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Intuitionistic fuzzy semi γ* generalized continuous mappings

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Abstract

In this paper we have introduced intuitionistic fuzzy semi γ* generalized continuous mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy semi γ* generalized continuous mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy semi γ^*_γ $T_{1/2}$ space, intuitionistic fuzzy semi γ* generalized continuous mappings, intuitionistic fuzzy semi γ* generalized irresolute mappings.

1. Introduction

Atanassov [2] introduced intuitionistic fuzzy sets. Coker [3] introduced intuitionistic fuzzy topological spaces. Abinaya and Jayanthi [1] introduced intuitionistic fuzzy semi γ* generalized closed sets. In this paper, we have introduced intuitionistic fuzzy semi γ* generalized continuous mappings and intuitionistic fuzzy semi γ* generalized irresolute mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy semi γ* generalized continuous mappings.

2. Preliminaries

Definition 2.1: [2] An **intuitionistic fuzzy set** (IFS) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2: [2] Let A and B be two IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$.

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_{\sim} = \langle x, 0, 1\rangle$ and $1_{\sim} = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An **intuitionistic fuzzy topology** (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_{\sim}, 1_{\sim} \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in I\} \in \tau$.

In this case the pair (X, τ) is called an **intuitionistic fuzzy topological space** (IFTS) and any IFS in τ is known as an **intuitionistic fuzzy open set** (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an **intuitionistic fuzzy closed set** (IFCS) in X.

Definition 2.4: [9] Two IFSs A and B are said to be **q-coincident** ($A_q B$) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5: [9] Two IFSs A and B are said to be **not q-coincident** ($A_{\bar{q}} B$) if and only if $A \subseteq B^c$.

Definition 2.6: [4] An **intuitionistic fuzzy point** (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(X) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.7: [1] An IFS A of an IFTS (X, τ) is said to be **an intuitionistic fuzzy semi γ* generalized closed set** (IF semi γ*GCS) if int(cl(A)) ∩ cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is an IFSOS in (X, τ).

The complement A^c of an IF semi γ*GCS A in an IFTS (X, τ) is called an **intuitionistic fuzzy semi γ* generalized open set** (IF semi γ*GOS) in X.

Definition 2.8: [1] An IFTS (X, τ) is an **intuitionistic fuzzy semi γ^*_γ $T_{1/2}$ space** (IF semi γ^*_γ $T_{1/2}$ space) if every IF semi γ*GCS is an IFγCS in X.

Definition 2.9: [1] An IFTS (X, τ) is an **intuitionistic fuzzy semi γ^*_c $T_{1/2}$ space** (IF semi γ^*_c $T_{1/2}$ space) if every IF semi γ*GCS is an IFCS in X.

Definition 2.10: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an **intuitionistic fuzzy** (IF) **continuous mapping** if $f^{-1}(V)$ is an IFCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.11: [8] Let (X, τ) be an IFTS and A = $\langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the **intuitionistic fuzzy γ interior** and **intuitionistic fuzzy γ closure** are defined by

γint(A) = ∪{ G/ G is an IFγOS in X and G ⊆ A },

γcl(A) = ∩{ K/ K is an IFγCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A^c) = (\gamma cl(A))^c$.

Definition 2.12: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) **intuitionistic fuzzy semi continuous** (IFS continuous) **mapping** if $f^{-1}(B) \in$ IFSO(X) for every B ∈ σ.

(ii) **intuitionistic fuzzy α continuous** (IFα continous) **mapping** if $f^{-1}(B) \in$ IFαO(X) for every B ∈ σ.

(iii) **intuitionistic fuzzy pre continuous** (IFP continuous) **mapping** if $f^{-1}(B) \in$ IFPO(X) for every B ∈ σ.

Definition 2.13: [6] A mapping f : (X, τ) → (Y, σ) is called an **intuitionistic fuzzy γ continuous** (IFγ continuous) **mapping** if $f^{-1}(B)$ is an IFγOS in (X, τ) for every B ∈ σ.

Definition 2.14: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an **intuitionistic fuzzy generalized continuous** (IFG continuous) **mapping** if $f^{-1}(B)$ ∈ IFGCS(X) for every IFCS B in Y.

3. Intuitionistic fuzzy semi γ* generalized continuous mappings

In this section we have introduced intuitionistic fuzzy semi γ* generalized continuous mappings and investigated some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an **intuitionistic fuzzy semi γ* generalized** (IF semi γ*G) **continuous mapping** if $f^{-1}(V)$ is an IF semi γ*GCS in (X, τ) for every IFCS V of (Y, σ).

For the sake of simplicity, we shall use the notation A = ⟨x, (μ_a, μ_b), (ν_a , ν_b)⟩ instead of A = ⟨x, (a/μ_a, b/μ_b), (a/ν_a , b/ν_b)⟩and B = ⟨y, (μ_u, μ_v), (ν_u ,ν_v)⟩ instead of B = ⟨y, (u/μ_u, v/μ_v), (u/ν_u , v/ν_v)⟩ in the following examples.

Example 3.2: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩ and G_2 = ⟨y, (0.8_u, 0.7_v), (0.2_u, 0.3_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_2^c = ⟨y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)⟩ is an IFCS in Y. Then f^{-1} (G_2^c) = ⟨x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)⟩ is an IFS in X.

Here IFSO(X) = { $0_\sim$,$1_\sim$, μ_a ∈ [0,1], μ_b ∈ [0,1], ν_a ∈ [0,1], ν_b ∈ [0,1] / $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $\nu_a \leq 0.5$, $\nu_b \leq 0.4$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Hence $f^{-1}(G_3^c)$ is an IF semi γ*GCS in (X, τ). Therefore f is an IF semi γ*G continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IF semi γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IF semi γ*GCS, $f^{-1}(V)$ is an IF semi γ*GCS in X. Hence f is an IF semi γ*G continuous mapping.

Example 3.4: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ and G_3 = ⟨y, (0.5_u, 0.6_v), (0.3_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here G_3^c = ⟨y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)⟩ is an IFCS in Y. Then $f^{-1}(G_3^c)$ = ⟨x, (0.3_a, 0.4_b), (0.5_a, 0.6_b)⟩ is an IFS in X.

IFSO(X) = { $0_\sim, 1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0.4 \leq \mu_a \leq 0.5$, $0.4 \leq \mu_b \leq 0.6$, $0.5 \leq \nu_a \leq 0.6$, $0.4 \leq \nu_b \leq 0.7$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Hence $f^{-1}(G_3^c)$ is an IF semi γ*GCS in (X, τ). Therefore f is an IF semi γ*G continuous mapping, but not an IF continuous mapping, since $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$.

Theorem 3.5: Every IFS continuous mapping is an IF semi γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IF semi γ*GCS, $f^{-1}(V)$ is an IF semi γ*GCS in X. Hence f is an IF semi γ*G continuous mapping.

Example 3.6: In example 3.4, f is an IF semi γ*G continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = int(G_1^c) = G_1 \not\subseteq f^{-1}(G_3^c)$, f is not an IFS continuous mapping.

Theorem 3.7: Every IFP continuous mapping is an IF semi γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IF semi γ*GCS, $f^{-1}(V)$ is an IF semi γ*GCS in X. Hence f is an IF semi γ*G continuous mapping.

Example 3.8: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here $G_3^c = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ is an IFS in X.

IFSO(X) = { $0_\sim, 1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0.4 \leq \mu_a \leq 0.5$, $0.4 \leq \mu_b \leq 0.6$, $0.5 \leq \nu_a \leq 0.6$, $0.4 \leq \nu_b \leq 0.7$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Then f is an IF semi γ*G continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int((f^{-1}(G_3^c))) = cl(G_1) = G_1^c \not\subseteq f^{-1}(G_3^c)$, f is not an IFP continuous mapping.

Theorem 3.9: Every IFα continuous mapping is an IF semi γ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IF semi γ*GCS, $f^{-1}(V)$ is an IF semi γ*GCS in X. Hence f is an IF semi γ*G continuous mapping.

Example 3.10: In example 3.4, f is an IF semi γ*G continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl((f^{-1}(G_3^c)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \not\subseteq f^{-1}(G_3^c)$, f is not an IFα continuous mapping.

Theorem 3.11: Every IFγ continuous mapping is an IF semi γ*G continuous mapping but not conversely in general.

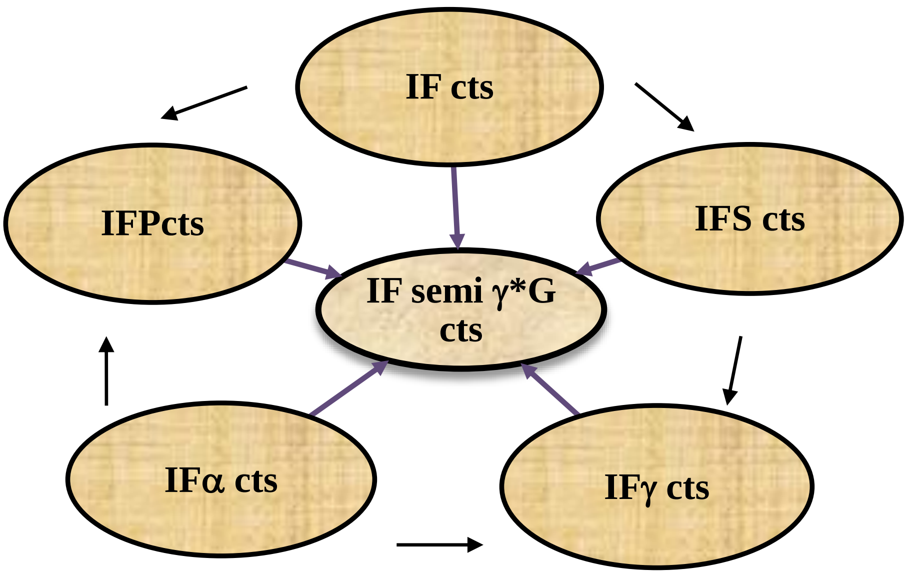
Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an IFγ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IF semi γ*GCS, $f^{-1}(V)$ is an IF semi γ*GCS in X. Hence f is an IF semi γ*G continuous mapping.

Example 3.12: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Here $G_3^c = \langle y, (0.5_u, 0.3_v), (0.5_u, 0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.6_b)\rangle$ is an IFS in X.

IFSO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, 0.4 \leq \mu_a \leq 0.5, 0.4 \leq \mu_b \leq 0.6, 0.5 \leq \nu_a \leq 0.6, 0.4 \leq \nu_b \leq 0.7$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

Then f is an IF semi γ*G continuous mapping, but since $f^{-1}(G_3^c)$ is not an IFγCS in X, as $cl(int(f^{-1}(G_3^c))) \cap int(cl(f^{-1}(G_3^c))) = cl(G_2) \cap int(G_1^c) = G_1^c \cap G_1 = G_1 \nsubseteq f^{-1}(G_3^c)$, f is not an IF$\gamma$ continuous mapping.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous.



The reverse implications are not true in general in the above diagram.

Theorem 3.13: A mapping f: (X, τ) $\rightarrow$ (Y, σ) is an IF semi γ*G continuous mapping if and only if the inverse image of each IFOS in Y is an IF semi γ*GOS in X.

Proof: Necessity: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Then $f^{-1}(A^c)$ is an IF semi γ*GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IF semi γ*GOS in X.

Sufficiency: Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is an IF semi γ*GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IF semi γ*GCS in X. Hence f is an IF semi γ*G continuous mapping.

Theorem 3.14: If f: (X, τ) → (Y, σ) is an IF semi γ*G continuous mapping. Then for each IFP $p_{(\alpha, \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha, \beta)}) \in A$, there exists an IF semi γ*GOS B of X such that $p_{(\alpha, \beta)} \in B$ and f(B) ⊆ A.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha, \beta)}) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is an IF semi γ*GOS in X such that $p_{(\alpha, \beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.15: If f: (X, τ) → (Y, σ) is an IF semi γ*G continuous mapping. Then for each IFP $p_{(\alpha, \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha, \beta)})\ _q\ A$, there exists an IF semi γ*GOS B of X such that $p_{(\alpha, \beta)}\ _q\ B$ and f(B) ⊆ A.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha, \beta)})\ _q\ A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is an IF semi γ*GOS in X such that $p_{(\alpha, \beta)}\ _q\ B$ and $f(B) = f\ (f^{-1}(A)) \subseteq A$.

Theorem 3.16: Let f: (X, τ) → (Y, σ) be an IF semi γ*G continuous mapping and g: (Y, σ) → (Z, δ) is an IF continuous mapping, then g o f: (X, τ) → (Z, δ) is an IF semi γ*G continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IF semi γ*G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IF semi γ*GCS in X. Hence g o f is an IF semi γ*G continuous mapping.

Theorem 3.17: Let f: (X, τ) → (Y, σ) be an IF semi γ*G continuous mapping, then f is an IFγ continuous mapping if X is an IF semi γ^*_γ $T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IF semi γ*GCS in X, by hypothesis. Since X is an IF semi γ^*_γ $T_{1/2}$ space, $f^{-1}(V)$ is an IFγCS in X. Hence f is an IFγ continuous mapping.

Theorem 3.18: Let f: (X, τ) → (Y, σ) be an IF semi γ*G continuous mapping, then f is an IF continuous mapping if X is an IF semi γ^*_c $T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IF semi γ*GCS in X, by hypothesis. Since X is an IF semi γ_c^* $T_{1/2}$ space, $f^{-1}(V)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.19: A mapping f: (X, τ) → (Y, σ) is an IF semi γ*G continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: Let A be an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. Therefore $f^{-1}(A)$ is an IFαCS and hence it is an IF semi γ*GCS. Thus f is an IF semi γ*G continuous mapping.

Theorem 3.20: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) = int(cl(f^{-1}(B)))$ for every IFS B in Y. Then f is an IF semi γ*G continuous mapping.

Proof: Let B be an IFOS in Y. Then int(B) = B and by hypothesis $f^{-1}(B) = int(cl(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFROS in X. Therefore it is an IF semi γ*GOS in X. Hence f is an IF semi γ*G continuous mapping.

Theorem 3.21: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IF semi γ_γ^* $T_{1/2}$ space:

(i) f is an IF semi γ*G continuous mapping,

(ii) If B is an IFOS in Y, then $f^{-1}(B)$ is an IF semi γ*GOS in X,

(iii) $f^{-1}(int(B)) \subseteq int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))$ for every IFS B in Y.

Proof: (i) ⇔ (ii) is obviously true by Theorem 3.13.

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IF semi γ*GOS in X. Since X is an IF semi γ_γ^* $T_{/1/2}$ space, $f^{-1}(int(B))$ is an IFγOS in X. Therefore $f^{-1}(int(B)) \subseteq int(cl(f^{-1}(int(B)))) \cup cl(int(f^{-1}(int(B)))) \subseteq int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))$.

(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement, say A is an IFOS in Y, then int(A) = A. Now by hypothesis $f^{-1}(A) = f^{-1}(int(A)) \subseteq int(cl(f^{-1}(A))) \cup cl(int(f^{-1}(A)))$. This implies $f^{-1}(A) \subseteq int(cl(f^{-1}(A))) \cup cl(int(f^{-1}(A)))$. Hence $f^{-1}(A)$ is an IFγOS in X. Since every IFγOS is an IF semi γ*GOS, $f^{-1}(A)$ is an IF semi γ*GOS in X. Thus $f^{-1}(B)$ is an IF semi γ*GCS in X, since $f^{-1}(A) = f^{-1}(B^c)$. Hence f is an IF semi γ*G continuous mapping.

Theorem 3.22: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IF semi γ^*_γ $T_{1/2}$ space:

(i) f is an IF semi γ*G continuous mapping,

(ii) $int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: (i) ⇒ (ii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IF semi γ*GCS in X. Since X is an IF semi γ^*_γ $T_{1/2}$ space, $f^{-1}(cl(A))$ is an IFγCS in X. Therefore $int(cl(f^{-1}(cl(A)))) \cap cl(int(f^{-1}(cl(A)))) \subseteq f^{-1}(cl(A))$. Now $int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))) \subseteq int(cl(f^{-1}(cl(A)))) \cap cl(int(f^{-1}(cl(A)))) \subseteq f^{-1}(cl(A))$.

(ii) ⇒ (i) Let A be an IFCS in Y. By hypothesis $int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFγCS in X and hence it is an IF semi γ*GCS. Thus f is an IF semi γ*G continuous mapping.

Theorem 3.23: If f: (X, τ) → (Y, σ) is an IF semi γ*G continuous mapping and g: (Y, σ) → (Z, δ) is an IFG continuous mapping and Y is an $IFT_{1/2}$ space, then g *o* f : (X, τ) → (Z, δ) is an IF semi γ*G continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGCS in Y, by hypothesis. Since Y is an $IFT_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IF semi γ*GCS in X, by hypothesis. Hence g *o* f is an IF semi γ*G continuous mapping.

Theorem 3.24: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IF semi γ^*_γ $T_{1/2}$ space:

(i) f is an IF semi γ*G continuous mapping,

(ii) $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$ for each IFCS B in Y,

(iii) $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$ for each IFOS B of Y,

(iv) $f(int(cl(A)) \cap cl(int(A))) \subseteq cl(f(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IF semi γ*GCS in X, by hypothesis. Since X is an IF semi γ^*_γ $T_{/1/2}$ space, $f^{-1}(B)$ is an IFγCS in X. Therefore $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(cl(B))$.

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let A ⊆ X. Then B = f(A) in Y and therefore $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(int(B)))) \cup cl(intf^{-1}(int(B))))) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$. Now $(int(cl(A^c)) \cup cl(int(A^c)))^c \subseteq (int(cl(f^{-1}(B)^c)) \cup cl(int(f^{-1}(B)^c)))^c \subseteq (f^{-1}(int(B^c)))^c$.

Therefore int(cl(A)) ∩ cl(int(A)) ⊆ f^{-1}(cl(B)). Now f(int(cl(A)) ∩ cl(int(A))) ⊆ f(f^{-1}(cl(B))) ⊆ cl(B) = cl(f(A)).

(iv) ⇒ (i) Let B be any IFCS in Y, then f^{-1}(B) is an IFS in X. By hypothesis f(int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B)))) ⊆ cl(f(f^{-1} (B))) ⊆ cl(B) = B. Now (int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B)))) ⊆ f^{-1} (f((int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B)))))) ⊆ f^{-1} (B). This implies f^{-1}(B) is an IFγCS and hence it is an IF semi γ*GCS in X. Thus f is an IF semi γ*G continuous mapping.

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