Some P-Fuzzifications of Gamma Soft Subgroup Structures VIA MIN-Operations

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Abstract

In this paper, the notion of P-Fuzzy M-Gamma soft subgroups of group is introduced and its basic properties are investigated. The study of the homomorphic and pre-image of P-Fuzzy M-Gamma soft subgroups are pursued. Using t-norm, the notion of sensible P-Fuzzy M-Gamma soft subgroups in a group is introduced and some related properties of M-Gamma soft subgroups are discussed.

Keywords: soft set, P–Fuzzy set, P - Fuzzy MΓ–soft subgroup, MΓ - group homomorphism, Imaginable, t-norm, characterized P-Fuzzy soft set.

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Section-1 INTRODUCTION

Molodtsov[15] initiated the concept of soft sets that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Maji et.al [16] gave the operations of soft sets and their properties. Furthermore, they [16] introduced fuzzy soft sets which combine the strength of both soft sets and fuzzy sets. As a generalization of the soft set theory, the fuzzy soft set theory makes description of the objective world more realistic, practical precise in some cases, making it very promising. Since the notion of soft groups was proposed by Aktas and Cagman[2], then the soft set theory is used a new tool to discuss algebraic structures. Cagman et.al[6] studied on soft int-group, which are different from the definitions of soft groups[2]. The new approach is based
on the inclusion relation and intersection of sets. It brings the soft set theory, the set theory, and the group theory together. On the basis of soft int-groups, Sezgin et.al[20] introduced the concept of soft intersection near-rings (soft int-near rings) by using intersection operation of sets and gave the applications of soft int near-rings to the near-ring theory. By introducing soft intersection, union products and soft characteristic functions, Sezer[20] made a new approach to the classical ring theory via the soft set theory, with the concepts of soft union rings, ideals and bi-ideal. Jun et.al[11] applied intersectional soft sets to BCK/BCI-algebras[12] an obtained many results. Liu et.al [14] further the investigated isomorphisms and fuzzy isomorphisms theorems of soft rings in [13] respectively. Soft sets were also applied to other algebraic structures such as near-rings[17], \( \Gamma \)-modulus and BCK/BCI-algebras[10].

The concept of fuzzy sets was first introduced by Zadeh [21]. Rosenfeld [18] used this concept to formulate the notion of Fuzzy groups. Since then, many other fuzzy algebraic concepts based on Rosenfeld's fuzzy groups were developed. Anthony and Sherwood [1] redefined fuzzy groups in terms of t-norms which is replaced the min operations of Rosenfeld's definition. Using this concept, chang [5] generalized some of the basic concepts of general topology, many Researchers [3] and [4] applied the concept of fuzzy sets to the elementary theory of \( \Gamma \)-rings. In [3] Booth introduced the concept of \( \Gamma \)-near rings which is due to satyanarayana [19]. Also Booth [4] studied radical theory of a \( \Gamma \)-near ring and introduced the notion of \( M \Gamma \)-group. The notion of Intuitionistic P-Fuzzy semi primality in a semi group is given by Kim[12]. In this paper, a new class of P-fuzzy M-Gamma soft subgroups of group are introduced and characterization of some properties of M-Gamma soft subgroups with respect to t-norms are discussed.

Section- 2 PRELIMINARIES

2.1 Definition: A non-empty set ‘R’ with two binary operations ‘+’ and ‘*’ is called a near-ring if it satisfies the following axioms.

1. \((R, +)\) is a group
2. \((R, \cdot)\) is a semi group
3. \((a+b)\cdot c = a\cdot c + b\cdot c\), for all \(a, b, c, \in R\).

Precisely speaking it is a right near-ring. Because it satisfies the right distributive law.

All near rings considered in this paper will be right distributive. A \( \Gamma \)-near ring is a triple \((M, +, \Gamma)\) Where,

(i) \((M, +)\) is a group (not necessarily abelian)

(ii) \( \Gamma \) is a non-empty set of binary operators on \( M \) such that for each \( \alpha \in \Gamma, (M, +, \alpha) \)
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is a near ring

(iii) \( a \alpha (b\beta c) = (a \alpha b)\beta c \) for all \( a, b, c, \in M \), \( \alpha, \beta \in \Gamma \). If, in addition, it holds that

(iv) \( a \alpha 0 = 0 \) for all \( a \in M \), then the \( \Gamma \)-near ring \( M \) is said to be zero- symmetric.

2.2 Definition: A soft set \( f_A \) over \( U \) is defined as \( f_A: E \rightarrow P(U) \) such that \( f_A(x) = \emptyset \) if \( x \notin A \). In other words, a soft set \( f_A \) over \( U \) is a parameterized family of subsets of the universe \( U \). For all \( \varepsilon \in A \), \( f_A(\varepsilon) \) may be considered as the set of \( \varepsilon \)-approximate elements of the soft set \( f_A \). A soft set \( f_A \) over \( U \) can be presented by the set of ordered pair:

\[
\begin{align*}
  f_A &= \{ (x, f_A(x)) / x \in E, f_A(x) = P(U) \} \\
  \text{................. (1)}
\end{align*}
\]

Clearly, a soft set is not a set. For illustration, Molodtsov consider several examples in [15].

If \( f_A \) is a soft set over \( U \), then the image of \( f_A \) is defined by \( \text{Im}(f_A) = \{ f_A(a) / a \in A \} \). The set of all soft sets over \( U \) will be denoted by \( S(U) \). Some of the operations of soft sets are listed as follows.

2.3 Definition: Let \( f_A, f_B \in S(U) \). If \( f_A(x) \subseteq f_B(x) \), for all \( x \in E \), then \( f_A \) is called a soft subset of \( f_B \) and denoted by \( f_A \subseteq f_B \). \( f_A \) and \( f_B \) are called soft equal, denoted by \( f_A = f_B \), if and only if \( f_A \subseteq f_B \) and \( f_B \subseteq f_A \).

2.4 Definition: Let \( f_A, f_B \in S(U) \) and \( \chi \) be a function from \( A \) to \( B \). Then the soft anti-image of \( f_A \) under \( \chi \), denoted by \( \chi f_A \), is a soft set over \( U \) defined by,

\[
\chi f_A(b) = \left\{ \begin{array}{ll}
  \bigcap \{ f_A(a) / a \in A, \chi(a) = b \}, & \text{if } \chi^{-1}(b) \neq \emptyset \\
  \emptyset, & \text{otherwise}
\end{array} \right. \quad \text{................. (2)}
\]

for all \( b \in B \) and the soft pre-image of \( f_B \) under \( \chi \), denoted by \( \chi^{-1} f_B \), is a soft set over \( U \) defined by \( \chi^{-1} f_B(a) = f_B(\chi(a)) \), for all \( a \in A \).

Note that the concept of level sets in the fuzzy set theory, Cagman et.al[6] initiated the concept of lower inclusions soft sets which serves as a bridge between soft sets and crisp sets.

2.5 Definition: Let \( G \) be an additive group. If, for all \( a, b \in M \), \( \alpha, \beta \in \Gamma \) and \( x \in G \) it holds that

(i) \( a \alpha x \in G \)
(ii) $a \alpha (b \beta c) = (a \alpha b) \beta x$

(iii) $(a + b) \alpha x = a \alpha x + b \alpha x$, then G is called an M-Gamma group or $M\Gamma$-group.

In what follows, let $M$ denotes the $\Gamma$-near ring and G denotes the $M\Gamma$-group unless or otherwise specified.

2.6 Definition : A subgroup H of G for which $a \alpha h \in H$ for $a \in M$, $\alpha \in \Gamma$, $h \in H$ is called an $M\Gamma$-subgroup of G.

We now review some fuzzy logic concepts. A fuzzy set in a set G is a function $A : G \rightarrow [0, 1]$.

2.7 Definition : Let P and G be a set and a group respectively. A mapping $A : G \times P \rightarrow [0, 1]$ is called a P-fuzzy soft set in G.

2.8 Definition : A P-fuzzy soft set 'A' in G is called a P-fuzzy $M\Gamma$-soft subgroup of G if

(i) $A(x-y, p) \geq \min \{A(x, p), A(y, p)\}$

(ii) $A(a \alpha x, p) \geq A(x, p)$, for $a \in M$, $p \in P$, $x \in G$ and $\alpha \in \Gamma$.

2.9 Definition : By a t-norm T, we mean a function $T : [0,1] \times [0,1] \rightarrow [0,1]$ satisfying the following conditions

$[T_1]$ $T(x, 1) = x$

$[T_2]$ $T(x, y) \leq T(x, z)$ if $y \leq z$

$[T_3]$ $T(x, y) = T(y, x)$

$[T_4]$ $T(x, T(y, z)) = T(T(x, y), z)$ for all $x, y, z \in [0,1]$

2.10 Proposition : For a t-norm, the following statement holds $T(x, y) \leq \min \{x, y\}$ for all $x, y \in [0,1]$. For a t-norm T on $[0,1]$, denoted by $\Delta T$, the set of elements $\alpha \in [0,1]$ such that $T(\alpha, \alpha) = \alpha$ (ie) $\Delta T = \{ \alpha \in [0,1] / T(\alpha, \alpha) = \alpha \}$.

2.11 Definition : Let T be a t-norm. A fuzzy set A in G is said to satisfy idempotent property with respect to T if $\text{Im}(A) \subseteq \Delta T$.

In definition (2.8), we use T operator for min operation.
Section – 3  Properties of P-Fuzzy $M\Gamma$-soft subgroups of Group

In this section, the notion of P-fuzzy $M\Gamma$-soft subgroups of $M\Gamma$-group are discussed.

3.1 Proposition : Let $T$ be a t-norm. If $'A'$ is idempotent P-fuzzy $M\Gamma$-soft subgroup of $G$, then we have $A(0,p) \geq A(x,p)$ for all $x \in G$, $p \in P$.

Proof: For every $x \in G$ and $p \in P$, we have

$$A(0,p) = A(x-x, p) \geq T\{A(x,p), A(x,p)\} = A(x,p).$$

This completes the proof.

3.2 Proposition : If $'A'$ is an idempotent P-fuzzy $M\Gamma$-soft subgroup of $G$, then the set $G^m = \{x \in G / A(x,p) \geq A(m,p)\}$ is an $M\Gamma$-soft subgroup of $G$.

Proof : Let $x,y \in G^m$, then $A(x,p) \geq A(m,p)$ and $A(y,p) \geq A(m,p)$. Since $'A'$ is an P-fuzzy $M\Gamma$-subgroup of $G$, it follows that $A(x-y,p) \geq \min \{A(x,p), A(y,p)\} \geq \min \{A(m,p), A(m,p)\} = A(m,p)$. Now let $a \in M$, $\alpha \in \Gamma$ and $h \in G^m$. Then $A(a \alpha h,p) \geq A(h,p) \geq A(m,p).$ Then we have $A(x-y,p) \geq A(m,p)$ and $A(aah,p) \geq A(m,p)$ that $x-y \in G^m$ and $aah \in G^m$. This completes the proof.

3.3 Corollary : Let $T$ be a t-norm. If $'A'$ is an idempotent P-fuzzy $M\Gamma$-soft subgroup of $G$, then the set $A_G = \{x \in G / A(x,p) = A(0,p)\}$ is an $M\Gamma$-soft subgroup of $G$.

Proof: From the proposition (3.1),

$$A_G = \{x \in G / A(x,p) = A(0,p)\}$$

$$= \{x \in G / A(x,p) \geq A(m,p)\}$$

Hence $A_G$ is an $M\Gamma$-soft subgroup of $G$ from the Proposition (3.2).

3.4 Definition : Let $G$ and $G'$ be $M\Gamma$-groups. A map $\theta : G \rightarrow G'$ is called a $M\Gamma$-group homomorphism if $\theta (x+y) = \theta (x) + \theta (y)$ and $\theta (a \alpha x) = a \alpha \theta (x)$ for all $a \in M$, $\alpha \in T$ and $x \in G$.

3.5 Definition : Let $\theta : G \rightarrow G'$ be an $M\Gamma$-group homomorphism of $M\Gamma$-groups. For a fuzzy soft set $A$ in $G'$, we define a characterized P-fuzzy set $A^\theta$ in $G$ by $A^\theta (x,p) = A (\theta (x,p))$ for all $x \in G$.

3.6 Propositions : Let $\theta : G \rightarrow G'$ be an $M\Gamma$-group homomorphism of $M\Gamma$-groups.
If 'A' is P-fuzzy $\Gamma$-soft subgroup of $G'$, then $A^0$ is an P-fuzzy $\Gamma$-soft subgroup of $G$.

**Proof:** For any $x, y \in G$ and $p \in P$, we have

$$A^0(x-y,p) = A(\theta(x-y,p))$$

$$= A(\theta(x) - \theta(y), p)$$

$$\geq T\{A(\theta(x,p)), A(\theta(y,p))\}$$

Let $a \in M$, $\alpha \in T$ and $x \in G$, then

$$A^0(\alpha x, p) = A(\theta(\alpha x,p))$$

$$= A(\alpha \theta(x,p))$$

$$\geq A(\theta(x,p))$$

$$\geq A^0(x, p)$$

This completes the proof.

3.7 Proposition: Let $I$ be an $\Gamma$-subgroup of $G$ and let 'A' be P-fuzzy soft set in G defined by

$$A(x,p) = \begin{cases} (a, p) & \text{if } x \in I \\ (b, p) & \text{otherwise} \end{cases}$$

for all $x \in G$, where $a,b \in [0,1]$ with $a > b$, then 'A' is P-fuzzy $\Gamma$-soft subgroup of $G$ where $\min\{a,b\} = \max\{a+b-1, 0\}$ for all $a,b \in [0,1]$.

**Proof:** Let $x, y \in G$. If $x, y \in I$, then

$$\min\{A(x,p), A(y,p)\} = \min\{a,a\} = \max\{2a-1,0\}$$

$$= \begin{cases} 2a & \text{if } a \geq 1/2 \\ b & \text{if } a < 1/2 \end{cases}$$

$$\leq a = A(x-y, p)$$

and for all $m \in M$ and $\alpha \in \Gamma$, we have

$$A(\max, p) = A(x,p) = a.$$  

If $y \in I$ and $x \notin I$ (or $x \in I$ and $y \notin I$), then

$$\min\{A(x,p), A(y,p)\} = \min\{a,b\} = \max\{a+b-1, 0\}$$
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\[ a + b - 1 \quad \text{if} \quad a + b \geq 1/2 \]
\[ b \quad \text{if} \quad a + b < 1/2 \]

\[ \leq b = A(x-y, p) \]

and for all \( m \in M \) and \( \alpha \in T \), we have

\[ A(\max, p) \geq b = A(y, p). \]

If \( y \not\in I \) and \( x \not\in I \), then

\[ \min \{ A(x, p), A(y, p) \} = A(b, b) = \max \{ 2b-1, 0 \} \]
\[ = \begin{cases} 2b - 1 & \text{if } b \geq 1/2 \\ 0 & \text{otherwise} \end{cases} \]

\[ \leq b = A(x-y, p) \]

and for all \( m \in M \) and \( \alpha \in T \), we have

\[ A(\max, p) \geq b = A(x, p). \]

Hence 'A' is P-fuzzy M\( \Gamma \)-soft subgroup of \( G \).

For any subset \( I \) of M\( \Gamma \)-subgroup of \( G \), \( \Phi \) denotes the characteristic function of \( I \).

3.8 Corollary : If \( I \subseteq G \), then \( I \) is an M\( \Gamma \)-subgroup of \( G \) if and only if \( \Phi \) is P-fuzzy M\( \Gamma \)-soft subgroup of \( G \).

Proof : Let \( I \) be an M\( \Gamma \)-subgroup of \( G \). Then it is easy to show that \( \Phi \) is P-fuzzy M\( \Gamma \)-soft subgroup of \( G \).

In fact, let \( x, y \in I \) and so \( x-y \in I \).

Hence we have \( \Phi(x-y, p) = 1 = T\{ \Phi(x, p), \Phi(y, p) \} = \min\{1,1\} \).

Assume that \( x \in I \) and \( y \in I \) or \( x \not\in I \) and \( y \in I \).

Then \( \Phi(x, p) = 1 > 0 = \Phi(y, p) \) (or \( \Phi(x, p) = 0 < 1 = \Phi(y, p) \)).

It follows that \( \Phi(x-y, p) \leq T\{ \Phi(x, p), \Phi(y, p) \} = \min\{1,0\} = 0 \).

Now let \( a \in M \) and \( \alpha \in \Gamma \).

If \( y \in I \), then we have \( a \alpha x \in I \). Hence \( \Phi(a \alpha x, p) = 1 = \Phi(y, p) \).

If \( y \not\in I \), then \( \Phi(a \alpha x, p) \geq \Phi(y, p) \).

Conversely, let \( \Phi \) be P-fuzzy M\( \Gamma \)-soft subgroup of \( G \).

Let \( x, y \in I \).
Then we have $\Phi(x-y, p) \geq T\{\Phi(x, p), \Phi(y, p)\} = \min\{1, 1\} = 1$ and so $x-y \in I$.

Now let $a \in M, a \in \Gamma$ and $y \in I$. Hence $\Phi(a \alpha x, p) \geq \Phi(y, p) = 1$ and so $a \alpha x \in I$.

3.9 Proposition : Let $T$ be a $t$-norm. Then every idempotent $P$-fuzzy $M\Gamma$-soft subgroup of $G$ is a fuzzy soft ideal of $G$.

Proof : Let 'A' be an idempotent $P$-fuzzy $M\Gamma$-soft subgroup of $G$. Then $A(x-y, p) \geq T\{A(x, p), A(y, p)\}$ for all $x, y \in G$. Since 'A' satisfies the idempotent property. We have,

$$\min\{A(x, p), A(y, p)\} = T\{\min\{A(x, p), A(y, p)\}, \min\{A(x, p), A(y, p)\}\}$$

$$\leq T\{A(x, p), A(y, p)\}$$

$$\leq \min\{A(x, p), A(y, p)\}.$$ 

It follows that $A(x-y, p) \geq T\{A(x, p), A(y, p)\} = \min\{A(x, p), A(y, p)\}$. So that 'A' is a fuzzy soft ideal of $G$.

3.10 Proposition : The family of $P$-fuzzy $M\Gamma$-soft subgroups of $G$ is completely distributive lattice with respect to meet '$\land$' and join '$\lor$'.

Proof : Since $[0,1]$ is a completely distributive lattice with respect to the usual ordering in $[0,1]$, it is sufficient to show that $\lor_{a \in \Lambda} A_a$ and $\land_{a \in \Lambda} A_a$ are $P$-fuzzy $M\Gamma$-soft subgroups of $G$ for a family of $P$-fuzzy $M\Gamma$-soft subgroups $\{A_a / a \in \Lambda\}$.

For any $x, y \in G$, we have

$$(\lor_{a \in \Lambda} A_a) (x-y, p) = \sup\{A_a (x-y, p) / a \in \Lambda\}$$

$$\geq \sup\{T(A_a (x, p), A_a (y, p)) / a \in \Lambda\}$$

$$\geq T\{\sup\{T(A_a (x, p)) / a \in \Lambda\} , \sup\{T(A_a (y, p)) / a \in \Lambda\}\}$$

$$= T\{(\lor_{a \in \Lambda} A_a) (x, p), (\lor_{a \in \Lambda} A_a) (y, p)\}$$

$$(\land_{a \in \Lambda} A_a) (x-y, p) = \inf\{A_a (x-y, p) / a \in \Lambda\}$$

$$\geq \inf\{T(A_a (x, p), A_a (y, p)) / a \in \Lambda\}$$

$$\geq T\{\inf\{T(A_a (x, p)) / a \in \Lambda\} , \inf\{T(A_a (y, p)) / a \in \Lambda\}\}$$

$$= T\{(\land_{a \in \Lambda} A_a) (x, p), (\land_{a \in \Lambda} A_a) (y, p)\}$$
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Now let \( a \in M, \ y \in I \) and \( \alpha \in \Gamma \). Then

\[
(V_{a \in \Lambda} A_a) (a \alpha x, p) = \sup \{ A_a (a \alpha x, p) / a \in \Lambda \}
\]

\[
\geq \sup \{ A_a (x, p) / a \in \Lambda \}
\]

\[
= (V_{a \in \Lambda} A_a) (x, p)
\]

\[
(\Lambda_{a \in \Lambda} A_a) (a \alpha x, p) = \inf \{ A_a (a \alpha x, p) / a \in \Lambda \}
\]

\[
\geq \inf \{ A_a (x, p) / a \in \Lambda \}
\]

\[
= (\Lambda_{a \in \Lambda} A_a) (x, p)
\]

Hence \( V_{a \in \Lambda} A_a \) and \( \Lambda_{a \in \Lambda} A_a \) are P-fuzzy \( M\Gamma \)-soft subgroups of \( G \).

This completes the proof.

**3.11 Proposition :** Let \( 'A' \) be P-fuzzy \( M\Gamma \)-soft subgroup of \( G \) and let \( \alpha \in \Gamma \) be such that \( T(\alpha, \alpha) = \alpha \). Then \( U(A; \alpha) \) is either empty or an \( M\Gamma \)-subgroup of \( G \) for all \( x \in G \).

**Proof :** Let \( x, y \in U(A; \alpha) \). Then we have

\[
A(x, p) \geq \alpha \quad \text{and} \quad A(y, p) \geq \alpha \quad \text{and so}
\]

\[
A(x-y, p) \geq T(A(x, p), A(y, p)) \geq T(\alpha, \alpha) = \alpha
\]

Which implies that \( x-y \in U(A; \alpha) \). Now let \( a \in M, y \in U(A; \alpha) \) and \( \gamma \in \Gamma \). Then we have

\[
A(a \gamma x, p) \geq A(x, p) \geq \alpha \quad \text{So} \quad a \gamma x \in U(A; \alpha). \quad \text{Hence} \quad U(A; \alpha) \quad \text{is} \quad M\Gamma \text{-subgroup of} \ G.
\]

Since \( T(1,1) = 1 \) we have the following corollary.

**3.12 Corollary :** If \( 'A' \) is P-fuzzy \( M\Gamma \)-soft subgroup of \( G \) , then \( U(A; 1) \) is either empty or an \( M\Gamma \)-subgroup of \( G \).

**Proof :** For a family \( \{ A_a / a \in \Lambda \} \) of P-fuzzy soft sets in \( G \), define the join \( V_{a \in \Lambda} A_a \) and the meet \( \Lambda_{a \in \Lambda} A_a \) as follows

\[
(V_{a \in \Lambda} A_a) (x, p) = \sup \{ A_a (x, p) / a \in \Lambda \}
\]

\[
(\Lambda_{a \in \Lambda} A_a) (x, p) = \inf \{ A_a (x, p) / a \in \Lambda \}
\]
for all \( x \in G \), where \( \Lambda \) is any index set.

Hence the proof.

**3.13 Proposition:** Let \( T \) be a t-norm and let \( 'A' \) be a P-fuzzy set in \( G \) with \( \text{Im}(A) = \{ \alpha_1, \alpha_2, \ldots, \alpha_n \} \) where \( \alpha_i < \alpha_j \) where \( i > j \). Suppose that there exists a chain of \( M\Gamma \)-subgroups of \( G \): \( G_0 \subset G_1 \subset \ldots \subset G_n = G \) such that \( A(\tilde{G}_k) = G_k \), where \( \tilde{G}_k = G_k / G_{k-1} \) and \( G_1 = 0 \) for \( k = 0, 1, 2, \ldots, n \). Then \( 'A' \) is P-fuzzy \( M\Gamma \)-soft subgroup of \( G \).

**Proof:** Let \( x, y \in G \). If \( x \) and \( y \) belong to the same \( \tilde{G}_k \), then \( A(x,p) = A(y,p) = G_k \) and \( x-y \in G_k \). Hence \( A(x-y,p) \geq G_k = \min \{ A(x,p), A(y,p) \} \)

\[ \geq T\{A(x,p), A(y,p)\} \]

Let \( x \in \tilde{G}_i \) and \( y \in \tilde{G}_j \) for every \( i \neq j \). Without loss of generality, we may assume that \( i > j \), then \( A(x,p) = G_i < G_j = A(y,p) \) and \( x-y \in G_i \). It follows that

\[ A(x-y,p) \geq G_i = \min \{ A(x,p), A(y,p) \} \]

\[ \geq T\{A(x,p), A(y,p)\} \]

Now let \( y \in G \), then there exists \( G_k \) such that \( y \in \tilde{G}_k \) for some \( k \in \{0, 1, 2, \ldots, n\} \). For any \( a \in M \), \( x \in \tilde{G}_k \) and \( \alpha \in \Gamma \). We have \( a \alpha x \in G_k \) and so

\[ A(a \alpha x,p) \geq G_k \geq A(x,p) \]. Hence \( 'A' \) is P-fuzzy \( M\Gamma \)-soft subgroup of \( G \).

**3.14 Proposition:** Let \( T \) be a t-norm. Then every imaginable P-fuzzy \( M\Gamma \)-soft subgroup of \( G \) is a fuzzy \( M\Gamma \)-soft subgroup of \( G \).

**Proof:** Assume \( 'A' \) is imaginable P-fuzzy \( M\Gamma \)-soft subgroup of \( G \), then we have

\[ A(x-y,p) \geq T\{A(x,p), A(y,p)\} \] and \( A(a \alpha x,p) \geq A(x,p) \) for all \( x, y \in G \).

Since \( A \) is imaginable, we have

\[ \min\{A(x,p), A(y,p)\} = T\{\min\{A(x,p), A(y,p)\}, \min\{A(x,p), A(y,p)\}\} \]

\[ \leq T\{A(x,p), A(y,p)\} \]

\[ \leq \min \{ A(x,p), A(y,p) \} \]

and so \( T(A(x,p), A(y,p)) = \min\{A(x,p), A(y,p)\} \).

It follows that \( A(x-y,p) \geq T\{A(x,p), A(y,p)\} = \min\{A(x,p), A(y,p)\} \) for all \( x, y \in G \). Hence \( 'A' \) is fuzzy \( M\Gamma \)-soft subgroup of \( G \).
3.15 Proposition: If 'A' is P-fuzzy $M\Gamma$-soft subgroup of a $M\Gamma$-group G and $\theta$ is an endomorphism of G, then $\Theta$ is P-fuzzy $M\Gamma$-soft subgroup of G.

Proof: For any $x, y \in G$, we have

1. $A[\Theta](x-y, p) = A[\Theta(x, p), \Theta(y, p)]$
   $\geq T\{A[\Theta(x, p)], A[\Theta(y, p)]\}$
   $= T\{A[\Theta(x, p)], A[\Theta(y, p)]\}$

2. $A[\Theta](a \cdot x, p) = A[\Theta(a \cdot x, p)]$
   $\geq A[\Theta(x, p)]$
   $= A[\Theta](x, p)$

Hence $A[\Theta]$ is P-fuzzy $M\Gamma$-soft subgroup of G.

3.16 Definition: Let $f : G \rightarrow G'$ be a group homomorphism and 'A' be P-fuzzy $M\Gamma$-subgroup of G'. Then $A(f(x, p)) = (A \circ f)(x, p) = f^{-1}(A)(x, p)$

3.17 Proposition: Let $f : G \rightarrow G'$ be a group homomorphism and 'A' be P-fuzzy $M\Gamma$-soft subgroup of G'. Then $f^{-1}(A)$ is P-fuzzy $M\Gamma$-soft subgroup of G.

Proof: Let $x, y \in G$. we have

$f^{-1}(A)(x-y, p) = (A \circ f)(x-y, p) = A((f(x)-f(y), p) \geq T\{A(f(x, p)), A(f(y, p))\}$

$\geq T\{(f \circ A)(x, y), A(y, p)\}$

$f^{-1}(A)(a \cdot x, p) = A(f(a \cdot x, p)) = A(f(a \cdot x, p)) \geq f^{-1}(A)(x, p)$

3.18 Proposition: Let 'A' be P-fuzzy $M\Gamma$-soft subgroup of G and $A^*$ be P-fuzzy soft set in G defined by $A^*(x, p) = A(x, p) + 1 - A(e, p)$. Then $A^*$ is P-fuzzy $M\Gamma$-soft subgroup of G containing A.

Proof: For $x, y \in G$, we have

$A^*(x-y, p) = A(x-y, p) + 1 - A(e, p)$

$\geq T\{A(x, p), A(y, p)\} + 1 - A(e, p)$

$\geq T\{A(x, p) + 1 - A(e, p), A(y, p) + 1 - A(e, p)\}$

$\geq T\{A^*(x, p), A^*(y, p)\}$

$A^*(a \cdot x, p) = A(a \cdot x, p) + 1 - A(e, p)$
\[ A(x,p) + 1 - A(e,p) \geq A^*(x,p) \]

Therefore \( A^* \) is P-fuzzy \( M\Gamma \)-soft subgroup of \( G \) containing \( A \).

3.19 Proposition: Let \( T \) be a continuous t-norm and let \( f \) be a homomorphism on a group \( G \). If \( 'A' \) is P-fuzzy \( M\Gamma \)-soft subgroup of \( G \), then \( A^f \) is P-fuzzy \( M\Gamma \)-soft subgroup of \( f(G) \).

Proof: Let \( A_1 = f^{-1}(y_1,p) \), \( A_2 = f^{-1}(y_2,p) \) and \( A_{12} = f^{-1}(y_{12},p) \), where \( y_1, y_2 \in f(R) \), \( p \in P \).

Consider the set
\[ A_1 - A_2 = \{ x \in S / (x,p) = (a_1,p) - (a_2,p) \} \]

for some \( (a_1,p) \in A_1 \) and \( (a_2,p) \in A_2 \).

If \( (x,p) \in A_1 - A_2 \), then \( (x,p) = (x_1,p) = (x_2,p) \) for some \( (x_1,p) \in A_1 \) and \( (x_2,p) \in A_2 \), so that we have
\[ f(x,p) = f(x_1,p) - f(x_2,p) = y_1 - y_2 \]

(ie) \( (x,p) \in f^{-1}((y_1,p) - (y_2,p)) = f^{-1}(y_1-y_2,p) = A_{12} \)

Thus \( A_1 - A_2 \subseteq A_{12} \).

It follows that
\[
(i) \quad A^f(y_1-y_2,p) = \sup \{ A(x,p) / (x,p) \in f^{-1}[(y_1,p) - (y_2,p)] \}
\]
\[ = \sup \{ A(x,p) / (x,p) \in A_{12} \} \geq \sup \{ A((x_1,p) - (x_2,p)) / (x_1,p) \in A_1 \text{ and } (x_2,p) \in A_2 \} \]
\[ \geq \sup \{ T\{A(x_1,p), A(x_2,p)\} / (x_1,p) \in A_1 \text{ and } (x_2,p) \in A_2 \} \]

Since \( T \) is continuous. For every \( \varepsilon > 0 \), we see that if
\[
\sup \{ A((x_1,p) - (x_1^*,p)) \leq \delta \}
\]
\[
\sup \{ A((x_2,p) - (x_2^*,p)) \leq \delta \}
\]
\[
T\{\sup\{A(x_1,p) / (x_1,p) \in A_1 \}, \sup\{A(x_2,p) / (x_2,p) \in A_2 \} \} - T((x_1^*,p), (x_2^*,p)) \leq \varepsilon
\]
Then we have
\[
\sup \{ A((x_1,p) - (a_1,p)) \leq \delta \}
\]
\[
\sup \{ A((x_2,p) - (a_2,p)) \leq \delta \}
\]
\[
T\{\sup\{A(x_1,p) / (x_1,p) \in A_1 \}, \sup\{A(x_2,p) / (x_2,p) \in A_2 \} \} - T((a_1,p),(a_2,p)) \leq \varepsilon
\]

Consequently, we have
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\[ A^f(y_1, y_2, p) \geq \sup \{ T[A(x_1, p), A(x_2, p)] / (x_1, p) \in A_1 \text{ and } (x_2, p) \in A_2 \} \]

\[ \geq T[\sup \{ A(x_1, p) / (x_1, p) \in A_1 \}, \sup \{ A(x_2, p) / (x_2, p) \in A_2 \}] \]

\[ \geq T[A^f(y_1, p), A^f(y_2, p)] \]

Similarly, we can show \( A^f(a \alpha x, p) \geq A^f(x, p) \). Hence \( A^f \) is P-fuzzy \( M\Gamma \)-subgroup of \( G \).

CONCLUSION

In this paper, we investigated the concept of P-fuzzy \( M\Gamma \)-soft subgroups with respect to t-norm and characterization of them.

References


