An Intuitionistic Fuzzy Join Semi $L$- Filter

Dr. B. Anandh
Assistant Professor, PG & Research Department of Mathematics
H.H. The Rajahs’ College, Pudukkottai- 622 001.

F. Benedict
Research Scholar, PG & Research Department of Mathematics
Sudharsan College of Arts & Science, Perumanadu, Pudukkottai- 622 104.

INTRODUCTION
In this paper contained in definition of intuitionistic fuzzy join semi $L$ – filter, intuitionistic fuzzy level join semi $L$ – filters. The properties of intuitionistic fuzzy join semi $L$ – filter and intuitionistic fuzzy level join semi $L$ – filters are established. 

Key words: Intuitionistic fuzzy set, Intuitionistic fuzzy join semi $L$ – ideal, Intuitionistic fuzzy semilattice. Intuitionistic fuzzy join semi $L$-filter.

Definition: 1.1
Let $X$ be a non-empty set. An intuitionistic fuzzy set $A$ of $X$ is an object of the following form $A= \{ < x, \mu_A(x), \nu_A(x) > \mid x \in X \}$, where $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$, respectively and for all $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition: 1.2
An intuitionistic fuzzy semilattice $A = < \mu_A, \nu_A >$ is called an intuitionistic fuzzy join semi $L$-ideal if for all $x, y \in A$,
(i) $\mu_A (x \vee y) \geq \min \{ \mu_A(x), \mu_A(y) \}$
(ii) $\nu_A(x \vee y) \leq \max \{ \nu_A(x), \nu_A(y) \}$.

Definition: 1.3
An intuitionistic fuzzy semilattice $A = < \mu_A, \nu_A >$ is called an intuitionistic fuzzy join semi $L$-filter if for all $x, y \in A$,
(i) $\mu_A (x \vee y) \leq \max \{ \mu_A(x), \mu_A(y) \}$
(ii) $\nu_A(x \vee y) \geq \min \{ \nu_A(x), \nu_A(y) \}$.

Definition: 1.4
If $A = \{ < x, \mu_A(x), \nu_A(x) > \mid x \in X \}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > \mid x \in X \}$ are any two
intuitionistic fuzzy join semi L-filter of X, then

(i) \( A \subseteq B \) iff \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \), for all \( x \in X \).

(ii) \( A = B \) iff \( \mu_A(x) = \mu_B(x) \) and \( \nu_A(x) = \nu_B(x) \), for all \( x \in X \).

(iii) \( \tilde{A} = \{ < x, \nu_A(x), \mu_A(x) > / x \in X \} \)

(iv) \( A \cap B = \{ < x, \min \{ \mu_A(x), \mu_B(x) \}, \max \{ \nu_A(x), \nu_B(x) \} > / x \in X \} \)

(v) \( A \cup B = \{ < x, \max \{ \mu_A(x), \mu_B(x) \}, \min \{ \nu_A(x), \nu_B(x) \} > / x \in X \} \)

(vi) \( [A] = \{ < x, \mu_A(x), 1 - \mu_A(x) > / x \in X \} \)

(vii) \( \langle A \rangle = \{ < x, \nu_A(x), 1 - \nu_A(x) > / x \in X \} \)

**Definition: 1.5**

Let \( A = < \mu_A, \nu_A > \) be an intuitionistic fuzzy join semi L-filter and \( t \in [0, 1] \). Then \( \mu_t = \{ x \in A / \mu(x) \leq t \} \) and \( \nu_t = \{ x \in A / \nu(x) \geq t \} \) is called intuitionistic fuzzy level join semi L-filter of \( A \).

**Definition: 1.6**

Let \( < \mu, \nu > \) be an intuitionistic fuzzy join semi L-filter of a intuitionistic fuzzy semilattice \( A \). The intuitionistic fuzzy level join semi L-filters are defined by \( \mu_t = \{ x \in A / \mu(x) \leq t \} \), \( \nu_t = \{ x \in A / \nu(x) \geq t \} \) and \( \mu_s = \{ x \in A / \mu(x) \leq s \} \), \( \nu_s = \{ x \in A / \nu(x) \geq s \} \). Clearly \( \mu_t \subseteq \mu_s \), whenever \( s < t \) and \( \nu_t \subseteq \nu_s \), whenever \( t < s \).

**Theorem: 1.7**

Intersection of two intuitionistic fuzzy join semi L-filter is also an intuitionistic fuzzy join semi L-filter of a intuitionistic fuzzy join semilattice.

**Proof:**

Let \( A \) be an intuitionistic fuzzy join semilattice

Let \( < \mu_1, \nu_1 > \) and \( < \mu_2, \nu_2 > \) be any two intuitionistic fuzzy join semi L-filter of an intuitionistic fuzzy join semilattice \( A \)

To prove that \( \mu_1 \cap \mu_2 \) and \( \nu_1 \cap \nu_2 \) is a intuitionistic fuzzy join semi L-filter of \( A \)

Let \( a, b \in \mu_1 \cap \mu_2 \)

Then \( a, b \in \mu_1 \) and \( a, b \in \mu_2 \)

\( \Rightarrow a \land b \in \mu_1 \) and \( a \land b \in \mu_2 \)

\( \Rightarrow a \land b \in \mu_1 \cap \mu_2 \)

Therefore \( \mu_1 \cap \mu_2 \) is an intuitionistic fuzzy join semi L-filter of \( A \)

Let \( c, d \in \nu_1 \cap \nu_2 \)

Then \( c, d \in \nu_1 \) and \( c, d \in \nu_2 \)

\( \Rightarrow c \land d \in \nu_1 \) and \( c \land d \in \nu_2 \)

\( \Rightarrow c \land d \in \nu_1 \cap \nu_2 \)

Therefore \( \nu_1 \cap \nu_2 \) is an intuitionistic fuzzy join semi L-filter of \( A \)

Hence \( \mu_1 \cap \mu_2 \) and \( \nu_1 \cap \nu_2 \) is an intuitionistic fuzzy join semi L-filter of \( A \)

**Theorem: 1.8**

The complement of an intuitionistic fuzzy join semi L-filter is an intuitionistic fuzzy join semi L-ideal.

**Proof:**

Let \( A = \{ < x, \mu_A(x), \nu_A(x) > / x \in X \} \) be an intuitionistic fuzzy join semi L-filter.
(ie) if (i) \( \mu_A(x \lor y) \leq \max \{\mu_A(x), \mu_A(y)\} \)
\( \nu_A(x \lor y) \geq \min \{\nu_A(x), \nu_A(y)\} \)
To prove that complement of A is an intuitionistic fuzzy join semi L- ideal.

(ie) (i) \( \mu_{\overline{A}}(x \lor y) \geq \max \{\mu_{\overline{A}}(x), \mu_{\overline{A}}(y)\} \)
(ii) \( \nu_{\overline{A}}(x \lor y) \leq \min \{\nu_{\overline{A}}(x), \nu_{\overline{A}}(y)\} \)

Now the complement of A is defined by \( \overline{A} = \{x, \nu_A(x), \mu_A(x) > / x \in X\} \)
Here \( \mu_{\overline{A}}(x) = \nu_A(x), \nu_{\overline{A}}(x) = \mu_A(x) \)

For (i):
\( \mu_{\overline{A}}(x \lor y) = \nu_A(x \lor y) \geq \max \{\nu_A(x), \nu_A(y)\} \)
\( = \min \{\mu_{\overline{A}}(x), \mu_{\overline{A}}(y)\} \)

For (ii):
\( \nu_{\overline{A}}(x \lor y) = \mu_A(x \lor y) \leq \max \{\mu_A(x), \mu_A(y)\} \)
\( = \max \{\nu_{\overline{A}}(x), \nu_{\overline{A}}(y)\} \)

Hence complement of A is an intuitionistic fuzzy join semi L-ideal.

**Theorem: 1.9**
If \( A = < \mu_A, \nu_A > \) is an intuitionistic fuzzy join semi L- filter. Then the \( [A] = < 1 - \mu_A, \nu_A > \) is an intuitionistic fuzzy join semi L- filter of A.

**Proof:**
Let A be intuitionistic fuzzy join semi L- filter
Let D=[A]
Then \( \mu_D = \mu_A, \nu_D = 1 - \mu_A \)
To prove that D is an intuitionistic fuzzy join semi L- filter.
(i) \( \mu_D(x \lor y) = \mu_A(x \lor y) \leq \max \{\mu_A(x), \mu_A(y)\} \)
\( = \max \{\mu_D(x), \mu_D(y)\} \)
\( \mu_B(x \lor y) \leq \max \{\mu_D(x), \mu_D(y)\} \)
(ii) \( \nu_D(x \lor y) = 1 - \mu_A(x \lor y) \)
\( \geq 1 - \max \{\mu_A(x), \mu_A(y)\} \)
\( = \min \{1 - \mu_A(x), 1 - \mu_A(y)\} \)
\( = \min \{\nu_D(x), \nu_D(y)\} \)
\( \nu_D(x \lor y) \geq \min \{\nu_D(x), \nu_D(y)\} \)

Hence D is an intuitionistic fuzzy join semi L- filter

**Theorem: 1.10**
If \( A = < \mu_A, \nu_A > \) is an intuitionistic fuzzy join semi L- filter. Then \( < A > = < 1 - \nu_A, \nu_A > \) is also an intuitionistic fuzzy join semi L- filter.

**Proof:**
Let A be an intuitionistic fuzzy join semi L- filter.
Then (i) \( \mu_A(x \lor y) \leq \max \{\mu_A(x), \mu_A(y)\} \)
\( \nu_A(x \lor y) \geq \min \{\nu_A(x), \nu_A(y)\} \)
To prove that \( < A > = < 1 - \nu_A, \nu_A > \) is an intuitionistic fuzzy join semi L- filter.
Let $D = \langle A \rangle$ (ie) $\mu_D = 1 - \nu_A$, $\nu_D = \nu_A$

(i) $\mu_D(x \lor y) = 1 - \nu_A(x \lor y)$

$\leq 1 - \min \{\nu_A(x), \nu_A(y)\}$
$= \max \{1 - \nu_A(x), 1 - \nu_A(y)\}$
$= \max \{\mu_D(x), \mu_D(y)\}$

$\mu_D(x \lor y) \leq \max \{\mu_D(x), \mu_D(y)\}$

(ii) $\nu_D(x \lor y) = \nu_A(x \lor y)$

$\geq \min \{\nu_A(x), \nu_A(y)\}$
$= \min \{\nu_D(x), \nu_D(y)\}$

$\nu_D(x \lor y) \geq \min \{\nu_D(x), \nu_D(y)\}$

Hence $D$ is an intuitionistic fuzzy join semi $L$-filter.

**Theorem: 1.11**

If $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy join semi $L$-filter of $X$, then $\mu_A$ and $1 - \nu_A$ are fuzzy join semi $L$-filter.

**Proof:**

Let $A$ be an intuitionistic fuzzy join semi $L$-filter of $X$.

(i) Let $D = \langle x, \mu_A \rangle$ be a fuzzy set.

Then $\mu_D = \mu_A$

$\mu_D(x \lor y) = \mu_A(x \lor y) \leq \max \{\mu_A(x), \mu_A(y)\} = \max \{\mu_D(x), \mu_D(y)\}$

$\mu_D(x \lor y) \leq \max \{\mu_D(x), \mu_D(y)\}$

Hence $\mu_D = \mu_A$ is a fuzzy join semi $L$-filter.

(ii) Let $C = \langle x, 1 - \nu_A \rangle$ be a fuzzy set.

Then $\mu_C = 1 - \nu_A$

$\mu_C(x \lor y) = 1 - \nu_A(x \lor y)$

$\leq 1 - \min \{\nu_A(x), \nu_A(y)\}$
$= \max \{1 - \nu_A(x), 1 - \nu_A(y)\}$
$= \max \{\nu_C(x), \nu_C(y)\}$

$\mu_C(x \lor y) \leq \max \{\nu_C(x), \nu_C(y)\}$

Hence $\mu_C = 1 - \nu_A$ is a fuzzy join semi $L$-filter.

**Theorem: 1.12**

Let $A$ be fuzzy join semilattice. If $\mu: A \to [0, 1], \nu: A \to [0, 1]$ is a intuitionistic fuzzy join semi $L$- filter, then the level subsets $\mu_t, \nu_t$ and $t \in [0, 1]$ is a intuitionistic fuzzy level join semi $L$- filter of $A$.

**Proof:**

Let $x, y \in \mu_t$. Then $\mu(x) \leq t$, $\mu(y) \leq t$

$\mu(x \lor y) \leq \max \{\mu(x), \mu(y)\} \leq t$

Therefore $x \lor y \in \mu_t$

Let $x, y \in \nu_t$. Then $\nu(x) \geq t$, $\nu(y) \geq t$

$\nu(x \lor y) \geq \min \{\nu(x), \nu(y)\} \geq t$

Therefore $x \lor y \in \nu_t$

Hence $\mu_t, \nu_t$ are intuitionistic fuzzy join semi $L$-filter of $A$. 
**Theorem: 1.13**
If $A$ is a fuzzy join semilattice. Then $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy join semi $L$-filter iff the level subsets $\mu_t, \nu_t$ and $t \in [0, 1]$ is an intuitionistic fuzzy level join semi $L$-filter of $A$.

**Proof:**
Let $A$ be a fuzzy join semilattice.
Assume that $A$ is intuitionistic fuzzy join semi $L$-filter.
Then $\mu_t, \nu_t$ are intuitionistic fuzzy level join semi $L$-filter of $A$.
Conversely, assume that $\mu_t, \nu_t$ are intuitionistic fuzzy level join semi $L$-filter of $A$.
To prove that $A$ is intuitionistic fuzzy join semi $L$-filter.
Let $x, y \in \mu_t$. Then $\mu(x) \leq t, \mu(y) \leq t$
$\max \{\mu(x), \mu(y)\} \leq t$
Therefore $x \nu y \in \mu_t$ (ie) $\mu(x \nu y) \leq t$
$\mu(x \nu y) \leq \max \{\mu(x), \mu(y)\}$
Let $x, y \in \nu_t$. Then $\nu(x) \geq t, \nu(y) \geq t$
$\min \{\nu(x), \nu(y)\} \geq t$
Therefore $x \nu y \in \nu_t$ (ie) $\nu(x \nu y) \geq t$
$\nu(x \nu y) \geq \min \{\nu(x), \nu(y)\}$
Hence $A$ is an intuitionistic fuzzy join semi $L$-filter.

**Theorem: 1.14**
The intersection of two intuitionistic fuzzy level join semi $L$ – filters of intuitionistic fuzzy join semi $L$ – filter $A$ is also a intuitionistic fuzzy level join semi $L$ – filter of $A$.

**Proof:**
Let $\langle \mu_t, \nu_t \rangle$ and $\langle \mu_s, \nu_s \rangle$ be two intuitionistic fuzzy level join semi $L$ – filters of intuitionistic fuzzy join semi $L$ – filter $A$.
Let $x, y \in \mu_t \cap \mu_s$.
Then $x, y \in \mu_t$ and $x, y \in \mu_s$
$\Rightarrow x \nu y \in \mu_t$ and $x \nu y \in \mu_s$
$\Rightarrow x \nu y \in \mu_t \cap \mu_s$
Therefore $\mu_t \cap \mu_s$ is an intuitionistic fuzzy level join semi $L$ – filter of $A$.
Let $x, y \in \nu_t \cap \nu_s$.
Then $x, y \in \nu_t$ and $x, y \in \nu_s$
$\Rightarrow x \nu y \in \nu_t$ and $x \nu y \in \nu_s$
$\Rightarrow x \nu y \in \nu_t \cap \nu_s$
Therefore $\nu_t \cap \nu_s$ is an intuitionistic fuzzy level join semi $L$ – filter of $A$.
Hence intersection of two intuitionistic fuzzy level join semi $L$ – filters is also a intuitionistic fuzzy level join semi $L$ – filter.

**Theorem: 1.15**
If $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy join semi $L$-filter of $X$, then $B = \langle \mu_A, 0 \rangle$ and $C = \langle 0, 1 - \mu_A \rangle$ are intuitionistic fuzzy join semi $L$-filter of $X$. 
Proof:
Given A is an intuitionistic fuzzy join semi L- filter of X.
To prove that B and C are intuitionistic fuzzy join semi L- filter.
If B = < µ, 0 > then µ = µ, ν = 0
Let x, y ∈ X.
Then µ(x ∧ y) = µ(x ∧ y) ≤ min {µ(x), µ(y)}
    = min {µ(x), µ(y)}
ν(x ∧ y) = 0 ≥ max {ν(x), ν(y)}
Hence B is an intuitionistic fuzzy join semi L- filter.
Let C = < 0, 1 - µ >. Then µ = 0, ν = 1 - µ
µ(x ∧ y) = 0 ≤ min {µ(x), µ(y)}
ν(x ∧ y) = 1 - µ(x ∧ y) ≥ max {1 - µ(x), 1 - µ(y)}
ν(x ∧ y) ≥ max {µ(x), µ(y)}.
Hence C is an intuitionistic fuzzy join semi L- filter.

REFERENCES