5, $(F_A - U_A) \cup (F_A - V_A)$ is soft $\partial$-closed. Thus,

$$k[(F_A - U_A) \cup (F_A - V_A)] \subseteq F_A - G_A.$$ 

Hence,

$$G_A \subseteq F_A - k[(F_A - U_A) \cup (F_A - V_A)] = F_A - k[F_A - (U_A \cap V_A)].$$

By theorem 3.12, $U_A \cap V_A$ is soft $\partial$-open. 

Theorem 3.15. Let $(F_A, k)$ be a soft Čech closure space. If $U_A$ is a soft $\partial$-open subset of $F_A$, then $G_A = F_A$ whenever $G_A$ is soft $g$-open and $(F_A - k[F_A - U_A]) \cup (F_A - U_A) \subseteq G_A$.

Proof. Assume that $U_A$ is soft $\partial$-open. Let $G_A$ be a soft $g$-open subset of $(F_A, k)$ such that $(F_A - k[F_A - U_A]) \cup (F_A - U_A) \subseteq G_A$. Then $(F_A - G_A) \subseteq F_A - ((F_A - k[F_A - U_A]) \cup (F_A - U_A))$. Therefore, $F_A - G_A \subseteq k[F_A - U_A] \cap U_A$ or equivalently, $F_A - G_A \subseteq k[F_A - U_A] - (F_A - U_A)$. But, $F_A - G_A$ is soft $g$-closed and $F_A - U_A$ is soft $\partial$-closed. By theorem 3.9, $F_A - G_A = \emptyset_A$. Consequently, $F_A = G_A$. 

Remark 3.16. The converse of the above theorem 3.15 is not true as shown in the following example.

Example 3.17. In example 3.4, take $U_A = F_8A$. Then, $(F_A - k[F_A - U_A]) \cup (F_A - U_A) = F_{13A} \subseteq G_A$, whenever $G_A$ is soft $g$-open. This implies, $G_A = F_A$, but $U_A$ is not soft $\partial$-open.

Theorem 3.18. Let $(F_A, k)$ be a soft Čech closure space and let $U_A \subseteq F_A$. If $U_A$ is a soft $\partial$-closed, then $k[U_A] - U_A$ is soft $\partial$-open.

Proof. Suppose that $U_A$ is soft $\partial$-closed. Let $G_A$ be a soft $g$-closed subset of $(F_A, k)$ such that $G_A \subseteq k[U_A] - U_A$. By theorem 3.9, $G_A = \emptyset_A$ and hence $G_A \subseteq F_A - k[F_A - (k[U_A] - U_A)]$. By theorem 3.12, $k[U_A] - U_A$ is soft $\partial$-open. 

Remark 3.19. The converse of the above theorem 3.18 is not true as shown in the following example.

Example 3.20. Let us consider the soft subsets of $F_A$ that are given in example 3.2. An operator $k : P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over $X$ as follows.

$$k(F_{1A}) = k(F_{5A}) = F_{8A}, k(F_{2A}) = F_{3A}, k(F_{3A}) = k(F_{9A})$$

$$= k(F_{13A}) = F_{13A}, k(F_{4A}) = F_{4A}, k(F_{6A}) = k(F_{8A})$$

$$= k(F_{11A}) = F_{11A}, k(F_{7A}) = F_{7A}, k(F_{10A}) = F_{14A}, k(F_{12A})$$

$$= k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.$$
Here, take $U_A = F_{2A}$. Then, $k[U_A] - U_A = F_{1A}$, which is soft $\partial$-open. But $U_A$ is not soft $\partial$-closed.

4. $T_2'$ and $T_2''$-soft Čech closure space

In this section, we introduce $T_2'$ and $T_2''$-soft Čech closure spaces and investigate some of their properties.

**Definition 4.1.** A soft Čech closure space $(F_A, k)$ is said to be a $T_2'$-space if every soft $\partial$-closed subset of $F_A$ is soft closed.

**Definition 4.2.** A soft Čech closure space $(F_A, k)$ is said to be a $T_2''$-space if every soft $g$-closed subset of $F_A$ is soft $\partial$-closed.

**Remark 4.3.** The following examples 4.4 and 4.5 shows that $T_2'$-space and $T_2''$-space does not implies each other.

**Example 4.4.** Let us consider the soft subsets of $F_A$ that are given in example 3.2. An operator $k : P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over $X$ as follows.

$$
k(F_{1A}) = k(F_{2A}) = k(F_{3A}) = F_{3A}, k(F_{5A}) = F_{5A}, k(F_{4A}) = k(F_{6A}) = F_{6A},
k(F_{8A}) = k(F_{10A}) = k(F_{14A}) = F_{14A}, k(F_{7A}) = k(F_{9A})
k = k(F_{11A}) = k(F_{12A}) = k(F_{13A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.
$$

Here, $(F_A, k)$ is $T_2'$-space but not $T_2''$-space. Since, $F_{1A} = \{(x_1, \{u_1\})\}$ is soft $g$-closed but not soft $\partial$-closed subset of $F_A$.

**Example 4.5.** In example 3.20, the soft Čech closure space $(F_A, k)$ is $T_2''$ but not $T_2'$-space. Since, $F_{6A} = \{(x_2, \{u_1, u_2\})\}$ is soft $\partial$-closed but not soft closed.

**Example 4.6.** In example 3.2, every soft $g$-closed subsets are soft $\partial$-closed and also every soft $\partial$-closed subsets are soft closed in $F_A$. Therefore, the soft Čech closure space $(F_A, k)$ is $T_2'$ and $T_2''$-space.

**Theorem 4.7.** Let $(F_A, k)$ be a soft Čech closure space. Then

(i) If $(F_A, k)$ is a $T_2'$-space then every soft singleton subset of $F_A$ is either soft $g$-closed or soft open.

(ii) If every soft singleton subset of $F_A$ is a soft $g$-closed, then $(F_A, k)$ is a $T_2'$-space.
Proof.

(i) Let \((F_A, k)\) be a \(T'^1\)-space. Let \((x, u) \in F_A\) and assume that \{(x, u)\} is not a soft \(g\)-closed subset of \((F_A, k)\). Then \(F_A - \{(x, u)\}\) is not a soft \(g\)-open subset of \(F_A\). This implies, \(F_A - \{(x, u)\}\) is soft \(\partial\)-closed. Then the only soft \(g\)-open subset containing \(F_A - \{(x, u)\}\) is \(F_A\). Since, \((F_A, k)\) is a \(T'_1\)-space, then \(F_A - \{(x, u)\}\) is soft closed. That is \{(x, u)\} is soft open.

(ii) Let \(U_A\) be a soft \(\partial\)-closed subset of \((F_A, k)\). Suppose that \((x, u) \notin U_A\). Then, \{(x, u)\} \subseteq F_A - U_A. That is, \(U_A \subseteq F_A - \{(x, u)\}\). Since, \(U_A\) is soft \(\partial\)-closed and \(F_A - \{(x, u)\}\) is soft \(g\)-open, \(k[U_A] \subseteq F_A - \{(x, u)\}\). That is, \{(x, u)\} \subseteq F_A - k[U_A]. Hence, \((x, u) \notin k[U_A]\) and then \(k[U_A] \subseteq U_A\). Therefore, \(U_A\) is a soft closed subset of \(F_A\). Hence, \((F_A, k)\) is a \(T'^1\)-space. ■

Theorem 4.8. Let \((F_A, k)\) be a soft Čech closure space. If \((F_A, k)\) is a \(T''_1\)-space, then every soft singleton subset of \(F_A\) is either soft \(\partial\)-open or soft closed.

Proof. It follows from the theorem 4.7 (i). ■

Result 4.9. If \((F_A, k)\) is a \(T'_1\)-space, then \((F_A, k)\) is a \(T''_1\)-space.

Proof. Let \((F_A, k)\) be a \(T'_1\)-space. Let \(U_A\) be a soft \(g\)-closed subset of \(F_A\), then \(U_A\) is soft closed. Since, every soft closed subset of \(F_A\) is soft \(\partial\)-closed. This implies, \(U_A\) is soft \(\partial\)-closed. Hence, \((F_A, k)\) is \(T''_1\)-space. ■

Remark 4.10. The converse of the above result 4.9 need not be true as shown in the following example.

Example 4.11. In example 3.20, the soft Čech closure space is \(T''_1\)-space but not \(T'_1\)-space. Since, \(F_{6A} = \{(x_2, \{u_1, u_2\})\}\) is soft \(g\)-closed subset of \(F_A\) but not soft closed.

Result 4.12. If \((F_A, k)\) is a \(T'_1\)-space, then \((F_A, k)\) is a \(T'_1\)-space.

Proof. Let \((F_A, k)\) be a \(T'_1\)-space. This implies, every soft \(g\)-closed subset of \(F_A\) is soft closed. Let \(U_A\) be a soft \(\partial\)-closed subset of \(F_A\). Since, every soft \(\partial\)-closed subset of \(F_A\) is soft \(g\)-closed. This implies, \(U_A\) is soft closed. Hence, \((F_A, k)\) is \(T'_1\)-space. ■

Remark 4.13. The converse of the above result 4.12 need not be true as shown in the following example. following example.

Example 4.14. In example 4.4, the soft Čech closure space is \(T'_1\)-space but not \(T'_2\)-space. Since, \(F_{1A} = \{(x_1, \{u_1\})\}\) is soft \(g\)-closed subset of \(F_A\) but not soft closed.
**Theorem 4.15.** Let \((F_A, k)\) be a soft Čech closure space. Then \((F_A, k)\) is a \(T^*\) space if and only if \((F_A, k)\) is both \(T^*\) space and a \(T^{**}\) space.

**Proof.** The proof is obvious. ■

5. Conclusion

In this paper, we introduced soft \(\partial\)-closed sets in soft Čech closure spaces which are defined over an initial universe with a fixed set of parameters. Also we investigated the behavior relative to union, intersection and soft subspaces of soft \(\partial\)-closed sets as well as soft \(\partial\)-open sets. Through soft \(\partial\)-closed sets, we introduced new separation axioms, namely \(T^*\) space and \(T^{**}\) space. Also, we discussed the relationship between \(T^*\), \(T^{**}\) and \(T^*\) spaces.

References


