A comparative study of fuzzy Polynomials and crisp Polynomials

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Abstract

One of the basic branches of pure Mathematics is Algebra. In Algebra study of polynomials is one of the most important parts. From our study we give a comparative analysis of fuzzy polynomials and crisp polynomials. In our paper we formulate the fuzzy polynomials by considering the variable of the polynomials as fuzzy number.

Keywords: Triangular Fuzzy Number, Crisp Polynomials, Fuzzy Polynomials, variable.

1. INTRODUCTION:

A basic form of mathematical expression, “Polynomials” which is used in all branches of advanced mathematics, from basic algebra to calculus and beyond. Polynomial functions play a role in economics, statistics, chemistry, numerical analysis and error correction in encoded material. In mathematics, a polynomial is an expression consisting of variables (or indeterminate) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents.

An example of a polynomial of a single variable x is $x^2 - 4x + 7$. Here all coefficient and variable are taken as crisp. Similarly we can define crisp polynomials of more than two variable in each case the coefficient and variables are taken as crisp. In our paper we extend this crisp coefficient and variables in fuzzy coefficient and fuzzy variables which leads to a new approach of defining fuzzy polynomials.
2. LITERATURE REVIEW


T. Allahviranloo, S. Asari has investigate fuzzy root of fuzzy polynomials (if exists) by using Newton-Raphson method by using parametric form of fuzzy coefficients of fuzzy polynomial and Newton-Raphson method they can find its fuzzy roots [5]. S. Abbasbandy, M. Otadi, M. Mosleh given, a new approach for solving systems of fuzzy polynomials based on fuzzy neural network [6]. In their worked, an architecture of fuzzy neural networks is also proposed to find a real root of a system of fuzzy polynomials (if exists) by introducing a learning algorithm. Puyin Liu introduced the fuzzy valued Bernstein polynomial to show the fact that continuous fuzzy valued functions can be approximately represented as the fuzzy valued polynomials on any compact set U ⊂ R^n [7].

3. BASIC THAT WE NEED : FROM FUZZY SET THEORY

Definition 3.1: Characteristic Function: It is a function on a crisp set assigns a value of either 1 or 0 to each individual in the universal set, thereby discriminating between members and non members of the crisp set under consideration.

Definition 3.2: Fuzzy set: Let X is a collection of objects denoted generally by x, then a fuzzy set of ordered pairs Ā in X is a set of order pairs

\[ \tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) : x \in X \} \]

\( \mu_{\tilde{A}} \) is called the membership function or grade of membership of x in Ā. The range of the membership function is a subset of the non-negative real number whose supremum is finite.

Definition 3.3: \( \alpha - \text{cut} \) of a fuzzy set: It is a crisp set defined on a fuzzy set given and denoted by

\[ \alpha_\tilde{A} = \{ x : \mu_{\tilde{A}}(x) \geq \alpha \} \]

where \( \mu_{\tilde{A}}(x) \) is \( \alpha - \text{cut} \) the corresponding membership function. When the inequality does not exist is called strong \( \alpha - \text{cut} \).
**Definition 3.4:**

**Height of a fuzzy set:** It is defined as the largest membership grade obtained by any element of a fuzzy set. i.e.

\[ h(A) = \sup_{x \in X} \mu_A(x). \]

**Definition 3.5:**

**Normal fuzzy set:** A fuzzy set A is said to be normal if \( h(A) = 1 \).

**Definition 3.6:**

**Triangular fuzzy set:** A fuzzy set is called triangular if the membership function of the set is given by

\[ \mu_A(x) = \begin{cases} 
  \frac{x-a}{b-a}, & a \leq x \leq b \\
  \frac{c-x}{c-b}, & b \leq x \leq c
\end{cases} \]

Simply it can be express as \( \tilde{A} = [a, b, c] \)

**Definition 3.7:**

**Operation of Triangular Fuzzy set:**

\( \tilde{A}_1 = [a_1, b_1, c_1] \) and \( \tilde{A}_2 = [a_2, b_2, c_2] \) are two triangular fuzzy number then Operation of these two are defined as

(a) Addition: \( \tilde{A}_1 + \tilde{A}_2 = [a_1, b_1, c_1] + [a_2, b_2, c_2] = [a_1 + a_2, b_1 + b_2, c_1 + c_2] \)

(b) Subtraction: \( \tilde{A}_1 - \tilde{A}_2 = [a_1, b_1, c_1] - [a_2, b_2, c_2] = [a_1 - a_2, b_1 - b_2, c_1 - c_2] \)

(c) Scalar Multiplication: \( \sigma \tilde{A}_1 = [\sigma a_1, \sigma b_1, \sigma c_1] \)

(d) Multiplication: \( \tilde{A}_1 \ast \tilde{A}_1 = [\min a_i b_i, \text{product of mid point}, \max a_i b_i] \)

**Our Approach with Example:**

A polynomial is an expression consisting of variables (or indeterminate) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents. Mathematically it can be written as

\[ f(x) = \sum_i a_i x^i \]

\[ = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \]

Which is a crisp polynomials with variables and as well as all the coefficient are crisp.
Fuzzy Polynomials: A polynomial is called fuzzy polynomials if the variable are a fuzzy variable.

\[ \hat{f}(\bar{x}) = \sum_{i} A_i \bar{x}^i = A_0 + A_1 \bar{x}^1 + A_2 \bar{x}^2 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]

Here all \( A_0, A_1, A_2, \ldots \ldots \ldots \ldots \ldots \) are all TFN which are consider as fuzzy variable of the polynomials \( \hat{f}(\bar{x}) \).

**Example:**

1. \( f(x) = 5x + 6 \)
   Then \( f(2) = 5 \times 2 + 6 = 16 \)

Extending these crisp polynomials to fuzzy polynomials we have the following expression

\[ \hat{f}(\bar{x}) = \sum_{i} A_i \bar{x}^i = A_1 \bar{x} + A_0 \]

Here \( A_0 = [6,6,6] \) and \( A_1 = [5,5,5] \)

\[ \hat{f}(\bar{2}) = \hat{f}([1,2,3]) = [5,5,5] \times [1,2,3] + [6,6,6] \]
\[ = [5,10,15] + [6,6,6] \]
\[ = [11,16,21] \]

Now the value of the polynomials is also a TFN and middle point is same as the crisp value.

**Example 2.** Consider

\( f(x) = x^2 + x \)

Then \( f(3) = 3^2 + 3 \)
\[ = 9 + 3 \]
\[ = 12 \]

Extending these crisp polynomials to fuzzy polynomials we have the following expression

\[ \hat{f}(\bar{x}) = \sum_{i} A_i \bar{x}^i = A_1 \bar{x} + A_0 \]

Here, \( A_0 = [1,1,1] \) and \( A_1 = [1,1,1] \)

\[ \hat{f}(\bar{3}) = \hat{f}(3) = 3^2 + 3 \]
\[ = [2,3,4] \times [2,3,4] + [2,3,4] \]
\[ = [4,9,16] + [2,3,4] \]
\[ = [6,12,20] \]
Similarly here we get a fuzzy value and middle point is same as the crisp value.

**Example 3** Consider a polynomials with more than one variables

\[ f(x, y) = 3xy^2 + 5x + 6y + 7 \]
\[ f(2,3) = 3 \times 2 \times 3^2 + 5 \times 2 + 6 \times 3 + 7 \]
\[ = 54 + 10 + 18 + 7 \]
\[ = 89 \]

Extending these crisp polynomials of two variables into fuzzy polynomials of two fuzzy variables we have the following expression.

\[ f(\tilde{x}, \tilde{y}) = \tilde{3} \tilde{x} \tilde{y}^2 + \tilde{5} \tilde{x} + \tilde{6} \tilde{y} + \tilde{7} \]
\[ f(2\tilde{3},3\tilde{3}) = 3 \times 2 \times 3^2 + 5 \times 2 + 6 \times 3 + 7 \]
\[ = 3 \times [1,2,3] \times [2,3,4]^2 + 5 \times [1,2,3] + 6 \times [2,3,4] + 7 \]
\[ = 3 \times [1,2,3] \times [4,9,16] + [5,10,15] + [12,18,24] + 7 \]
\[ = [3,3,3] \times [4,18,48] + [17,28,39] + [7,7,7] \]
\[ = [12,54,144] + [24,35,46] \]
\[ = [36,89,190] \]

Similarly here we get a fuzzy value and middle point is same as the crisp value.

**REFERENCES:**


