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An Explicit Method for Solving Intuitionistic Fuzzy Heat Equation

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Abstract

In this paper, an explicit method is used for the solution of the intuitionistic fuzzy heat equation. We first describe the necessary notations and definitions. A finite difference method for one dimensional intuitionistic fuzzy heat equation is considered. Stability has been checked using Fourier or Von Neumann analysis. The explicit scheme is conditionally stable. In final part we present examples with numerical results.

Keywords : Intuitionistic Fuzzy Number, Fuzzy Heat Equation, Finite difference Scheme, Stability.

1. INTRODUCTION

The topics of different numerical methods for solving fuzzy differential and fuzzy partial differential equations have been rapidly growing in recent years. Researchers are trying to solve real life problems by modelling them mathematically. Many of those real life problems ultimately arise as a problem of solving partial differential equation. Most of such models contain parameters which involve uncertainties. One of the consequences of that is to consider the fuzzy partial differential equations (FPDE) as it can deal with uncertainties. So FPDE are more useful for such problems compared to PDE and they have various applications in different fields such as fluid mechanics, heat & mass transfer and electromagnetic theory. Intuitionistic fuzzy set is a generalization of fuzzy set. So we may get better result using intuitionistic fuzzy partial differential equations rather than fuzzy partial differential equations.

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The concept of fuzzy set and fuzzy number was first introduced by Zadeh(1965).There are several generalizations of fuzzy set; Intuitionstic Fuzzy set (IFS) is one of them. IFS was first introduced by Atanassov(1983). The concept of fuzzy derivative has been discussed by S.Seikkla in [14]. It was followed up by Dubois and Prade in [17], who defined and used extension principle. In [2] T.Allahviranloo used a numerical method to solve fuzzy partial differential equation, that was based on the derivative defined by Seikkla.

In this work, we solve heat equation with intuitionistic fuzzy parameters using finite difference method. We propose a new approach for finding the numerical solution of intuitionistic fuzzy partial differential equation, which has been applied for the solution of the proposed model.

2. PRELIMINARIES

Definition 2.1. Intuitionistic Fuzzy Sets (IFS): Let X be a finite set of elements. An IFS $\tilde{A}$ in X is defined as

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\}, \tag{2.1}$$

where the functions $\mu_{\tilde{A}} : X \mapsto [0,1]$ and $\nu_{\tilde{A}} : X \mapsto [0,1]$ respectively represent the degrees of membership and non-membership of the element $x \in X$ to the set $\tilde{A}$ such that $\mu_{\tilde{A}}(x) \in [0,1],\ \nu_{\tilde{A}}(x) \in [0,1],\ 0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$.

Definition 2.2. (α, β)**-cuts:** A subset (α, β)-cut of X, generated by IFS $\tilde{A}$, where $\alpha \in [0,1],\ \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$ defined as

$$\tilde{A}_{\alpha,\beta} = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha,\ \nu_{\tilde{A}}(x) \leq \beta\} \tag{2.2}$$

$\tilde{A}_{\alpha,\beta}$ is a crisp set of elements which belong to $\tilde{A}$ at least to the degree of α and which does not belong to $\tilde{A}$ at most to the degree of β.

Definition 2.3. Intuitionistic Fuzzy Number (IFN): An IFS $\tilde{A}$ on real line is called IFN if it satisfies the following conditions

1. there exists $x_o \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1 \ and \ \nu_{\tilde{A}}(x_0) = 0$
2. the membership function $\mu_{\tilde{A}}$ is convex
   i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq$ min$\{\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2);\ \forall x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]\}$
3. the non-membership function $\nu_{\tilde{A}}$ is concave
   i.e., $\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq$ max$\{\nu_{\tilde{A}}(x_1),\ \nu_{\tilde{A}}(x_2)\};\ \forall x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]\}$.

Definition 2.4. Triangular Intuitionistic Fuzzy Number (TIFN): A triangular intuitionistic fuzzy number $\tilde{A}$ is denoted by $\tilde{A} = \langle (a_1, a_2, a_3), (a_1', a_2, a_3') \rangle$. An IFN $\tilde{A}$ is called TIFN if its membership and non-membership functions follow the following rules:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \quad x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, \ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, \ a_2 \leq x \leq a_3 \\ 0, \quad x \geq a_3 \end{cases} \tag{2.3}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} 1, \quad x \leq a_1' \\ \frac{a_2-x}{a_2-a_1'}, \ a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2}, \ a_2 \leq x \leq a_3' \\ 1, \quad x \geq a_3' \end{cases} \tag{2.4}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$.
(α, β)-cut of $\tilde{A} = \langle (a_1, a_2, a_3), (a_1', a_2, a_3') \rangle$ may be represented as

$$\tilde{A}_{\alpha,\beta} = \langle [\underline{A}(\alpha), \overline{A}(\alpha)], [\underline{A'}(\beta), \overline{A'}(\beta)] \rangle,$$

where

$$\underline{A}(\alpha) = a_1 + \alpha(a_2 - a_1),$$

$$\overline{A}(\alpha) = a_3 - \alpha(a_3 - a_2),$$

$$\underline{A}'(\beta) = a_2 - \beta(a_2 - a_1')$$

and

$$\overline{A}'(\beta) = a_2 + \beta(a_3' - a_2).$$

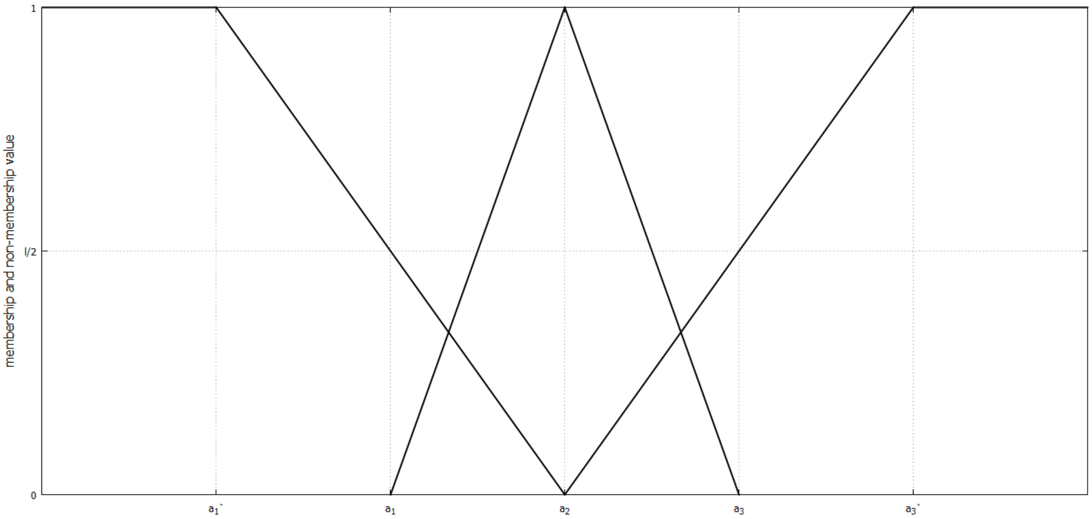

Figure 1: TIFN$\langle(a_1, a_2, a_3), (a'_1, a_2, a'_3)\rangle$

Definition 2.5. Fuzzy Arithmetic :

As discussed above , fuzzy numbers may be transformed into an interval through parametric form.

So, for any arbitrary fuzzy number $\tilde{x} = [\underline{x}(\alpha), \overline{x}(\alpha)]$,
$\tilde{y} = [\underline{y}(\alpha), \overline{y}(\alpha)]$, and scalar k, we have the interval based fuzzy arithmetic as

i) $\tilde{x} = \tilde{y}$ if and only if $\underline{x}(\alpha) = \underline{y}(\alpha)$ and $\overline{x}(\alpha) = \overline{y}(\alpha)$

ii) $\tilde{x} + \tilde{y} = [\underline{x}(\alpha) + \underline{y}(\alpha), \overline{x}(\alpha) + \overline{y}(\alpha)]$

iii) $\tilde{x} - \tilde{y} = [\underline{x}(\alpha), \overline{x}(\alpha)] - [\underline{y}(\alpha), \overline{y}(\alpha)] = [\underline{x}(\alpha) - \overline{y}(\alpha), \overline{x}(\alpha) - \underline{y}(\alpha)]$

iv) $\tilde{x} \times \tilde{y} = [\text{min}(\underline{x}(\alpha) \times \underline{y}(\alpha)),\ \underline{x}(\alpha) \times \overline{y}(\alpha),\ \overline{x}(\alpha) \times \underline{y}(\alpha),\ \overline{x}(\alpha) \times \overline{x}(\alpha)),\ \text{max}(\underline{x}(\alpha) \times \underline{y}(\alpha)),\ \underline{x}(\alpha) \times \overline{y}(\alpha), \overline{x}(\alpha) \times \underline{y}(\alpha),\ \overline{x}(\alpha) \times \overline{x}(\alpha)]$

v)

$$k\tilde{x} = \begin{cases} [k\underline{x}(\alpha), k\overline{x}(\alpha)],\ k \geq 0 \\ [k\overline{x}(\alpha), k\underline{x}(\alpha)],\ k < 0 \end{cases} \tag{2.5}$$

3. FINITE DIFFERENCE METHOD

In this section we solve the intuitionistic fuzzy heat equation by explicit method.
Now we consider the heat equation

$$(D_t - a^2 D_x^2)\tilde{U} = \tilde{0} \tag{3.1}$$

with boundary condition

$$\tilde{U}(0, t) = \tilde{U}(l, t) = \tilde{0}, \tag{3.2}$$

and initial condition

$$\tilde{U}(x,0)=\tilde{f}(x). \tag{3.3}$$

Let us consider $\tilde{U}$ be a fuzzy function of the independent crisp variables x and t. We define the domain

$$I=\{(x,t):0\le x\le l,0\le t\le T\}. \tag{3.4}$$

A (α,β)-cuts of $\tilde{U}(x,t)$ and its parametric form will be

$$\tilde{U}(x,t)[\alpha,\beta]=<[\underline{U}(x,t;\alpha),\overline{U}(x,t;\alpha)],\ [\underline{U'}(x,t;\beta),\overline{U'}(x,t;\beta)]> \tag{3.5}$$

We suppose that $\underline{U}(x,t;\alpha),\overline{U}(x,t;\alpha)),\underline{U'}(x,t;\beta)$ and $\overline{U'}(x,t;\beta))$ have continuous partial derivatives with respect to x and t, therefore $(D_t-a^2D_x^2)\underline{U}(x,t;\alpha)$,
$(D_t-a^2D_x^2)\overline{U}(x,t;\alpha)),(D_t-a^2D_x^2)\underline{U'}(x,t;\beta)$ and $(D_t-a^2D_x^2)\overline{U'}(x,t;\beta))$ are continuous for all $(x,t)\in I$, all $\alpha\in[0,1]$, $\beta\in[0,1]$.
Equation (3.1) can be decomposed as

$$(D_t)\underline{U}-a^2(D_x^2)\overline{U}=\underline{0}, \tag{3.6}$$

$$(D_t)\overline{U}-a^2(D_x^2)\underline{U}=\overline{0}, \tag{3.7}$$

$$(D_t)\underline{U'}-a^2(D_x^2)\overline{U'}=\underline{0}, \tag{3.8}$$

$$(D_t)\overline{U'}-a^2(D_x^2)\underline{U'}=\overline{0}. \tag{3.9}$$

We subdivide the x-t plane into sets of equal rectangles of sides $\delta x=h$, $\delta t=k$, by equally spaced grid lines parallel to t-axis, defined by $x_i=ih,i=0,1,2,.....$ and equally spaced grid lines parallel to x-axis, defined by $t_j=jk,j=0,1,2,......$
Denote the value of $\tilde{U}$ at the representative mesh point $P(ih,jk)$ by

$$\tilde{U_P}=\tilde{U}(ih,jk)=\tilde{U}_{i,j} \tag{3.10}$$

and also denote the parametric form of intuitionistic fuzzy number $\tilde{U}_{i,j}$, involving the parameters α and β, as

$$\tilde{U}_{i,j}=<[\underline{u}_{i,j}(\alpha),\overline{u}_{i,j}(\alpha)],[\underline{u'}_{i,j}(\beta),\overline{u'}_{i,j}(\beta)]>. \tag{3.11}$$

Then, we have

$$(D_t)\tilde{U}(x,t)=<[D_t\underline{U}(x,t;\alpha),D_t\overline{U}(x,t;\alpha)],[D_t\underline{U'}(x,t;\beta),D_t\overline{U'}(x,t;\beta)]>, \tag{3.12}$$

$$D_x^2\tilde{U}(x,t) =< [D_x^2\underline{U}(x,t;\alpha), D_x^2\overline{U}(x,t;\alpha)], [D_x^2\underline{U'}(x,t;\beta), D_x^2\overline{U'}(x,t;\beta)] > . \tag{3.13}$$

Following Farajzadeh et al (2010), using Taylor's theorem and definition of standard difference formula we obtain (see definition 2.5)

$$D_x^2\underline{U}(x,t;\alpha)|_{i,j} \simeq \frac{\underline{u}_{i+1,j} - 2\overline{u}_{i,j} + \underline{u}_{i-1,j}}{h^2}, \tag{3.14}$$

$$D_x^2\overline{U}(x,t;\alpha)|_{i,j} \simeq \frac{\overline{u}_{i+1,j} - 2\underline{u}_{i,j} + \overline{u}_{i-1,j}}{h^2}, \tag{3.15}$$

$$D_x^2\underline{U'}(x,t;\beta)|_{i,j} \simeq \frac{\underline{u'}_{i+1,j} - 2\overline{u'}_{i,j} + \underline{u'}_{i-1,j}}{h^2}, \tag{3.16}$$

$$D_x^2\overline{U'}(x,t;\beta)|_{i,j} \simeq \frac{\overline{u'}_{i+1,j} - 2\underline{u'}_{i,j} + \overline{u'}_{i-1,j}}{h^2}, \tag{3.17}$$

with a leading error of $O(h^2)$. The notation of forward-difference approximation for $(D_t)\tilde{U}$ at P, we have

$$D_t\underline{U}(x,t;\alpha)|_{i,j} \simeq \frac{\underline{u}_{i,j+1} - \overline{u}_{i,j}}{k}, \tag{3.18}$$

$$D_t\overline{U}(x,t;\alpha)|_{i,j} \simeq \frac{\overline{u}_{i,j+1} - \underline{u}_{i,j}}{k}, \tag{3.19}$$

$$D_x^2\underline{U'}(x,t;\beta)|_{i,j} \simeq \frac{\underline{u'}_{i,j+1} - \overline{u'}_{i,j}}{k}, \tag{3.20}$$

$$D_x^2\overline{U'}(x,t;\beta)|_{i,j} \simeq \frac{\overline{u'}_{i,j+1} - \underline{u'}_{i,j}}{k}, \tag{3.21}$$

with a leading error of O(k).

Using (3.16), (3.17), (3.18), (3.19), (3.20), (3.21), (3.22) and (3.23) the difference scheme of heat equation is

$$\frac{\underline{u}_{i,j+1} - \overline{u}_{i,j}}{k} - a^2\frac{\overline{u}_{i+1,j} - 2\underline{u}_{i,j} + \overline{u}_{i-1,j}}{h^2} = 0, \tag{3.22}$$

$$\frac{\overline{u}_{i,j+1} - \underline{u}_{i,j}}{k} - a^2\frac{\underline{u}_{i+1,j} - 2\overline{u}_{i,j} + \underline{u}_{i-1,j}}{h^2} = 0, \tag{3.23}$$

$$\frac{\underline{u}'_{i,j+1} - \overline{u'}_{i,j}}{k} - a^2 \frac{\overline{u'}_{i+1,j} - 2\underline{u}'_{i,j} + \overline{u'}_{i-1,j}}{h^2} = 0, \tag{3.24}$$

$$\frac{\overline{u'}_{i,j+1} - \underline{u}'_{i,j}}{k} - a^2 \frac{\underline{u}'_{i+1,j} - 2\overline{u'}_{i,j} + \underline{u}'_{i-1,j}}{h^2} = 0. \tag{3.25}$$

This can be written as

$$\underline{u}_{i,j+1} = r\overline{u}_{i-1,j} + (1-2r)\underline{u}_{i,j} + r\overline{u}_{i+1,j}, \tag{3.26}$$

$$\overline{u}_{i,j+1} = r\underline{u}_{i-1,j} + (1-2r)\overline{u}_{i,j} + r\underline{u}_{i+1,j}, \tag{3.27}$$

$$\underline{u}'_{i,j+1} = r\overline{u'}_{i-1,j} + (1-2r)\underline{u}'_{i,j} + r\overline{u'}_{i+1,j}, \tag{3.28}$$

$$\overline{u'}_{i,j+1} = r\underline{u}'_{i-1,j} + (1-2r)\overline{u'}_{i,j} + r\underline{u}'_{i+1,j}, \tag{3.29}$$

where $r = \frac{a^2 k}{h^2}$.

4. STABILITY OF HEAT EQUATION

The Von Neumann method is based on the decomposition of the errors into Fourier series.
Consider the 1D fuzzy heat equation

$$\frac{\partial \tilde{U}}{\partial t}(x,t) - a\frac{\partial^2 \tilde{U}}{\partial x^2}(x,t) = 0, 0 < x < 1, t > 0. \tag{4.1}$$

Which can be discretized as the equation

$$u_{i,j+1} = u_{i-1,j} + (1-2r)u_{i,j} + u_{i+1,j} \tag{4.2}$$

where $r = \frac{\alpha k}{h^2}$ and the solution $u_{i,j}$ of the discrete equation approximates the analytical solution $U(x,t)$ of the PDE on the grid. The round of error $\epsilon_{i,j}$ as

$$\epsilon_{i,j} = N_{i,j} - u_{i,j} \tag{4.3}$$

where $u_{i,j}$ is the solution of the discretized equation (4.1). The exact solution $u_{i,j}$ must satisfy the discretized equation exactly, the error $\epsilon_{i,j}$ also satisfy this equation. Thus

$$\epsilon_{i,j+1} = \epsilon_{i-1,j} + (1-2r)\epsilon_{i,j} + \epsilon_{i+1,j} \tag{4.4}$$

is recurrence relation for the error. Equation (4.2) and (4.4) shows that both the error and numerical solution have the same growth with respect to time. For periodic boundary condition we may use the finite Fourier series with respect to x,

$$\epsilon(x,t) = \sum_{m=M} E_m(t)e^{ik_m x} \tag{4.5}$$

where $k_m = \frac{\pi m}{2}$ with $m = -M, ..., -2, -1, 0, 1, 2, ...M$, $M = \frac{L}{h}$
If the boundary condition is not periodic then we use the finite Fourier Integral with respect to x

$$\epsilon(x,t) = \int_{-\frac{\pi}{h}}^{-\frac{\pi}{h}} K_k(t)e^{ikx}dk \tag{4.6}$$

Consider the growth of error of a typical term

$$\epsilon_m(x,t) = E_m(t)e^{ik_m x} \tag{4.7}$$

if a Fourier series is used.

$$\epsilon_k(x,t) = K_k(t)e^{ikx} \tag{4.8}$$

if a Fourier series is used. the characteristic of the stability can be studied using just this form for the error with no loss in generality. To find out how error varies in steps of time.
Using equation (4.7) into equation (4.4)

$$\epsilon_{i,j} = E_m(t)e^{ik_m x} \tag{4.9}$$

$$\epsilon_{i,j+1} = E_m(t+k)e^{ik_m x} \tag{4.10}$$

$$\epsilon_{i+1,j} = E_m(t)e^{ik_m(x+h)} \tag{4.11}$$

$$\epsilon_{i-1,j} = E_m(t)e^{ik_m(x-h)} \tag{4.12}$$

Then after simplification

$$\frac{E_m(t+k)}{E_m(t)} = 1 + r(e^{ik_m h} + e^{-ik_m h} - 2) \tag{4.13}$$

Introducing $\theta = K_m h$, $Sin(\frac{\theta}{2}) = \frac{e^{\frac{i\theta}{2}} - e^{-\frac{i\theta}{2}}}{2i} \Rightarrow Sin^2(\frac{\theta}{2}) = \frac{e^{i\theta}+e^{-i\theta}-2}{4}$

$$\frac{E_m(t+k)}{E_m(t)} = 1 - 4rSin^2(\frac{\theta}{2}) \tag{4.14}$$

The amplification factor

$$G = \frac{E_m(t+k)}{E_m(t)} \tag{4.15}$$

The necessary and sufficient condition for the error remain bounded is that $\mid G \mid \leq 1$
$\Rightarrow \mid 1 - 4rSin^2\frac{\theta}{2} \mid \leq 1$
$\Rightarrow 4rSin^2\frac{\theta}{2} \leq 2$

$$r = \frac{ak}{h^2} \leq \frac{1}{2} \tag{4.16}$$

Therefore our difference scheme is conditionally stable.

5. NUMERICAL EXAMPLE

Example 5.1. This example can be found in [7]

$$\frac{\partial \tilde{U}}{\partial t}(x,t) - \frac{\partial^2 \tilde{U}}{\partial x^2}(x,t) = 0, \quad 0 < x < l, \quad t > 0, \tag{5.1}$$

with the boundary conditions

$$\tilde{U}(0,t) = \tilde{U}(l,t) = 0, \quad t > 0 \tag{5.2}$$

and initial condition

$$\tilde{U}(x,0) = \tilde{f}(x) = \tilde{K}\cos(\pi x - \pi/2), \quad 0 \leq x \leq 1, \tag{5.3}$$

and

$$\tilde{K}[\alpha,\beta] =< [\underline{K}(\alpha), \overline{K}(\alpha)], [\underline{K}(\beta), \overline{K}(\beta)] >=< [\alpha - 1, 1 - \alpha], [-1.25\beta, 1.25\beta] > \tag{5.4}$$

The exact solutions for

$$\frac{\partial \underline{U}}{\partial t}(x,t;\alpha) - \frac{\partial^2 \underline{U}}{\partial x^2}(x,t;\alpha) = 0, \; 0 < x < 1, \; t > 0, \tag{5.5}$$

$$\frac{\partial \overline{U}}{\partial t}(x,t;\alpha) - \frac{\partial^2 \overline{U}}{\partial x^2}(x,t;\alpha) = 0, \; 0 < x < 1, \; t > 0, \tag{5.6}$$

$$\frac{\partial \underline{U'}}{\partial t}(x,t;\beta) - \frac{\partial^2 \underline{U'}}{\partial x^2}(x,t;\beta) = 0, \; 0 < x < 1, \quad t > 0, \tag{5.7}$$

$$\frac{\partial \overline{U'}}{\partial t}(x,t;\beta) - \frac{\partial^2 \overline{U'}}{\partial x^2}(x,t;\beta) = 0, \; 0 < x < 1, \; t > 0 \tag{5.8}$$

are, respectively

$$\underline{U}(x,t;\alpha) = \underline{K}(\alpha)e^{-\pi^2 t}\cos(\pi x - \pi/2) \tag{5.9}$$

$$\overline{U}(x,t;\alpha) = \overline{K}(\alpha)e^{-\pi^2 t}\cos(\pi x - \pi/2) \tag{5.10}$$

$$\underline{U'}(x,t;\beta) = \underline{K}(\beta)e^{-\pi^2 t}\cos(\pi x - \pi/2) \tag{5.11}$$

$$\overline{U'}(x,t;\beta) = \overline{K}(\beta)e^{-\pi^2 t}\cos(\pi x - \pi/2) \tag{5.12}$$

It is clear that the partial derivatives of $\frac{\partial \overline{U}}{\partial t}$ and $\frac{\partial^2 \overline{U}}{\partial x^2}$ exist. Using the equations (3.28), (3.29), (3.30) and (3.31) the exact solution with $h = 0.1$ and $k = 0.000001$, therefore $r = 0.0001$ is presented below. This figure shows the approximate and exact solution at the point $(0.3, 0.000002)$.

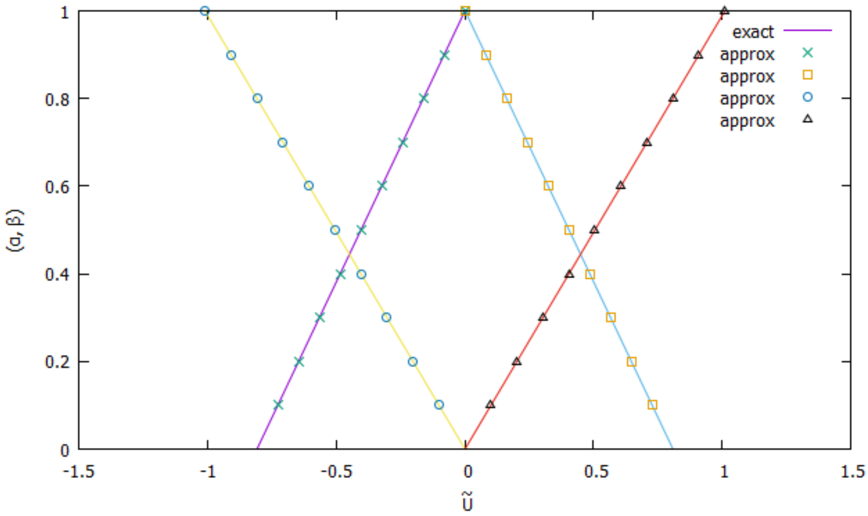

Figure 2: $h = 0.1, k = 0.000001, r = 0.0001$

Example 5.2. This example can be found in [1]

$$\frac{\partial \tilde{U}}{\partial t}(x,t) = 4\frac{\partial^2 \tilde{U}}{\partial x^2}(x,t), \quad 0 < x < 1, \quad t > 0, \tag{5.13}$$

with the boundary conditions

$$\tilde{U}(0,t) = \tilde{U}(1,t) = 0, \quad t > 0 \tag{5.14}$$

and initial condition

$$\tilde{U}(x,0) = \tilde{f}(x) = \frac{2}{\pi}\tilde{K}\sin\pi x, \quad 0 \leq x \leq 1, \tag{5.15}$$

and

$$\tilde{K}[\alpha,\beta] =< [\underline{K}(\alpha), \overline{K}(\alpha)], [\underline{K}(\beta), \overline{K}(\beta)] >=< [\alpha - 1, 1-\alpha], [-1.2\beta, 1.2\beta] > \tag{5.16}$$

The exact solutions for

$$\frac{\partial \underline{U}}{\partial t}(x,t;\alpha) = 4\frac{\partial^2 \underline{U}}{\partial x^2}(x,t;\alpha), \quad 0 < x < 1, \quad t > 0, \tag{5.17}$$

$$\frac{\partial \overline{U}}{\partial t}(x,t;\alpha) = 4\frac{\partial^2 \overline{U}}{\partial x^2}(x,t;\alpha), \quad 0 < x < 1, \quad t > 0, \tag{5.18}$$

$$\frac{\partial \underline{U'}}{\partial t}(x,t;\beta) = 4\frac{\partial^2 \underline{U'}}{\partial x^2}(x,t;\beta), \quad 0 < x < 1, \quad t > 0, \tag{5.19}$$

$$\frac{\partial \overline{U'}}{\partial t}(x,t;\beta) = 4\frac{\partial^2 \overline{U'}}{\partial x^2}(x,t;\beta) = 0, \quad 0 < x < 1, \quad t > 0 \tag{5.20}$$

are, respectively

$$\underline{U}(x,t;\alpha) = \frac{2}{\pi}\underline{K}(\alpha)e^{-4\pi^2 t}\sin\pi x \tag{5.21}$$

$$\overline{U}(x,t;\alpha) = \frac{2}{\pi}\overline{K}(\alpha)e^{-4\pi^2 t}\sin\pi x \tag{5.22}$$

$$\underline{U'}(x,t;\beta) = \frac{2}{\pi}\underline{K}(\beta)e^{-4\pi^2 t}\sin\pi x \tag{5.23}$$

$$\overline{U'}(x,t;\beta) = \frac{2}{\pi}\overline{K}(\beta)e^{-4\pi^2 t}\sin\pi x \tag{5.24}$$

It is clear that the partial derivatives of $\frac{\partial \overline{U}}{\partial t}$ and $\frac{\partial^2 \overline{U}}{\partial x^2}$ exist. Using the equations (3.28), (3.29), (3.30) and (3.31) the exact solution with $h = 0.01$ and $k = 0.00001$, therefore $r = 0.4$ is presented below . This figure shows the approximate and exact solution at the point $(0.04, 0.00002)$.

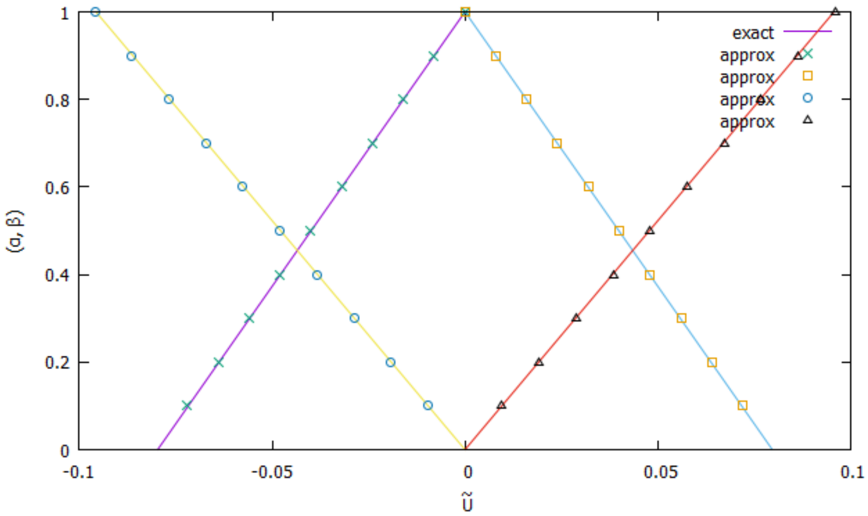

Figure 3: $h = 0.01, k = 0.00001, r = 0.4$

6. CONCLUSION

The intuitionistic fuzzy partial differential equation can be applied for modeling in physics, engineering and mechanical system. In this paper we applied an explicit finite difference method to solve intuitionistic fuzzy heat equation. Future work may focus on intuitionistic fuzzy wave equation.

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