An improved ranking for Fuzzy Transportation Problem using Symmetric Triangular Fuzzy Number

G. Uthra¹, K. Thangavelu¹ and B. Amutha²

¹P.G.and Research Department of Mathematics, Pachaiyappa’s College, Chennai-600 030, Tamil Nadu, India.
²Department of Mathematics, Jeppiaar Engineering College, Chennai - 600 119, Tamil Nadu, India.

Abstract

The aim of this paper is to obtain the Optimal Solution for the Fuzzy Transportation Problem using Symmetric Triangular Fuzzy Numbers. The cost, supply and demand values of the Fuzzy Transportation Problem are taken as Symmetric Triangular Fuzzy Numbers. The Symmetric Triangular Fuzzy Numbers are converted into crisp values using proposed ranking. The Initial solution is then obtained by Vogel’s Approximation Method (VAM) and the Optimal Solution is obtained by Modified Distribution Method (MODI). A numerical example is given to illustrate the proposed method.

Keywords: Symmetric Triangular Fuzzy Number, Fuzzy Transportation Problem, Yager’s Ranking, Proposed Ranking

1. INTRODUCTION

Transportation problem is a particular class of linear programming problem, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one
place to another. The goods are transported from a set of sources (e.g., factory) to a set of destinations (e.g., warehouse) to meet the specific requirements. In other words, transportation problems deal with the transportation of a single product manufactured at different plants (supply - origin) to a number of different warehouses (demand - destination). The objective is to satisfy the demand at destinations from the supply constraints at the minimum transportation cost possible. To achieve this objective, we must know the quantity of available supplies and the quantities demanded. In addition, we must also know the location, to find the cost of transporting one unit of commodity from the place of origin to the destination. The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centers.

Suppose the transportation cost of transporting goods from each of the origin to each of the destination being different and known. The problem is to transport the goods from various origins to different destinations in such a manner that the cost of shipping or transportation is a minimum. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors.


In this paper a Fuzzy Transportation problem is considered. The cost, supply and demand values of the Fuzzy Transportation Problem are taken as Symmetric Triangular Fuzzy Numbers. The Symmetric Triangular Fuzzy Numbers are converted into crisp values using the proposed ranking. The problem is then solved by the usual VAM method to get the Initial basic feasible solution and MODI method to get the Optimal Solution.

The rest of this paper organized as follows. Section 2, some basic definitions, Yager’s Ranking of Symmetric Triangular Fuzzy Numbers and the proposed ranking of Symmetric Triangular Fuzzy Numbers are given. Section 3 presents introduction of Fuzzy Transportation Problem. Section 4, procedure, numerical example for the proposed method and the comparative study are given followed by conclusion in section-5.

2. DEFINITIONS

Definition 2.1. Fuzzy number

Let A be a classical set, \( \mu_A(x) \) be a function from A to \([0,1]\). A fuzzy set A with the membership function \( \mu_A(x) \) is defined as \( A = \{ x, \mu_A(x); x \in A \text{ and } \mu_A(x) \in [0,1] \} \).

Definition 2.2: Triangular Fuzzy Number

A fuzzy number \( A = (a_1, a_2, a_3) \) is defined to be a triangular fuzzy number if its membership functions \( \mu_A : \mathbb{R} \rightarrow [0,1] \) is equal to

\[
\mu_A(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1} & \text{if } x \in [a_1, a_2] \\
1 & \text{if } x = a_2 \\
\frac{a_3 - x}{a_3 - a_2} & \text{if } x \in [a_2, a_3] \\
0 & \text{otherwise}
\end{cases}
\]

It is denoted by \( A = (a_1, a_2, a_3) \) where \( a_1 \) is Core (A), \( a_2 \) is left width and \( a_3 \) is right width. The geometric representation of Triangular Fuzzy Number is shown in figure-1. Since, the shape of the Triangular Fuzzy Number A is usually in triangle it is called so.
The Parametric form of a Triangular Fuzzy Number is represented by
\[ A = [a_1 - a_2(1 - r), a_1 + a_3(1 - r)] \]

**Definition 2.3. Symmetric Triangular Fuzzy Number**

If \( a_2 = a_3 \), then the Triangular Fuzzy Number \( A = (a_1, a_2, a_3) \) is called Symmetric Triangular Fuzzy Number. It is denoted by \( A = (a_1, a_2) \) where \( a_1 \) is Core (A), \( a_2 \) is left width and right width of Core(A).

The Parametric form of a Symmetric Triangular Fuzzy Number is represented by
\[ A = [a_1 - a_2(1 - r), a_1 + a_2(1 - r)] \]

**Definition 2.4. Yager’s Ranking of Triangular Fuzzy Number**

Yager’s ranking technique [11] which satisfy compensation, linearity, additively properties and provides results which consists of human intuition. If \( A = (a_1, a_2, a_3) \) is a Triangular Fuzzy Number then the Yager’s ranking is defined by
\[ R(A) = \int_{0}^{1} 0.5(a_{l}^U, a_{u}^U) d\alpha \]
An improved ranking for Fuzzy Transportation Problem using Symmetric...

Where \((a^L_\alpha, a^U_\alpha) = \{(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha\}\)

For Symmetric Triangular Fuzzy Number, \(a_2 = a_3\).

**Definition 2.5. Proposed Ranking**

If \(A = (a_1, a_2, a_3)\) is a Triangular Fuzzy Number then the average ranking is defined by

\[
R(A) = \frac{a_1 + a_2 + a_3}{3}
\]

For Symmetric Triangular Fuzzy Number, \(a_2 = a_3\).

**Definition 2.6. Arithmetic Operations of Symmetric Triangular Fuzzy Number**

The Fuzzy Number is fully and uniquely represented by its r-cut, since the r-cut of each fuzzy number are closed interval of real numbers for all \(r \in [0, 1]\). This enables us to define arithmetic operations on Fuzzy number in terms of arithmetic operations on their r-cut.

Let \(A^*\) and \(B^*\) by arbitrary fuzzy numbers with the r-cut \(A^* = [A(r), A^*(r)]\) and \(B^* = [B(r), B^*(r)]\). Then the arithmetic operations between \(A^*\) and \(B^*\) are denoted by

(i) \(A^* + B^* = [A(r) + B(r), A^*(r) + B^*(r)]\)
(ii) \(A^* - B^* = [A(r) - B^*(r), A^*(r) - B(r)]\)
(iii) \(A^*B^* = H = [H(r)H^*(r)]\)

Where \(H(r) = \min \{A(r)B(r), A^*(r)B^*(r), A^*(r)B(r), A(r)B^*(r)\}\)

\(H^*(r) = \max \{A(r)B(r), A^*(r)B^*(r), A^*(r)B(r), A(r)B^*(r)\}\)

(iv) \(A^*/B^* = H = H(r)H^*(r)\)

Where \(H(r) = \min \{A(r)/B(r), A^*(r)/B^*(r), A^*(r)/B(r), A(r)/B^*(r)\}\)

\(H^*(r) = \max \{A(r)/B(r), A^*(r)/B^*(r), A^*(r)/B(r), A(r)/B^*(r)\}\)

(v) \(KA = \begin{cases} [KA(r), KA^*(r)], & \text{if } K \geq 0 \\ [KA^*(r), KA(r)], & \text{if } K < 0 \end{cases} \)
3. FUZZY TRANSPORTATION PROBLEM

Consider a transportation problem with ‘m’ Sources and ‘n’ destinations. Let $a_i^*$ be the number of supply units available at source $i$ ($i=1,2,...,m$) and let $b_j^*$ be the number of demand units required at destination $j$ ($j=1,2,...,n$). Let $C_{ij}^*$ represent the unit transportation cost for transporting the units from Source $i$ to Destination $j$. The objective is to determine the number of units to be transported from Source $i$ to destination $j$ so that the total transportation cost is minimized. In addition, the supply limits at the sources and the demand requirements at the destinations must be satisfied exactly.

The mathematical model of a Fuzzy Transportation Problem is given by

Minimize $Z^* = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^* x_{ij}$

Subject to

$\sum_{j=1}^{n} x_{ij} = a_i^*$, for $i = 1,2,...,m$

$\sum_{i=1}^{m} x_{ij} = b_j^*$, for $j = 1,2,...,n$

Where $x_{ij} \geq 0$

In the above model the transportation cost $C_{ij}^*$, Supplies $a_i^*$, and the demands $b_j^*$ are Symmetric Triangular Fuzzy Number.

4. PROCEDURE

1. First convert the cost, supply and demand values of the fuzzy transportation problem which are all symmetric triangular fuzzy numbers into crisp values by using proposed ranking.

2. Check the condition that the fuzzy transportation problem is balanced.
   (i) If balanced go to step 4 (Total supply = Total Demand)
   (ii) If not balanced go to step 3 (Total Supply ≠ Total Demand)

3. If the given Fuzzy Transportation problem is not balanced then add dummy row (or) dummy column with cost value as zero and supply value or demand value as (according to row or column) the difference between the total supply and total demand value.

4. Obtain the Initial basic feasible solution by VAM method and the optimal solution by MODI method.
**Example**

Consider a Fuzzy transportation problem whose cost, supply and demand values are Symmetric Triangular Fuzzy Numbers.

<table>
<thead>
<tr>
<th></th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
<th>D₄</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>(2,3,3)</td>
<td>(2,3,3)</td>
<td>(2,3,3)</td>
<td>(1,4,4)</td>
<td>(0,3,3)</td>
</tr>
<tr>
<td>S₂</td>
<td>(4,9,9)</td>
<td>(4,8,8)</td>
<td>(2,5,5)</td>
<td>(1,4,4)</td>
<td>(2,13,13)</td>
</tr>
<tr>
<td>S₃</td>
<td>(2,7,7)</td>
<td>(0,5,5)</td>
<td>(0,5,5)</td>
<td>(4,8,8)</td>
<td>(2,8,8)</td>
</tr>
<tr>
<td>Demand</td>
<td>(1,4,4)</td>
<td>(0,9,9)</td>
<td>(1,4,4)</td>
<td>(2,7,7)</td>
<td></td>
</tr>
</tbody>
</table>

By proposed ranking, the crisp values of the Symmetric Triangular Fuzzy Number are as follows.

<table>
<thead>
<tr>
<th></th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
<th>D₄</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>2.667</td>
<td>2.667</td>
<td>2.667</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>S₂</td>
<td>7.333</td>
<td>6.667</td>
<td>4</td>
<td>3</td>
<td>9.333</td>
</tr>
<tr>
<td>S₃</td>
<td>5.333</td>
<td>3.333</td>
<td>3.333</td>
<td>6.667</td>
<td>6</td>
</tr>
<tr>
<td>Demand</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>5.333</td>
<td></td>
</tr>
</tbody>
</table>

Here, Total Supply = Total Demand. The given problem is balanced.

The initial basic feasible solution is obtained by VAM method. The following table illustrate the initial basic feasible solution for the fuzzy transportation problem.

<table>
<thead>
<tr>
<th></th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
<th>D₄</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>2.667</td>
<td>2.667</td>
<td>2.667</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>S₂</td>
<td>7.333</td>
<td>6.667</td>
<td>4</td>
<td>3</td>
<td>9.333</td>
</tr>
<tr>
<td>S₃</td>
<td>5.333</td>
<td>3.333</td>
<td>3.333</td>
<td>6.667</td>
<td>6</td>
</tr>
<tr>
<td>Demand</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>5.333</td>
<td></td>
</tr>
</tbody>
</table>

Here \( m + n - 1 \neq \text{No. of allocations} \)
Hence there is a degeneracy in the basic feasible solution. To resolve this degeneracy, allocate a very small quantity $\varepsilon$ to the non-allocated cell with smallest cost value. The following table provides the non degenerate basic feasible solution to the above problem.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>2.667 $(2)$</td>
<td>2.667 $(\varepsilon)$</td>
<td>2.667</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$S_2$</td>
<td>7.333 $(1)$</td>
<td>6.667</td>
<td>4 $(3)$</td>
<td></td>
<td>3 $(5.333)$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>5.333</td>
<td>3.333 $(6)$</td>
<td>3.333</td>
<td>6.667</td>
<td>6</td>
</tr>
</tbody>
</table>

Demand: 3 6 3 5.333

Now $m + n - 1 = $ No. of allocations.

The Initial Basic Feasible Solution = $2.667 \times 2 + 2.667 \varepsilon + 7.333 \times 1 + 4 \times 3 + 3 \times 5.333 + 3.333 \times 6 = 60.664$

To get the Optimal Solution of the transportation problem apply MODI method. The following table provides the optimal allocation for the fuzzy transportation problem.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>2.667 $(2)$</td>
<td>2.667</td>
<td>2.667</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>$S_2$</td>
<td>7.333 $(1)$</td>
<td>6.667 $(\varepsilon)$</td>
<td>4 $(3)$</td>
<td></td>
<td>3 $(5.333)$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>5.333</td>
<td>3.333 $(6)$</td>
<td>3.333</td>
<td>6.667</td>
<td>6</td>
</tr>
</tbody>
</table>

Demand: 3 6 3 5.333

The Optimal Transportation Cost = $2.667 \times 2 + 6.667 \varepsilon + 7.333 \times 1 + 4 \times 3 + 3 \times 5.333 + 3.333 \times 6 = 60.664.$
Comparative Study:

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Ranking Methods</th>
<th>Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Proposed Ranking</td>
<td>60.664</td>
</tr>
<tr>
<td>2</td>
<td>Yager’s Ranking</td>
<td>72.5625</td>
</tr>
<tr>
<td>3</td>
<td>S.Naresh Kumar &amp; S. Kumara Ghuru Ranking</td>
<td>84.33</td>
</tr>
</tbody>
</table>

5. CONCLUSION:

In this paper a Fuzzy Transportation Problem whose cost values are taken as Symmetric Triangular Fuzzy Numbers are considered. The Symmetric Triangular Fuzzy Numbers are converted into crisp values using a proposed ranking function. The initial basic feasible solution and optimal solution is obtained by the usual VAM and MODI method respectively. The proposed ranking proves to be a better one from the comparative study.

REFERENCES:


