An Optimal Solution for Generalized Trapezoidal 
Intuitionistic Fuzzy Transportation Problem

G.Uthra¹, K.Thangavelu¹ and R.M.Umamageswari²

¹ P.G and Research Department of Mathematics, Pachaiyappa’s college, 
Chennai-600030, India.
² Jeppiaar Engineering College, Chennai-600 119, India.

Abstract

This paper proposes the method to solve “Transportation problem whose costs are taken as Generalized Trapezoidal Intuitionistic Fuzzy Numbers”. The Generalized Trapezoidal Intuitionistic Fuzzy Numbers [GTIFN] are transformed to crisp data by a new ranking technique. We yield the initial basic feasible solution and optimal solution by conventional optimization process. The numerical example illustrates the efficiency of the proposed technique.

1. INTRODUCTION

In today's real life, there are many complex situations in engineering and business, in which experts and decision makers struggle with uncertainty and hesitation. In many practical situations, collection of crisp data of various parameters is difficult due to lack of exact communications, error in data, market knowledge and customer's satisfaction. The information available is sometimes vague and insufficient. The real life problems, when defined by the decision maker with uncertainty leads to the notion of fuzzy sets. Due to imprecise information, the exact evaluation of membership values is not possible. Moreover, the evaluation of the non-membership values is always impossible. This leads to an indeterministic environment where hesitation survives. Dealing with inexact information while making decisions, the concept of fuzziness was introduced by Bellman and Zadeh [1]. K. T. Atanassov [2] introduced the concept of Intuitionistic fuzzy set theory, which is more apt to deal with such problems. Chetia. K and P. K. Das [3] proved some results on intuitionistic fuzzy soft matrix. Intuitionistic fuzzy sets [4], [5], [6] found to be highly effective in

This paper formulates the transportation problem with generalized trapezoidal intuitionistic fuzzy number as costs to deal with uncertainty and hesitation in supply and demand. The new ranking measure proposed in this paper proves to be efficient over the other fuzzy ranking existing techniques.

The organization of this paper is as follows: In Section 2, the basic concepts of generalized trapezoidal intuitionistic fuzzy numbers, Arithmetic addition and new ranking method of generalized trapezoidal intuitionistic fuzzy numbers are reviewed. Section 3, presents the fuzzy transportation problem and its mathematical formulation. Section 4, briefly the proposed algorithm. Section 5, illustrates the numerical example proving the efficiency of proposed approach with comparative analysis. Finally, Section 6 exposes the conclusion.

2. BASIC DEFINITION

Definition 2.1 Intuitionistic Fuzzy Set:

Let $X$ be the universal set. An intuitionistic fuzzy set (IFS) $\tilde{A}^I$ in $X$ is given by

$$
\tilde{A}^I = \{ (x, \mu_\tilde{A}^I(x), \nu_\tilde{A}^I(x)) : x \in X \}
$$

where the functions $\mu_\tilde{A}^I(x), \nu_\tilde{A}^I(x)$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set $\tilde{A}^I$, which is a subset of $X$ and for every $x \in X, 0 \leq \mu_\tilde{A}^I(x) + \nu_\tilde{A}^I(x) \leq 1$.

Obviously, every intuitionistic fuzzy set has the form $\tilde{A}^I = \{ (x, \mu_\tilde{A}^I(x), \nu_\tilde{A}^I(x)) : x \in X \}$ in $X$. For each intuitionistic fuzzy set $\tilde{A}^I = \{ (x, \mu_\tilde{A}^I(x), \nu_\tilde{A}^I(x)) : x \in X \}$, $\pi_\tilde{A}^I(x) = 1 - \mu_\tilde{A}^I(x) - \nu_\tilde{A}^I(x)$ is called the hesitancy degree of $x$ to lie in $\tilde{A}^I$. If $\tilde{A}^I$ is a fuzzy set, then $\pi_\tilde{A}^I(x) = 0$ for all $x \in X$. 

Definition 2.2 Intuitionistic Fuzzy Number:

An IFS \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) : x \in X\} \) of the real line \( R \) is called an intuitionistic fuzzy number (IFN) if

(a) \( \tilde{A} \) is convex for the membership function \( \mu_{\tilde{A}}(x) \), i.e., if \( \mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mu_{\tilde{A}}(x_1) \land \mu_{\tilde{A}}(x_2) \) for all \( x_1, x_2 \in R, \lambda \in [0,1] \).

(b) \( \tilde{A} \) is concave for the non-membership function \( \nu_{\tilde{A}}(x) \), i.e., if \( \nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \nu_{\tilde{A}}(x_1) \lor \nu_{\tilde{A}}(x_2) \) for all \( x_1, x_2 \in R, \lambda \in [0,1] \).

(c) \( \tilde{A} \) is normal, that is, there is some \( x_0 \in R \) such that \( \mu_{\tilde{A}}(x_0) = 1 \) and \( \nu_{\tilde{A}}(x_0) = 0 \).

Definition 2.3 Generalized Trapezoidal Intuitionistic Fuzzy Number:

An intuitionistic fuzzy number \( \tilde{A} \) is said to be generalized trapezoidal intuitionistic fuzzy number (GTIFN) with parameters \( b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4 \), and denoted by \( \tilde{A} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4; \omega_{\tilde{A}}, \nu_{\tilde{A}}) \) or \( \tilde{A} = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_{\tilde{A}}, \nu_{\tilde{A}}) \) if its membership and non-membership functions are as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0, & \text{if } x < a_1 \\
\omega_{\tilde{A}} \left( \frac{x - a_1}{a_2 - a_1} \right), & \text{if } a_1 \leq x \leq a_2 \\
\omega_{\tilde{A}}, & \text{if } a_2 \leq x \leq a_3 \\
\omega_{\tilde{A}} \left( \frac{a_4 - x}{a_4 - a_3} \right), & \text{if } a_3 \leq x \leq a_4 \\
0, & \text{if } x > a_4.
\end{cases}
\]

and

\[
\nu_{\tilde{A}}(x) = \begin{cases} 
1, & \text{if } x < b_1 \\
\frac{(b_2 - x) + u_{\tilde{A}}(x - b_1)}{b_2 - b_1}, & \text{if } b_1 \leq x \leq b_2 \\
u_{\tilde{A}}, & \text{if } b_2 \leq x \leq b_3 \\
\frac{(x - b_3) + u_{\tilde{A}}(b_4 - x)}{b_4 - b_3}, & \text{if } b_3 \leq x \leq b_4 \\
1, & \text{if } x > b_4.
\end{cases}
\]

Where \( 0 < \omega_{\tilde{A}} \leq 1, 0 \leq u_{\tilde{A}} \leq 1 \) and \( 0 < \omega_{\tilde{A}} + u_{\tilde{A}} \leq 1 \).
Definition 2.4 Arithmetic addition on Generalized Trapezoidal Intuitionistic Fuzzy Number:

Let \( \tilde{A}^l = ((a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \omega_{\tilde{A}^l}, u_{\tilde{A}^l}) \) and
\( \tilde{B} = ((a_5, a_6, a_7, a_8), (b_5, b_6, b_7, b_8); \omega_{\tilde{B}^l}, u_{\tilde{B}^l}) \) be two GTIFNs. Then

Addition: \( \tilde{A} + \tilde{B} = ((a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4), (a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8); \omega, u) \)

where \( \omega = \min\{\omega_{\tilde{A}^l}, \omega_{\tilde{B}^l}\} \) and \( u = \max\{u_{\tilde{A}^l}, u_{\tilde{B}^l}\} \).

Definition 2.5 Let \( \tilde{A}^l \) and \( \tilde{B}^l \) be two TrIFNs. The ranking of \( \tilde{A}^l \) and \( \tilde{B}^l \) by the R (.) on \( E \), the set of TrIFNs is defined as follows:

i. \( R(\tilde{A}^l) > R(\tilde{B}^l) \) iff \( \tilde{A}^l > \tilde{B}^l \)

ii. \( R(\tilde{A}^l) < R(\tilde{B}^l) \) iff \( \tilde{A}^l < \tilde{B}^l \)

iii. \( R(\tilde{A}^l) = R(\tilde{B}^l) \) iff \( \tilde{A}^l = \tilde{B}^l \)

Definition 2.6 The ordering \( \geq \) and \( \leq \) between any two TrIFNs \( \tilde{A}^l \) and \( \tilde{B}^l \) are defined as follows:

i. \( \tilde{A}^l \geq \tilde{B}^l \) iff \( \tilde{A}^l > \tilde{B}^l \) or \( \tilde{A}^l = \tilde{B}^l \)

ii. \( \tilde{A}^l \leq \tilde{B}^l \) iff \( \tilde{A}^l < \tilde{B}^l \) or \( \tilde{A}^l = \tilde{B}^l \)

Definition 2.7 Let \( \{\tilde{A}_i^l, i = 1,2,\ldots, n\} \) be a set of TrIFNs. If \( R(\tilde{A}_1^l) \leq R(\tilde{A}_k^l) \) for all \( i \), then the TrIFN \( \tilde{A}_k^l \) is the minimum of \( \{\tilde{A}_i^l, i = 1,2,\ldots, n\} \).

Definition 2.8 Let \( \{\tilde{A}_i^l, i = 1,2,\ldots, n\} \) be a set of TrIFNs. If \( R(\tilde{A}_1^l) \geq R(\tilde{A}_i^l) \) for all \( i \), then the TrIFN \( \tilde{A}_i^l \) is the maximum of \( \{\tilde{A}_i^l, i = 1,2,\ldots, n\} \).

Definition 2.9 Ranking Technique:

Let \( \tilde{A}^l = ((a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2); \omega_{\tilde{A}^l}, u_{\tilde{A}^l}) \) be a Generalized Trapezoidal Intuitionistic Fuzzy Number, then we define the new ranking of \( \tilde{A}^l \) as,
An Optimal Solution for Generalized Trapezoidal Intuitionistic Fuzzy...

\[ R_i(\tilde{A}') = \omega_A \left( \frac{a_i + b_i + c_i + d_i}{4} \right) \quad \text{and} \quad R_2(\tilde{A}') = \nu_A \left( \frac{a_2 + b_2 + c_2 + d_2}{4} \right), \quad \text{and} \]

\[ R(\tilde{A}') = \frac{R_i(\tilde{A}') + 2R_2(\tilde{A}')} {3}. \]

3. MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM:

Let us assume that there are \( m \) sources and \( n \) destinations. Let \( \tilde{a}_i \) be the fuzzy supply at \( i \), \( \tilde{b}_j \) be the fuzzy demand at destination \( j \), \( \tilde{c}_{ij} \) be the unit fuzzy transportation cost from source \( i \) to destination \( j \) and \( \tilde{x}_{ij} \) be the number of units shifted from source \( i \) to destination \( j \).

The fuzzy transportation problem can be mathematically expressed as

Minimize \( \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \)

Subject to the constraints \( \sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{a}_i, i = 1,2,3,...,m \), \( \sum_{i=1}^{m} \tilde{x}_{ij} \leq \tilde{b}_j, j = 1,2,3,...,n \) and \( \tilde{x}_{ij} \geq 0 \) for all \( i \) and \( j \).

A fuzzy transportation problem is said to be balanced if the total fuzzy supply from all sources equal to the total fuzzy demand in all destination \( \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \), otherwise it is called unbalanced.

4. PROPOSED APPROACH OF FUZZY TRANSPORTATION PROBLEM

Step 1: Find the rank of each cell \( \tilde{c}_{ij} \) of the chosen fuzzy cost matrix by using the ranking function as mentioned in section 2. If the number of sources is equal to the number of destinations, go to step 3. If the number of sources is not equal to the number of destinations go to step 2.

Step 2: Introduce dummy rows or dummy columns with zero fuzzy costs to form a balanced one.

Step 3: Proceed by the VAM method to find the initial basic feasible solution and if \( m+n-1 = \) number of allocations, then proceed by MODI method to obtain the optimal solution.

Step 4: Add the optimal fuzzy cost using fuzzy addition mentioned in section 2, to optimize the cost.
5. NUMERICAL EXAMPLE

1. Consider a fuzzy transportation problem with four origins \( O_1, O_2, O_3, O_4 \) and four destinations \( D_1, D_2, D_3, D_4 \) whose costs are considered to be generalized trapezoidal intuitionistic fuzzy numbers. The problem is to find the optimal transportation in an efficient way.

\[
\begin{array}{cccccc}
\text{Supply} & \text{D}_1 & \text{D}_2 & \text{D}_3 & \text{D}_4 & \text{Demand} \\
\hline
\text{O}_1 & (3.5,6,8; 2.4,7,10; 0.6,0.1) & (5.8,11,13; 4.6,12,14; 0.7,0.2) & (8.10,11,15; 7.9,13,17; 0.5,0.3) & (5.8,10,12; 4.7,11,13; 0.5,0.3) & (5.8,10,12; 4.7,11,14; 0.5,0.3) \\
\text{O}_2 & (7.9,10,12; 6.8,11,13; 0.7,0.1) & (3.5,6,8; 1.4,7,10; 0.4,0.3) & (6.8,10,12; 5.7,11,13; 0.7,0.1) & (5.8,10,12; 4.6,11,13; 0.8,0.1) & (5.8,9,11; 4.6,10,12; 0.4,0.3) \\
\text{O}_3 & (2.4,5,7; 1.3,6,8; 0.6,0.1) & (5.7,10,12; 4.6,11,14; 0.7,0.1) & (8.11,13,15; 7.9,14,16; 0.6,0.2) & (4.6,7,10; 2.5,8,11; 0.8,0.1) & (5.7,9,11; 4.6,10,12; 0.6,0.2) \\
\text{O}_4 & (6.8,10,12; 5.7,11,13; 0.8,0.1) & (2.5,6,8; 1.3,7,9; 0.7,0.1) & (5.7,10,14; 4.6,12,15; 0.6,0.2) & (2.4,5,7; 1.3,6,8; 0.7,0.1) & (4.6,8,10; 3.5,9,11; 0.6,0.2) \\
\text{Demand} & (5.7,8,10; 4.6,9,11; 0.6,0.1) & (5.7,8,10; 4.6,9,11; 0.6,0.1) & (10.8,6,4; 12.9,5,3; 0.4,0.3) & (7.9,11,14; 6.8,13,15; 0.5,0.3) & (4.7,8,10; 3.5,9,11; 0.5,0.3) \\
\end{array}
\]

**Solution:** The given generalized trapezoidal intuitionistic fuzzy cost table is balanced one. Using ranking technique, the rank of generalized trapezoidal intuitionistic fuzzy cost matrix is obtained.

**Table 1. Rank Table.**

<table>
<thead>
<tr>
<th></th>
<th>D(_1)</th>
<th>D(_2)</th>
<th>D(_3)</th>
<th>D(_4)</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{O}_1)</td>
<td>1.4833</td>
<td>3.3583</td>
<td>4.1333</td>
<td>3.2083</td>
<td>3.2583</td>
</tr>
<tr>
<td>(\text{O}_2)</td>
<td>2.85</td>
<td>1.8333</td>
<td>2.7</td>
<td>4.35</td>
<td>2.7</td>
</tr>
<tr>
<td>(\text{O}_3)</td>
<td>1.2</td>
<td>3.85</td>
<td>3.8833</td>
<td>2.2333</td>
<td>2.6667</td>
</tr>
<tr>
<td>(\text{O}_4)</td>
<td>3</td>
<td>1.5583</td>
<td>2.0333</td>
<td>1.35</td>
<td>2.3333</td>
</tr>
<tr>
<td>Demand</td>
<td>2.1</td>
<td>1.6583</td>
<td>3.8083</td>
<td>2.6083</td>
<td>2.6667</td>
</tr>
</tbody>
</table>

Proceeding by VAM method, \(m+n-1\) = the number of allocations, the initial basic feasible solution is non-degenerate.

Transportation Cost = 21.4510.
Proceeding by MODI method, the optimum solution is:

<table>
<thead>
<tr>
<th></th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
<th>D₄</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>O₁</td>
<td>1.483</td>
<td>3</td>
<td>1.0413</td>
<td>4.1333</td>
<td>3.2083</td>
</tr>
<tr>
<td>O₂</td>
<td>2.85</td>
<td>1.8333</td>
<td>2.7</td>
<td>4.35</td>
<td>2.7</td>
</tr>
<tr>
<td>O₃</td>
<td>1.2</td>
<td>0.0584</td>
<td>3.85</td>
<td>2.2333</td>
<td>2.6083</td>
</tr>
<tr>
<td>O₄</td>
<td>3</td>
<td>1.5583</td>
<td>2.0333</td>
<td>1.35</td>
<td>2.3333</td>
</tr>
<tr>
<td>Demand</td>
<td>2.1</td>
<td>1.6583</td>
<td>3.8083</td>
<td>2.6083</td>
<td>2.6667</td>
</tr>
</tbody>
</table>

Therefore, the transportation cost = 21.4492

**Comparative Analysis**

<table>
<thead>
<tr>
<th>RANKING TECHNIQUE</th>
<th>VAM/MODI</th>
<th>BEST CANDIDATE METHOD</th>
<th>ZERO TERMINATION METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>21.4492</td>
<td>25.9436</td>
<td>23.7357</td>
</tr>
<tr>
<td>A. Nagoor Gani &amp; V.N.Mohammed</td>
<td>27.6425</td>
<td>29.5942</td>
<td>32.5278</td>
</tr>
</tbody>
</table>

**6. CONCLUSION**

This paper proposes an optimal solution of a fuzzy transportation problem whose costs are taken as generalized trapezoidal intuitionistic fuzzy numbers. The proposed new ranking technique is better and effective compared to other methods in dealing such complex fuzzy transportation problems. As a future extension, the proposed algorithm may be used to solve, assignment problems using linguistic variables, transportation problems involving linguistic expressions, higher order intuitionistic transportation problem etc.,
REFERENCES


