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α, β, γ, δ Parametric form for Solving Intuitionistic Fuzzy Linear System of Equations

Bidhan Chandra Saw* and Subhendu Bikash Hazra

Department of Mathematics, Bankura University, Bankura, West Bengal-722101, India.

Abstract

In this paper we have developed a method to solve intuitionistic fuzzy linear system of equations (IFLS). In this method, the intuitionistic fuzzy system has been converted to a crisp system of linear equations with the help of four parametric form of intuitionistic fuzzy numbers. Using four parametric form, the $n \times n$ IFLS has been converted to two $n \times n$ crisp systems, which then have been solved using a new method.

Keywords : Non-negative Intuitionistic Fuzzy Number, Parametric form of Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Linear Systems(IFLS)

1. INTRODUCTION

System of linear equations is one of the most important topics in research due to its applicability in mathematics, physics, and other science as well as engineering subjects. In real life situation we often get parameters as a result of some experiment, estimation, observation or modelling. Since all these processes involve uncertainty at some point or the other, the natural choice is to consider fuzzy parameters instead of crisp ones. Fuzzy parameters handle uncertainty and vaguness. In fuzzy set theory, membership of an element plus non-membership of that element is equal to one. In real life situation it may not be always true that the degree of membership of an element plus the degree of non-membership of that element is equal to one, because there may exist some hesitation degree. In intuitionistic fuzzy set this hesitation degree is also considered. So intuitionistic fuzzy set(number) seems to fit more suitably to describe uncertainty, therefore it is more useful to take intuitionistic fuzzy set(number) rather than fuzzy set(number) .

*Corresponding author

The concept of fuzzy set and fuzzy numbers has been first introduced by Zadeh [25]. Since then, there have several generalizations of fuzzy set; intuitionistic fuzzy set (IFS) is one of them. IFS was first introduced by Atanassov [5–8]. Intuitionistic fuzzy linear systems (IFLS) are the linear systems whose parameters are all or partially represented by intuitionistic fuzzy numbers (IFN). Friedman et al. [17] first proposed a general model for solving $n \times n$ fuzzy linear systems (FLS) with the coefficient matrix consisting of crisp numbers and right hand side column vector consisting of arbitrary fuzzy numbers. By using parametric form of fuzzy number they converted $n \times n$ FLS into $2n \times 2n$ crisp systems. Behera et al. [15] gave a new model for solving FLS. They converted $n \times n$ fuzzy system into $n \times n$ crisp system by using double parametric form. Fuzzy system of linear equations has been studied by several authors [1–4, 11–13, 16, 18–22, 24]. Banerjee et al. [10] developed an approach to solve intuitionistic fuzzy linear systems (IFLS). They converted $m \times n$ system of IFLS into two $2m \times 2n$ crisp linear systems. Atti et al. [9] also developed an approach to solve IFLS. They converted $n \times n$ system of equation into four $n \times n$ crisp linear systems. Solvability of system of intuitionistic fuzzy linear equations was studied by Pradhan et al. [23].

In this paper, we have developed an approach to solve IFLS following Behera et al. [15],which originally was developed for FLS. The paper is organised as follows: In the next section we review some definitions, in section (3) we discuss in detail the proposed method of solution and in section (4) numerical examples have been included which are solved using the new method. The last section includes the conclusion.

2. PRELIMINARIES

Definition 2.1. Intuitionistic Fuzzy Sets: Let X be a finite set of elements. An IFS $\tilde{A}$ in X is defined as

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\},$$

where the functions $\mu_{\tilde{A}} : X \mapsto [0,1]$ and $\nu_{\tilde{A}} : X \mapsto [0,1]$ respectively represent the degrees of membership and non-membership of the element $x \in X$ to the set $\tilde{A}$ such that $\mu_{\tilde{A}}(x) \in [0,1],\ \nu_{\tilde{A}}(x) \in [0,1],\ 0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1.$

Definition 2.2. (α, β)**-cuts:** A subset (α, β)-cut of X, generated by IFS $\tilde{A}$, where $\alpha \in [0,1],\ \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$ defined as

$$\tilde{A}_{\alpha,\beta} = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha,\ \nu_{\tilde{A}}(x) \leq \beta\}$$

$\tilde{A}_{\alpha,\beta}$ is a crisp set of elements which belong to $\tilde{A}$ at least to the degree of α and which does not belong to $\tilde{A}$ at most to the degree of β.

Definition 2.3. Intuitionistic Fuzzy Number(IFN): An IFS $\tilde{A}$ on real line is called IFN if it satisfies the following conditions

1. there exists $x_o \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1 \; and \; \nu_{\tilde{A}}(x_0) = 0$
2. the membership function $\mu_{\tilde{A}}$ is convex
   i.e, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2);\} \, \forall x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]$
3. the non-membership function $\nu_{\tilde{A}}$ is concave
   i.e, $\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max\{\nu_{\tilde{A}}(x_1),\ \nu_{\tilde{A}}(x_2)\};\, \forall x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]$.

Definition 2.4. Triangular Intuitionistic Fuzzy Number (TIFN): A triangular intuitionistic fuzzy number $\tilde{A}$ is denoted by $\tilde{A} = \langle (l_1, l_2, l_3), (l'_1, l_2, l'_3) \rangle$. An IFN $\tilde{A}$ is called TIFN if its membership and non-membership functions follow the following rules:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \quad x \leq l_1 \\ \frac{x-l_1}{l_2-l_1},\ l_1 \leq x \leq l_2 \\ \frac{l_3-x}{l_3-l_2},\ l_2 \leq x \leq l_3 \\ 0, \quad x \geq l_3 \end{cases}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} 1, \quad x \leq l'_1 \\ \frac{l_2-x}{l_2-l'_1},\ l'_1 \leq x \leq l_2 \\ \frac{x-l_2}{l'_3-l_2},\ l_2 \leq x \leq l'_3 \\ 1, \quad x \geq l'_3 \end{cases}$$

where $l'_1 \leq l_1 \leq l_2 \leq l_3 \leq l'_3$.
(α, β)-cut of $\tilde{A} = \langle (l_1, l_2, l_3), (l'_1, l_2, l'_3) \rangle$ may be represented as
$\tilde{A}_{\alpha,\beta} = \langle [\underline{A}(\alpha), \overline{A}(\alpha)], [\underline{A'}(\beta), \overline{A'}(\beta)] \rangle$, where $\underline{A}(\alpha) = l_1 + \alpha(l_2 - l_1)$, $\overline{A}(\alpha) = l_3 - \alpha(l_3 - l_2)$, $\underline{A}'(\beta) = l_2 - \beta(l_2 - l'_1)$ and $\overline{A}'(\beta) = l_2 + \beta(l'_3 - l_2)$.

Definition 2.5. Trapezoidal Intuitionsitic fuzzy number (TrIFN): A trapezoidal intuitionistic fuzzy number $\tilde{A}$ is denoted by $\tilde{A} = \langle (l_1, l_2, l_3, l_4), (l'_1, l_2, l_3, l'_4) \rangle$. An IFN $\tilde{A}$ is called TrIFN if its membership and non-membership functions follow the following rule:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \quad x \leq l_1 \\ \frac{x-l_1}{l_2-l_1},\ l_1 \leq x \leq l_2 \\ 1,\ l_2 \leq x \leq l_3 \\ \frac{l_4-x}{l_4-l_3},\ l_3 \leq x \leq l_4 \\ 0, \quad x \geq l_4 \end{cases}$$

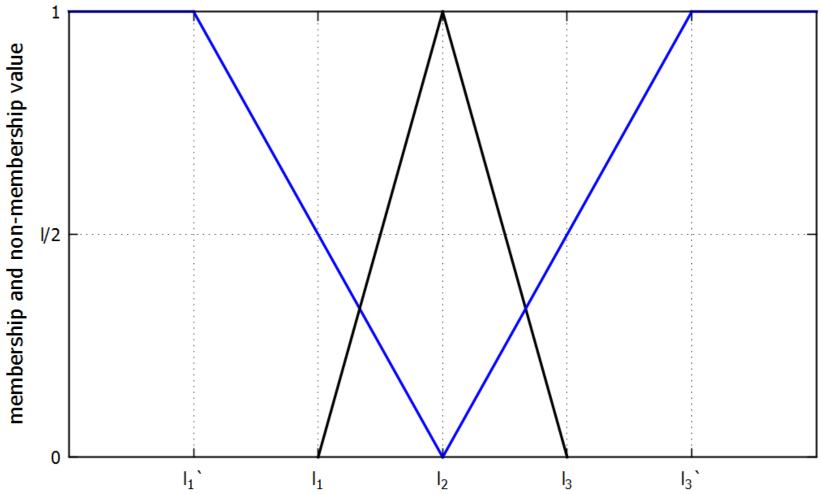

Figure 1: TIFN$\langle(l_1, l_2, l_3), (l'_1, l_2, l'_3)\rangle$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} 1, \quad x \leq l'_1 \\ \frac{l_2 - x}{l_2 - l'_1}, \; l'_1 \leq x \leq l_2 \\ 0, l_2 \leq x \leq l_3 \\ \frac{x - l_3}{l'_4 - l_3}, \; l_3 \leq x \leq l'_4 \\ 1, \quad x \geq l'_4 \end{cases}$$

where $l'_1 \leq l_1 \leq l_2 \leq l_3 \leq l_4 \leq l'_4$.
(α, β)-cut of $\tilde{A} = \langle(l_1, l_2, l_3, l_4), (l'_1, l_2, l_3, l'_4\rangle)$ may be represented as
$\tilde{A}_{\alpha,\beta} = \langle[\underline{A}(\alpha), \overline{A}(\alpha)], [\underline{A'}(\beta), \overline{A'}(\beta)]\rangle$, where $\underline{A}(\alpha) = l_1 + \alpha(l_2 - l_1)$, $\overline{A}(\alpha) = l_4 - \alpha(l_4 - l_3)$, $\underline{A}'(\beta) = l_2 - \beta(l_2 - l'_1)$ and $\overline{A}'(\beta) = l_3 + \beta(l'_4 - l_3)$.

Note:

1. If $l_2 = l_3$ then trapezoidal intuitionistic fuzzy number (TrIFN) $\langle(l_1, l_2, l_3, l_4), (l'_1, l_2, l_3, l'_4\rangle$ is transformed into triangular intuitionistic fuzzy number (TIFN) $\langle(l_1, l_2, l_4), (l'_1, l_2, l'_4)\rangle$.

Definition 2.6. Non-Negative TrIFN : A trapezoidal intuitionistic fuzzy number $\tilde{A} = \langle(l_1, l_2, l_3, l_4), (l'_1, l_2, l_3, l'_4)\rangle$ is said to be non-negative if $l'_1 \geq 0$.

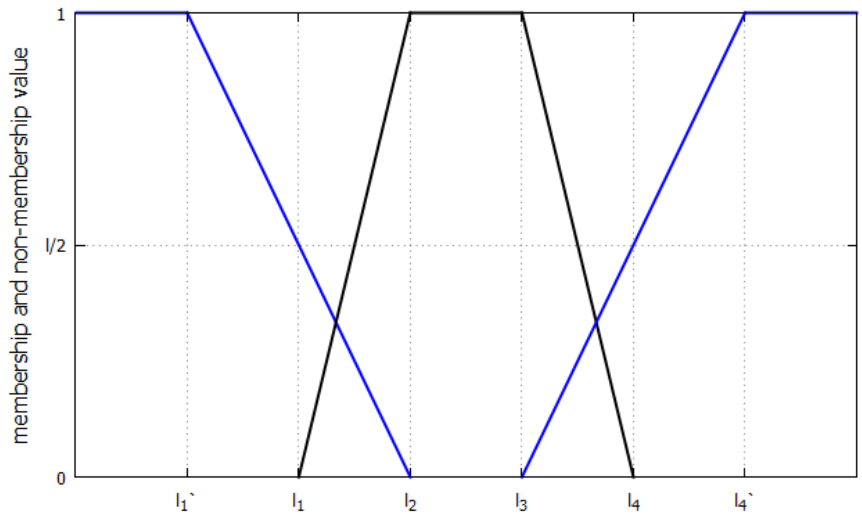

Figure 2: TrIFN$\langle(l_1, l_2, l_3, l_4), (l'_1, l_2, l_3, l'_4\rangle$

Definition 2.7. Non-Negative TIFN : A triangular intuitionistic fuzzy Number $\tilde{A} = \langle(l_1, l_2, l_3), (l'_1, l_2, l'_3)\rangle$ is said to be non-negative if $l'_1 \geq 0$.

3. INTUITIONISTIC FUZZY SYSTEM OF LINEAR EQUATIONS

The $n \times n$ intuitionistic fuzzy system of linear equations may be written as

$$\begin{aligned}
\tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + ... + \tilde{a}_{1n}\tilde{x}_n &= \tilde{b}_1 \\
\tilde{a}_{21}\tilde{x}_2 + \tilde{a}_{22}\tilde{x}_2 + ... + \tilde{a}_{2n}\tilde{x}_n &= \tilde{b}_2 \\
&\vdots \\
\tilde{a}_{n1}\tilde{x}_n + \tilde{a}_{n2}\tilde{x}_2 + ... + \tilde{a}_{nn}\tilde{x}_n &= \tilde{b}_n.
\end{aligned}$$

In matrix-vector form the above system may be written as
$A\tilde{X} = \tilde{B}$, where the coefficient matrix $A = (a_{kj}),\ 1 \leq k \leq n,\ 1 \leq j \leq n$ is a crisp real $n \times n$ matrix, $\tilde{B} = (\tilde{b}_k),\ 1 \leq k \leq n$, is a column vector of fuzzy numbers and $\tilde{X} = (\tilde{x}_j),\ 1 \leq j \leq n$, is the vector of fuzzy unknowns.

3.1 Solution Method Using Four Parametric Form With Crisp Coefficient:

The system of equations $A\tilde{X} = \tilde{B}$ can be represented as

$$\sum_{j=1}^{n} \tilde{a}_{kj}\tilde{x_j} = \tilde{b_k} \text{ for } k = 1, 2, ..., n. \tag{3.1}$$

Using parametric form of intuitionistic fuzzy number we may write

$$\tilde{x_j} = \langle[\underline{x_j}(\alpha), \overline{x_j}(\alpha)], [\underline{x'_j}(\beta), \overline{x'_j}(\beta)]\rangle \text{ and } \tilde{b_k} = \langle[\underline{b_k}(\alpha), \overline{b_k}(\alpha)], [\underline{b'_k}(\beta), \overline{b'_k}(\beta)]\rangle.$$

Substituting the above expression in equation(3.1), we get

$$\sum_{j=1}^{n} a_{kj}\langle[\underline{x_j}(\alpha), \overline{x_j}(\alpha)], [\underline{x'_j}(\beta), \overline{x'_j}(\beta)]\rangle = \langle[\underline{b_k}(\alpha), \overline{b_k}(\alpha)], [\underline{b'_k}(\beta), \overline{b'_k}(\beta)]\rangle, \\ \text{for } k = 1, 2, ..., n.$$

From equation(3.1), one may get two $n \times n$ system of equations as

$$\sum_{j=1}^{n} a_{kj}\langle[\underline{x_j}(\alpha), \overline{x_j}(\alpha)]\rangle = \langle[\underline{b_k}(\alpha), \overline{b_k}(\alpha)]\rangle, \text{ for } k = 1, 2, ..., n. \tag{3.2}$$

and

$$\sum_{j=1}^{n} a_{kj}\langle[\underline{x'_j}(\beta), \overline{x'_j}(\beta)]\rangle = \langle[\underline{b'_k}(\beta), \overline{b'_k}(\beta)]\rangle, \text{ for } k = 1, 2, ..., n. \tag{3.3}$$

Let us define

$$\tilde{x}_j(\alpha, \gamma) = \gamma(\overline{x_j}(\alpha) - \underline{x_j}(\alpha)) + \underline{x_j}(\alpha), \text{ for } j = 1, 2, ..., n. \tag{3.4}$$

$$\tilde{b}_k(\alpha, \gamma) = \gamma(\overline{b_k}(\alpha) - \underline{b_k}(\alpha)) + \underline{b_k}(\alpha), \text{ for } k = 1, 2, ..., n. \tag{3.5}$$

and

$$\tilde{x}_j(\beta, \delta) = \delta(\overline{x'_j}(\beta) - \underline{x'_j}(\beta)) + \underline{x'_j}(\beta), \text{ for } j = 1, 2, ..., n. \tag{3.6}$$

$$\tilde{b}_k(\beta, \delta) = \delta(\overline{b'_k}(\beta) - \underline{b'_k}(\beta)) + \underline{b'_k}(\beta), \text{ for } k = 1, 2, ..., n. \tag{3.7}$$

Substituting the above expressions in equation (3.2) and in equation (3.3) we get,

$$\sum_{j=1}^{n} a_{kj}\tilde{x}_j(\alpha,\gamma) = \tilde{b}_k(\alpha,\gamma), \text{ for } k = 1,2,...,n. \tag{3.8}$$

$$\sum_{j=1}^{n} a_{kj}\tilde{x}_j(\beta,\delta) = \tilde{b}_k(\beta,\delta), \text{ for } k = 1,2,...,n. \tag{3.9}$$

The above system of equation (3.8) is now solved to obtain $\tilde{x}_j(\alpha,\gamma)$. After getting $\tilde{x}_j(\alpha,\gamma)$ one can put $\gamma = 0$ to get $\underline{x_j}(\alpha)(= \tilde{x}_j(\alpha,0))$ and $\gamma = 1$ to get $\overline{x_j}(\alpha)(= \tilde{x}_j(\alpha,1))$.
Similarly, solving equations (3.9) one can get $\tilde{x}_j(\beta,\delta)$. After getting $\tilde{x}_j(\beta,\delta)$ one can put $\delta = 0$ to get $\underline{x'_j}(\beta)(= \tilde{x}_j(\beta,0))$ and $\delta = 1$ to get $\overline{x'_j}(\beta)(= \tilde{x}_j(\beta,1))$.

Definition 3.1. If $\tilde{X} = (\tilde{x_1},\tilde{x_2},...,\tilde{x_n})^T$ is a solution set of (3.1) and for each j, $1 \leq j \leq n$ the inequalities $\underline{x_j}(\alpha) \leq \overline{x_j}(\alpha)$, $\underline{x'_j}(\beta) \leq \overline{x'_j}(\beta)$ hold, then the solution $\tilde{X}$ is called a strong solution of the system (3.1).

Definition 3.2. If $\tilde{X} = (\tilde{x_1},\tilde{x_2},...,\tilde{x_n})^T$ is a solution set of (3.1) and for some j, $1 \leq j \leq n$ the inequalities $\underline{x_j}(\alpha) > \overline{x_j}(\alpha)$ or $\underline{x'_j}(\beta) > \overline{x'_j}(\beta)$ hold, then the solution $\tilde{X}$ is called a weak solution of the system (3.1).

4. NUMERICAL EXAMPLE ON INTUITIONISTIC FUZZY SYSTEM OF LINEAR EQUATIONS WITH CRISP COEFFICIENTS

Example 4.1. Let us consider 2×2 intuitionistic fuzzy system of linear equations ([10])
$4\tilde{x}_1 + (-5)\tilde{x}_2 = \langle(25,35,50,67),(20,35,50,73)\rangle$
$7\tilde{x}_1 + (-3)\tilde{x}_2 = \langle(27,37,48,65),(22,37,48,72)\rangle$.

Using parametric form of intuitionistic fuzzy number we have,
$\tilde{b}_{1\alpha,\beta} = \langle[\underline{b_1}(\alpha),\overline{b_1}(\alpha)],[\underline{b'_1}(\beta),\overline{b'_1}(\beta)]\rangle = \langle[25+10\alpha,67-17\alpha],[35-15\beta,50+23\beta]\rangle$,
$\tilde{b}_{2\alpha,\beta} = \langle[\underline{b_2}(\alpha),\overline{b_2}(\alpha)],[\underline{b'_2}(\beta),\overline{b'_2}(\beta)]\rangle = \langle[27+10\alpha,65-17\alpha],[37-15\beta,48+24\beta]\rangle$.
and $\tilde{x}_{1\alpha,\beta} = \langle[\underline{x_1}(\alpha),\overline{x_1}(\alpha)],[\underline{x'_1}(\beta),\overline{x'_1}(\beta)]\rangle$,
$\tilde{x}_{2\alpha,\beta} = \langle[\underline{x_2}(\alpha),\overline{x_2}(\alpha)],[\underline{x'_2}(\beta),\overline{x'_2}(\beta)]\rangle$.

Let us define
$\tilde{x}_j(\alpha,\gamma) = \gamma(\overline{x_j}(\alpha) - \underline{x_j}(\alpha)) + \underline{x_j}(\alpha)$,
$\tilde{b}_k(\alpha,\gamma) = \gamma(\overline{b_k}(\alpha) - \underline{b_k}(\alpha)) + \underline{b_k}(\alpha)$.

and

$$\tilde{x}_j(\beta,\delta) = \delta(\overline{x'_j}(\beta) - \underline{x'_j}(\beta)) + \underline{x'_j}(\beta),$$
$$\tilde{b}_k(\beta,\delta) = \delta(\overline{b'_k}(\beta) - \underline{b'_k}(\beta)) + \underline{b'_k}(\beta).$$

Then we get the 1^{st} 2×2 system as
$4\tilde{x}_1(\alpha,\gamma) + (-5)\tilde{x}_1(\alpha,\gamma) = \gamma[67 - 17\alpha - 25 - 10\alpha] + 25 + 10\alpha,$
$7\tilde{x}_2(\alpha,\gamma) + (-3)\tilde{x}_2(\alpha,\gamma) = \gamma[65 - 17\alpha - 27 - 10\alpha] + 27 + 10\alpha.$
i.e,
$4\tilde{x}_1(\alpha,\gamma) + (-5)\tilde{x}_1(\alpha,\gamma) = \gamma[42 - 27\alpha] + 25 + 10\alpha,$
$7\tilde{x}_2(\alpha,\gamma) + (-3)\tilde{x}_2(\alpha,\gamma) = \gamma[38 - 27\alpha] + 27 + 10\alpha.$

Writing in matrix-vector form we have

$$\begin{pmatrix} 4 & -5 \\ 7 & -3 \end{pmatrix} \begin{pmatrix} x_1(\alpha,\gamma) \\ x_2(\alpha,\gamma) \end{pmatrix} = \begin{pmatrix} \gamma[42 - 27\alpha] + 25 + 10\alpha \\ \gamma[38 - 27\alpha] + 27 + 10\alpha \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1(\alpha,\gamma) \\ x_2(\alpha,\gamma) \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 7 & -3 \end{pmatrix}^{-1} \begin{pmatrix} \gamma[42 - 27\alpha] + 25 + 10\alpha \\ \gamma[38 - 27\alpha] + 27 + 10\alpha \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1(\alpha,\gamma) \\ x_2(\alpha,\gamma) \end{pmatrix} = \begin{pmatrix} -\frac{3}{23} & \frac{5}{23} \\ -\frac{7}{23} & \frac{4}{23} \end{pmatrix} \begin{pmatrix} \gamma[42 - 27\alpha] + 25 + 10\alpha \\ \gamma[38 - 27\alpha] + 27 + 10\alpha \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1(\alpha,\gamma) \\ x_2(\alpha,\gamma) \end{pmatrix} = \begin{pmatrix} \dfrac{60 + 20\alpha + 64\gamma - 54\alpha\gamma}{23} \\ \dfrac{-67 - 30\alpha - 142\gamma + 81\alpha\gamma}{23} \end{pmatrix}.$$

Now we get the solution as

$$\tilde{x}_1(\alpha,\gamma) = \frac{60 + 20\alpha + 64\gamma - 54\alpha\gamma}{23} \text{ and } \tilde{x}_2(\alpha,\gamma) = \frac{67 + 30\alpha + 142\gamma - 81\alpha\gamma}{23}.$$

Putting $\gamma = 0$ in $\tilde{x}_1(\alpha,\gamma)$ and $\tilde{x}_2(\alpha,\gamma)$ we get respectively

$$\tilde{x}_1(\alpha,0) = \underline{x_1}(\alpha) = \frac{60 + 20\alpha}{23} \text{ and } \tilde{x}_2(\alpha,0) = \underline{x_2}(\alpha) = \frac{-67 - 30\alpha}{23}.$$

Putting $\gamma = 1$ in $\tilde{x}_1(\alpha,\gamma)$ and $\tilde{x}_2(\alpha,\gamma)$ we get respectively

$$\tilde{x}_1(\alpha,1) = \overline{x_1}(\alpha) = \frac{124-34\alpha}{23}, \text{ and } \tilde{x}_2(\alpha,1) = \overline{x_2}(\alpha) = \frac{-209+51\alpha}{23}.$$

Then we get the 2^{nd} 2×2 system as
$4\tilde{x}_1(\beta,\delta) + (-5)\tilde{x}_1(\beta,\delta) = \delta[50+23\beta-35+15\beta]+35-15\beta,$
$7\tilde{x}_2(\beta,\delta) + (-3)\tilde{x}_2(\beta,\delta) = \delta[48+24\beta-37+15\beta]+37-15\beta.$
i.e,
$4\tilde{x}_1(\beta,\delta) + (-5)\tilde{x}_1(\beta,\delta) = \delta[15+38\beta]+35-15\beta,$
$7\tilde{x}_2(\beta,\delta) + (-3)\tilde{x}_2(\beta,\delta) = \delta[11+39\beta]+37-15\beta.$

Writing in matrix-vector form we have

$$\begin{pmatrix} 4 & -5 \\ 7 & -3 \end{pmatrix} \begin{pmatrix} x_1(\beta,\delta) \\ x_2(\beta,\delta) \end{pmatrix} = \begin{pmatrix} \delta[15+38\beta]+35-15\beta \\ \delta[11+39\beta]+37-15\beta \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1(\beta,\delta) \\ x_2(\beta,\delta) \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 7 & -3 \end{pmatrix}^{-1} \begin{pmatrix} \delta[15+38\beta]+35-15\beta \\ \delta[11+39\beta]+37-15\beta \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1(\beta,\delta) \\ x_2(\beta,\delta) \end{pmatrix} = \begin{pmatrix} -\frac{3}{23} & \frac{5}{23} \\ -\frac{7}{23} & \frac{4}{23} \end{pmatrix} \begin{pmatrix} \delta[15+38\beta]+35-15\beta \\ \delta[11+39\beta]+37-15\beta \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1(\beta,\delta) \\ x_2(\beta,\delta) \end{pmatrix} = \begin{pmatrix} \dfrac{80-30\beta+10\delta+81\beta\delta}{23} \\ \dfrac{-97+45\beta-61\delta-110\beta\delta}{23} \end{pmatrix}.$$

Now we get the solution as

$$\tilde{x}_1(\beta,\delta) = \frac{80-30\beta+10\delta+81\beta\delta}{23} \text{ and } \tilde{x}_2(\beta,\delta) = \frac{-97+45\beta-61\delta-110\beta\delta}{23}.$$

Putting $\delta = 0$ in $\tilde{x}_1(\beta,\delta)$ and $\tilde{x}_2(\beta,\delta)$ we get respectively

$$\tilde{x}_1(\beta,0) = \underline{x_1'}(\beta) = \frac{80-30\beta}{23} \text{ and } \tilde{x}_2(\beta,0) = \underline{x_2'}(\beta) = \frac{-97+45\beta}{23}.$$

Putting $\delta = 1$ in $\tilde{x}_1(\beta,\delta)$ and $\tilde{x}_2(\beta,\delta)$ we get respectively

$\tilde{x}_1(\beta,1) = \overline{x'_1}(\beta) = \dfrac{90+51\beta}{23}$ and $\tilde{x}_2(\beta,1) = \overline{x'_2}(\beta) = \dfrac{-158-65\beta}{23}$.

Here we see that,
$\tilde{x}_{1\alpha,\beta} = \langle[\underline{x_1}(\alpha), \overline{x_1}(\alpha)], [\underline{x'_1}(\beta), \overline{x'_1}(\beta)]\rangle$,
and
$\tilde{x}_{2\alpha,\beta} = \langle[\underline{x_2}(\alpha), \overline{x_2}(\alpha)], [\underline{x'_2}(\beta), \overline{x'_2}(\beta)]\rangle$,
are both trapezoidal shaped Intuitionistic Fuzzy Numbers.
Here $\underline{x_2}(\alpha) > \overline{x_2}(\alpha)$ and $\underline{x'_2}(\beta) > \overline{x'_2}(\beta)$.
Hence the solution $\tilde{x} = (\tilde{x}_1, \tilde{x}_2)^t$ is a weak solution of the system .

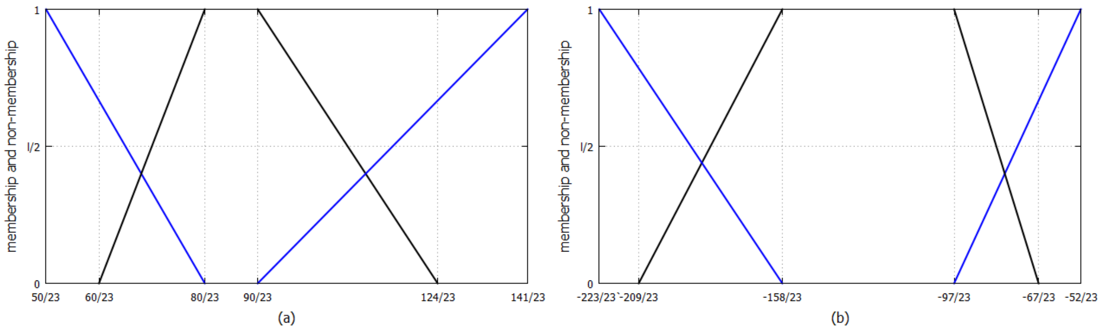

Figure 3: fig(a):$\tilde{x}_1$ fig(b): $\tilde{x}_2$

Example 4.2. Let us consider 2×2 intuitionistic fuzzy system of linear equations
$(-4)\tilde{x}_1 + 2\tilde{x}_2 = \langle(2,3,4,5),(1,3,4,6)\rangle$,
$3\tilde{x}_1 + (-1)\tilde{x}_2 = \langle(4,5,6,7),(3,5,6,8)\rangle$.

Using parametric form of intuitionistic fuzzy number we have
$\tilde{b}_{1\alpha,\beta} = \langle[\underline{b_1}(\alpha), \overline{b_1}(\alpha)], [\underline{b'_1}(\beta), \overline{b'_1}(\beta)]\rangle = \langle[2+\alpha, 5-\alpha], [3-2\beta, 4+2\beta]\rangle$,
$\tilde{b}_{2\alpha,\beta} = \langle[\underline{b_2}(\alpha), \overline{b_2}(\alpha)], [\underline{b'_2}(\beta), \overline{b'_2}(\beta)]\rangle = \langle[4+\alpha, 7-\alpha], [5-2\beta, 6+2\beta]\rangle$,
and
$\tilde{x}_{1\alpha,\beta} = \langle[\underline{x_1}(\alpha), \overline{x_1}(\alpha)], [\underline{x'_1}(\beta), \overline{x'_1}(\beta)]\rangle$,
$\tilde{x}_{2\alpha,\beta} = \langle[\underline{x_2}(\alpha), \overline{x_2}(\alpha)], [\underline{x'_2}(\beta), \overline{x'_2}(\beta)]\rangle$.
Let us define
$\tilde{x}_j(\alpha,\gamma) = \gamma(\overline{x_j}(\alpha) - \underline{x_j}(\alpha)) + \underline{x_j}(\alpha)$,
$\tilde{b}_k(\alpha,\gamma) = \gamma(\overline{b_k}(\alpha) - \underline{b_k}(\alpha)) + \underline{b_k}(\alpha)$,

and

$\tilde{x}_j(\beta,\delta) = \delta(\overline{x'_j}(\beta) - \underline{x'_j}(\beta)) + \underline{x'_j}(\beta)$,
$\tilde{b}_k(\beta,\delta) = \delta(\overline{b'_k}(\beta) - \underline{b'_k}(\beta)) + \underline{b'_k}(\beta)$.

Then we get the 1^{st} 2×2 system as
$(-4)\tilde{x}_1(\alpha,\gamma)+2\tilde{x}_1(\alpha,\gamma)=\gamma[5-\alpha-2-\alpha]+2+\alpha,$
$3\tilde{x}_2(\alpha,\gamma)+(-1)\tilde{x}_2(\alpha,\gamma)=\gamma[7-\alpha-4-\alpha]+4+\alpha.$
i.e,
$(-4)\tilde{x}_1(\alpha,\gamma)+2\tilde{x}_1(\alpha,\gamma)=\gamma[3-2\alpha]+2+\alpha,$
$3\tilde{x}_2(\alpha,\gamma)+(-1)\tilde{x}_2(\alpha,\gamma)=\gamma[3-2\alpha]+4+\alpha.$

Writing in matrix-vector form we have

$$\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}\begin{pmatrix}x_1(\alpha,\gamma)\\ x_2(\alpha,\gamma)\end{pmatrix}=\begin{pmatrix}\gamma[3-2\alpha]+2+\alpha\\ \gamma[3-2\alpha]+4+\alpha\end{pmatrix}.$$

$$\Rightarrow\begin{pmatrix}x_1(\alpha,\gamma)\\ x_2(\alpha,\gamma)\end{pmatrix}=\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}^{-1}\begin{pmatrix}\gamma[3-2\alpha]+2+\alpha\\ \gamma[3-2\alpha]+4+\alpha\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1(\alpha,\gamma)\\ x_2(\alpha,\gamma)\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & 1\\ \frac{3}{2} & 2\end{pmatrix}\begin{pmatrix}\gamma[3-2\alpha]+2+\alpha\\ \gamma[3-2\alpha]+4+\alpha\end{pmatrix}.$$

$$\Rightarrow\begin{pmatrix}x_1(\alpha,\gamma)\\ x_2(\alpha,\gamma)\end{pmatrix}=\begin{pmatrix}\dfrac{10+3\alpha+9\gamma-6\alpha\gamma}{2}\\ \dfrac{22+7\alpha+21\gamma-14\alpha\gamma}{2}\end{pmatrix}.$$

Now we get the solution as

$$\tilde{x}_1(\alpha,\gamma)=\frac{10+3\alpha+9\gamma-6\alpha\gamma}{2}, \text{ and } \tilde{x}_2(\alpha,\gamma)=\frac{22+7\alpha+21\gamma-14\alpha\gamma}{2}.$$

Putting $\gamma=0$ in $\tilde{x}_1(\alpha,\gamma)$ and $\tilde{x}_2(\alpha,\gamma)$ we get respectively

$$\tilde{x}_1(\alpha,0)=\underline{x_1}(\alpha)=\frac{10+3\alpha}{2} \text{ and } \tilde{x}_2(\alpha,0)=\underline{x_2}(\alpha)=\frac{22+7\alpha}{2}.$$

Putting $\gamma=1$ in $\tilde{x}_1(\alpha,\gamma)$ and $\tilde{x}_2(\alpha,\gamma)$ we get respectively

$$\tilde{x}_1(\alpha,1)=\overline{x_1}(\alpha)=\frac{19-3\alpha}{2} \text{ and } \tilde{x}_2(\alpha,1)=\overline{x_2}(\alpha)=\frac{43-7\alpha}{2}.$$

Then we get the 2^{nd} 2×2 system as
$(-4)\tilde{x}_1(\beta,\delta)+2\tilde{x}_1(\beta,\delta)=\delta[4+2\beta-3+2\beta]+3-2\beta,$
$3\tilde{x}_2(\beta,\delta)+(-1)\tilde{x}_2(\beta,\delta)=\delta[6+2\beta+5-2\beta]+5-2\beta,$

i.e,
$(-4)\tilde{x}_1(\beta,\delta)+2\tilde{x}_1(\beta,\delta)=\delta[1+4\beta]+3-2\beta,$
$3\tilde{x}_2(\beta,\delta)+(-1)\tilde{x}_2(\beta,\delta)=\delta[1+4\beta]+5-2\beta.$
Writing in matrix-vector form we have

$$\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}\delta[1+4\beta]+3-2\beta\\ \delta[1+4\beta]+5-2\beta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}-4 & 2\\ 3 & -1\end{pmatrix}^{-1}\begin{pmatrix}\delta[1+4\beta]+3-2\beta\\ \delta[1+4\beta]+5-2\beta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & 1\\ \frac{3}{2} & 2\end{pmatrix}\begin{pmatrix}\delta[1+4\beta]+3-2\beta\\ \delta[1+4\beta]+5-2\beta\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1(\beta,\delta)\\ x_2(\beta,\delta)\end{pmatrix}=\begin{pmatrix}\frac{13-6\beta+3\delta+12\beta\delta}{2}\\ \frac{29-14\beta+7\delta+28\beta\delta}{2}\end{pmatrix}.$$

Now we get the solution as

$$\tilde{x}_1(\beta,\delta)=\frac{13-6\beta+3\delta+12\beta\delta}{2},\text{ and }\tilde{x}_2(\beta,\delta)=\frac{29-14\beta+7\delta+28\beta\delta}{2}.$$

Putting $\delta=0$ in $\tilde{x}_1(\beta,\delta)$ and $\tilde{x}_2(\beta,\delta)$ we get respectively

$$\tilde{x}_1(\beta,0)=\underline{x'_1}(\beta)=\frac{13-6\beta}{2},\text{ and }\tilde{x}_2(\beta,0)=\underline{x'_2}(\beta)=\frac{29-14\beta}{2}.$$

Putting $\delta=1$ in $\tilde{x}_1(\beta,\delta)$ and $\tilde{x}_2(\beta,\delta)$ we get respectively

$$\tilde{x}_1(\beta,1)=\overline{x'_1}(\beta)=\frac{16+6\beta}{2},\text{ and }\tilde{x}_2(\beta,1)=\overline{x'_2}(\beta)=\frac{36+14\beta}{2}.$$

Here we see that,
$\tilde{x}_{1\alpha,\beta}=\langle[\underline{x_1}(\alpha),\overline{x_1}(\alpha)],[\underline{x'_1}(\beta),\overline{x'_1}(\beta)]\rangle,$
and
$\tilde{x}_{2\alpha,\beta}=\langle[\underline{x_2}(\alpha),\overline{x_2}(\alpha)],[\underline{x'_2}(\beta),\overline{x'_2}(\beta)]\rangle,$
are both trapezoidal shaped Intuitionistic Fuzzy Numbers.
The solution $\tilde{x}=(\tilde{x_1},\tilde{x_2})^t$ is a strong solution of the system .

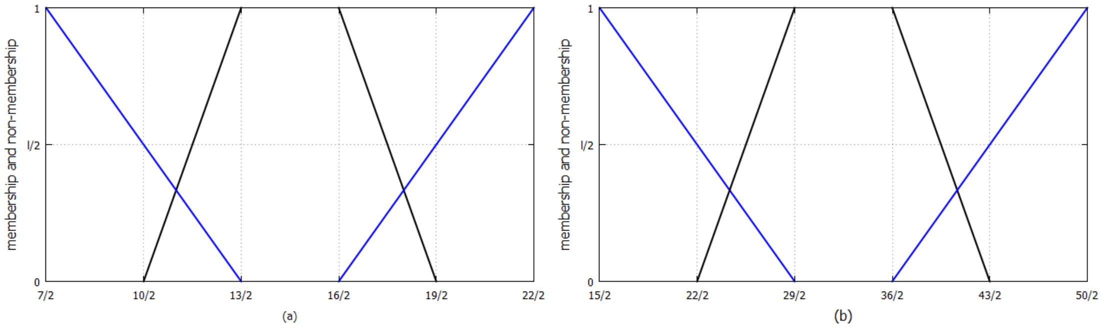

Figure 4: fig(a):$\tilde{x_1}$ fig(b): $\tilde{x_2}$

5. CONCLUSION

In this paper we have developed a new approach to solve intuitionistic fuzzy linear systems (IFLS) with crisp coefficients. In this approach $n \times n$ IFLS is transformed into two $n \times n$ crisp systems. This $\alpha, \beta, \gamma, \delta$ parametric approach is easy, straight forward and requires much lesser effort than the other existing methods. Future work may focus on fully intuitionistic fuzzy linear systems (FIFLS).

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