Application of Triangular Fuzzy Membership Matrix in the Study of Medical Diagnostic Model

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Abstract

Fuzzy set theory is highly suitable and applicable for developing knowledge based system in medicine for the tasks of medical findings. In this paper, we have extended Sanchez’s approach for medical diagnosis using the representation of triangular fuzzy membership matrices. A definition of union and intersection of triangular fuzzy membership numbers is introduced. A procedure is proposed for the fuzzy medical diagnostic model. Example is illustrated for the presented approach along with the simulation result using Dot Net. The result obtained in the current work is compared with the existing earlier result to point out the conclusion.

Keywords and Phrases: Triangular fuzzy number, Triangular fuzzy membership matrix, Union and Intersection of triangular fuzzy membership numbers, Max-min relations, Fuzzy medical diagnostic model, Decision maker.

AMS Subject Classification: 03B52, 92C50, 91B06.

1. INTRODUCTION

Uncertainty is a challenging part in Human’s everyday life. Since the future cannot be predicted, it is impossible to be certain about what exactly is going to happen day to day. The main cause of uncertainty is the information deficiency. Information may be incomplete, not fully reliable, vague, contradictory or deficient in some other way. These various information deficiencies may result in fuzziness or vagueness. Fuzzy set theory introduced by professor Zadeh [8] in 1965 acts as a qualitative computational approach which describes uncertainty.

The paper is organized as follows: In section 2, some basic definitions of triangular fuzzy number (TFN) and triangular fuzzy membership number (TFMN) are reviewed. In section 3, a definition of union and intersection of triangular fuzzy membership numbers (TFMNs) is introduced. Using the definition, some fundamental properties of set operations and propositions are verified. In section 4, a procedure is proposed for the fuzzy medical diagnostic model. Example is illustrated with the simulation result using Dot Net. For the sake of verification, the result obtained using proposed method is compared with the existing result. Section 5 concludes the paper.

2. PRE-REQUISITES

Definition 2.1. (Triangular fuzzy number) Triangular fuzzy number is denoted as
\[ \tilde{A} = (a_1, a_2, a_3), a_1, a_2, a_3 \in \mathbb{R}, a_1 < a_2 < a_3. \]

Definition 2.2. (Conversion of triangular fuzzy number to triangular fuzzy membership number [1]) Let \( \tilde{A} = (a_1, a_2, a_3) \) be a triangular fuzzy number. Then \( \mu_{\tilde{A}} = (a_1/10, a_2/10, a_3/10) \), where \( 0 \leq a_1 \leq a_2 \leq a_3 \leq 10 \). Thus \( 0 \leq a_1/10 \leq a_2/10 \leq a_3/10 \leq 1 \).

Definition 2.3. (Arithmetic mean(AM) for a triangular fuzzy membership number) Let \( \mu_{\tilde{A}} = (a_1/10, a_2/10, a_3/10) \) be a triangular fuzzy membership number then \( \text{AM}(\mu_{\tilde{A}}) = (a_1 + a_2 + a_3) / 30. \)

Definition 2.4. (Triangular fuzzy number matrix) A triangular fuzzy number matrix of order m×n is defined as \( \tilde{A} = (a_{ij})_{m \times n} \) where \( a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU}) \) is the \( ij^{th} \) element of \( \tilde{A} \). \( a_{ijL}, a_{ijU} \) are the left and right spreads of \( a_{ij} \) respectively and \( a_{ijM} \) is the mean value.
Definition 2.5. (Subtraction operation on triangular fuzzy number matrix) Let $\mathbf{A} = (a_{ij})_{n \times n}$ and $\mathbf{B} = (b_{ij})_{n \times n}$ be two triangular fuzzy number matrices of same order. Then $\mathbf{A}(-)\mathbf{B} = (a_{ij} - b_{ij})_{n \times n}$ where $a_{ij} - b_{ij} = (a_{ijL} - b_{ijU}, a_{ijM} - b_{ijM}, a_{ijU} - b_{ijL})$ is the $ij$th element of $\mathbf{A}(-)\mathbf{B}$. The same condition holds for triangular fuzzy membership number matrix.

Definition 2.6. (Max-Min Composition on fuzzy membership value matrices) Let $F_{mn}$ denote the set of all $m \times n$ matrices over $F$. Elements of $F_{mn}$ are called as fuzzy membership value matrices. For $\mathbf{A} = (a_{ij}) \in F_{mp}$ and $\mathbf{B} = (b_{ij}) \in F_{pn}$ the max-min product $\mathbf{A}(.\mathbf{B}) = (\sup_k [\inf (a_{ik}, b_{kj})]) \in F_{mn}$.

3. UNION AND INTERSECTION OF TRIANGULAR FUZZY MEMBERSHIP NUMBERS

The definition of union and intersection for the fuzzy soft matrix is given by Sarala and Prabhavathi [6]. This forms the basis for the definition of union and intersection for triangular fuzzy membership numbers.

Definition 3.1. (Union and Intersection of TFMNs) Let $\mathbf{A} = (\mu_{AL}, \mu_{AM}, \mu_{AU})$.

$\mathbf{B} = (\mu_{BL}, \mu_{BM}, \mu_{BU})$ be two triangular fuzzy membership numbers, then the union and intersection of $\mathbf{A}$ and $\mathbf{B}$ is given by

a) $\mathbf{A} \cup \mathbf{B} = (\mu_{AL}, \mu_{AM}, \mu_{AU}) \cup (\mu_{BL}, \mu_{BM}, \mu_{BU})$

$= (\mu_{AL} + \mu_{BL} - \mu_{AL} \times \mu_{BL}, \mu_{AM} + \mu_{BM} - \mu_{AM} \times \mu_{BM}, \mu_{AU} + \mu_{BU} - \mu_{AU} \times \mu_{BU}).$

b) $\mathbf{A} \cap \mathbf{B} = (\mu_{AL}, \mu_{AM}, \mu_{AU}) \cap (\mu_{BL}, \mu_{BM}, \mu_{BU}) = (\mu_{AL} \times \mu_{BL}, \mu_{AM} \times \mu_{BM}, \mu_{AU} \times \mu_{BU}).$

Example: Let $\mathbf{A} = (0.6, 0.7, 0.9), \mathbf{B} = (0.5, 0.6, 0.8)$ be two triangular fuzzy membership numbers. Then $\mathbf{A} \cup \mathbf{B} = (0.6+0.5, 0.6\times0.5, 0.7+0.6, 0.7\times0.6, 0.9+0.8-0.9\times0.8) = (0.8, 0.88, 0.98)$.

$\mathbf{A} \cap \mathbf{B} = (0.6\times0.5, 0.7\times0.6, 0.9\times0.8) = (0.3, 0.4, 0.7)$.

3.1 Fundamental properties of set operations: Let $\mathbf{A}, \mathbf{B}, \mathbf{C}$ be TFMNs. The following properties are satisfied using the definition of union and intersection.

1. Involution: $\overline{\mathbf{A}} = \mathbf{A}, \overline{\mathbf{A}} = \mathbf{1}(-)\mathbf{A}$, where $\mathbf{1} = (1,1,1)$
2. Commutativity: a) $\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}$; b) $\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$

3. Associativity: a) $(\tilde{A} \cup \tilde{B}) \cup \tilde{C} = \tilde{A} \cup (\tilde{B} \cup \tilde{C})$; b) $(\tilde{A} \cap \tilde{B}) \cap \tilde{C} = \tilde{A} \cap (\tilde{B} \cap \tilde{C})$

4. Distributive laws: a) $\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$; b) $\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$

5. Demorgans laws: a) $\tilde{A} \cap \tilde{B} = \tilde{A} \cup \tilde{B}$; b) $\tilde{A} \cup \tilde{B} = \tilde{A} \cap \tilde{B}$

The following properties are not satisfied using the definition of union and intersection of TFMNs.

1. a) $\tilde{A} \cup \tilde{A} \neq \tilde{A}$; b) $\tilde{A} \cap \tilde{A} \neq \tilde{A}$

   The property of Idempotence is not satisfied.

2. a) $\tilde{A} \cup (\tilde{A} \cap \tilde{B}) \neq \tilde{A}$; b) $\tilde{A} \cap (\tilde{A} \cup \tilde{B}) \neq \tilde{A}$

   The property of Absorption is not satisfied.

3.2 Proposition: Let $\tilde{A}$ be TFMN and $\tilde{1} = (1,1,1)$, $\tilde{0} = (0,0,0)$

1. a) $\tilde{A} \cup \tilde{1} = \tilde{1}$  
   b) $\tilde{A} \cap \tilde{1} = \tilde{A}$

2. a) $\tilde{A} \cup \tilde{0} = \tilde{A}$  
   b) $\tilde{A} \cap \tilde{0} = \tilde{0}$

3. a) $\tilde{A} \cup \tilde{A} \neq \tilde{1}$  
   b) $\tilde{A} \cap \tilde{A} \neq \tilde{0}$

4. APPLICATION OF TRIANGULAR FUZZY MEMBERSHIP MATRIX IN MEDICAL DIAGNOSTIC MODEL

Let $P$ be the set of patients, $S$ be the set of symptoms and $D$ be the set of diseases. The elements of triangular fuzzy number matrix are defined as $\tilde{A} = (a_{ij})_{m \times n}$ where $a_{ij} = (a_{ijL}, a_{ijM}, a_{ijU})$ is the $ij^{th}$ element of $\tilde{A}$. Here $0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 10$, where $a_{ijL}$ is the lower bound, $a_{ijM}$ is the moderate value and $a_{ijU}$ is the upper bound.

4.1 Procedure for the Fuzzy Medical Diagnostic Model

Step 1: Construct a triangular fuzzy number matrix $(F, D)$ over $S$, where $F$ is a mapping given by

$F: D \rightarrow \tilde{F}(S)$, $\tilde{F}(S)$ is the set all triangular fuzzy sets of $S$. This matrix is denoted by $\tilde{R}_0$ which is the fuzzy occurrence matrix or symptom - disease triangular fuzzy number matrix.

Step 2: Construct another triangular fuzzy number matrix $(F_1, S)$ over $P$, where $F_1$ is a mapping given by $F_1: S \rightarrow \tilde{F}(P)$. This matrix is denoted by $\tilde{R}_s$ which is the patient - symptom triangular fuzzy number matrix.
Step 3: The matrices $\overline{R}_0$ and $\overline{R}_s$ are converted into triangular fuzzy membership matrices namely $(R_0)_{\text{mem}}$ and $(R_s)_{\text{mem}}$ using the definition 2.2.

Step 4: Convert $(R_0)_{\text{mem}}$ and $(R_s)_{\text{mem}}$ into fuzzy matrices $R_0$ and $R_s$ by taking the arithmetic mean for triangular fuzzy membership number using the definition 2.3.

Step 5: Compute the following relation matrices to diagnose the disease

i) $R_1 = (R_s)(.)R_0$,  
ii) $R_2 = (1-R_s)(.)R_0$,

iii) $R_3 = (R_s)(1-R_0)$,  
iv) $R_4 = (1-R_s)(1-R_0)$,

v) $R_5 = \max(R_1, R_2)$,  
vi) $R_6 = \min(R_3, R_4)$,

vii) $R_7 = R_5 - R_6$

Find the maximum value in each row of $R_7$ matrix, which gives the strong diagnosis of the disease to the patient.

4.2 Illustrative example

Let $P = (p_1, p_2, p_3, p_4)$ denote the set of patients.

Let $S = (s_1, s_2, s_3, s_4, s_5, s_6)$ denote the set of symptoms.

$s_1$= Fatigue, weight loss, polydipsia, polyuria, irritations, extreme lethargy, weight around waist, high triglyceride (> 150), low level HDL (<40 for men, <50 for women);

$s_2$= Polyphagia (Excessive eating);

$s_3$= Nausea and Vomiting, Dehydration, Abdominal pain, Low blood pressure;

$s_4$= Loss of appetite, Increased heart rate;

$s_5$= Swelling of legs, Puffing around eyes, High blood pressure (130/85 mm HG or higher), Itching, Bleeding, Bone pain, A1C ≥ 8;

$s_6$= Blurry vision, Retinal swelling, Leaking blood vessels.

Let $D = (d_1, d_2, d_3, d_4)$ denote the diseases diabetic general, DKA( diabetic ketoacidosis), diabetic nephropathy and diabetic retinopathy respectively.

Step 1: We consider the set $S$ as the universal set and the set $D = (d_1, d_2, d_3, d_4)$ representing various types of diabetics. The triangular fuzzy number matrix $(F,D)$ is a parametrized family $(F(d_1), F(d_2),F(d_3),F(d_4))$ of all triangular fuzzy number matrix
over the set $S$ and are determined from expert medical documentation. Thus the triangular fuzzy number matrix $(F, D)$ represents a relation matrix $\tilde{R}_0$ which gives medical knowledge of four kinds of diseases and their symptoms.

\[
\tilde{R}_0 = \begin{bmatrix}
  d_1 & d_2 & d_3 & d_4 \\
 s_1 & (10,10,10) & (4,6,8) & (2,4,6) & (2,4,6) \\
 s_2 & (10,10,10) & (4,5,6) & (2,4,6) & (4,6,8) \\
 s_3 & (1,2,3) & (9,9,9) & (1,2,3) & (1,1,1) \\
 s_4 & (1,1,1) & (10,10,10) & (1,2,3) & (1,1,1) \\
 s_5 & (1,1,1) & (0,0,0) & (10,10,10) & (1,1,1) \\
 s_6 & (1,1,1) & (1,1,1) & (0,0,0) & (10,10,10)
\end{bmatrix}
\]

**Step 2:** We take $P = (p_1, p_2, p_3, p_4)$ as the universal set representing the patients and $S = (s_1, s_2, s_3, s_4, s_5, s_6)$ as the set of parameters. The triangular fuzzy number matrix $(F_1, S)$ is another parametrized family of triangular fuzzy number matrix and represents a relation matrix $\tilde{R}_s$ called patient - symptom matrix given by

\[
\tilde{R}_s = \begin{bmatrix}
  & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\
 p_1 & (9,9,9) & (9,9,9) & (1,1,1) & (1,1,1) & (1,1,1) & (1,2,3) \\
 p_2 & (4,6,8) & (4,5,6) & (10,10,10) & (9,9,9) & (0,0,0) & (1,1,1) \\
 p_3 & (4,5,6) & (2,4,6) & (1,1,1) & (0,0,0) & (10,10,10) & (1,1,1) \\
 p_4 & (2,4,6) & (4,5,6) & (1,1,1) & (1,1,1) & (0,0,0) & (10,10,10)
\end{bmatrix}
\]
Proceeding with the procedure 4.1, we arrive at the following result.

\[ R_7 = \begin{bmatrix}
  d_1 & d_2 & d_3 & d_4 \\
  p_1 & 0.7 & 0.4 & 0.3 & 0.2 & 0.7 \\
  p_2 & -0.3 & 0.4 & 0.2 & 0 & 0.4 \\
  p_3 & -0.3 & 0.1 & 0.5 & 0 & 0.5 \\
  p_4 & -0.3 & 0 & 0.2 & 0.6 & 0.6 \\
\end{bmatrix} \quad \text{Row}_i = \text{Maximum of } i^{\text{th}} \text{ row}
\]

From \( R_7 \), we can conclude that the patient \( p_1 \) is suffering from the disease \( d_1 \), \( p_2 \) from \( d_2 \), \( p_3 \) from \( d_3 \) and \( p_4 \) from \( d_4 \).

4.3 Simulation result using Dot Net Program
4.4 Verification using the method proposed in [2]

The relation matrices are

i) \((R_s)_{mem} \cdot (R_0)_{mem}\)

ii) \((R_s)_{mem} \cdot (1 - R_0)_{mem}\)

iii) \(AM((R_s)_{mem} \cdot (R_0)_{mem}) - AM((R_s)_{mem} \cdot (1 - R_0)_{mem})\)

The maximum value in each row of (iii) confirms the disease to the patient.
5. CONCLUSION

In this paper, we have applied the notion of triangular fuzzy membership matrix in a medical diagnostic model. The advantage of this model is, if the patient-symptom and the symptom-disease matrices are known, then it is possible to find which patient is suffering from what kind of disease.

One way to verify the solution obtained is to make an exhaustive comparison.

- In this work, we have considered the case study taken by Vivek Raich et.al [7] and arrived at the same conclusion.
- The proposed method is also verified with the existing method of Elizabeth and Sujatha [2]. The result obtained coincides with the proposed method.

The comparison reveals the method developed in this paper is an alternative way to solve the medical diagnosis problem under fuzzy environment.

REFERENCES


