Semi-Pre Open Sets and Semi-Pre Continuity in Gradation of Openness

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Abstract

Intuitionistic fuzzy topological spaces have been extended in Sostak sense by Coker and Dimirci (see [5]). In the present paper, we introduce intuitionistic fuzzy semi-pre open sets and study their characterizations under the concept of gradation of openness in I-fuzzy topological spaces. Further we introduce and study the intuitionistic fuzzy semi-pre continuous mappings in the I-fuzzy topological spaces.

AMS subject classification:

Keywords: Fuzzy sets, intuitionistic fuzzy sets, gradation of openness, intuitionistic gradation of openness, fuzzy topological space.

1. Introduction

The notion of fuzzy sets was introduced by Zadeh [9] and later fuzzy topological spaces were introduced by Chang [2]. In [8], Sostak generalized the fuzzy topological spaces using the idea of gradation of openness. Further Chattopdhyay, Hazra and Samanta (see [3]) defined the concept of gradation of openness of fuzzy sets on X by a mapping \( \tau : I^X \rightarrow I \) (\( I \equiv [0, 1] \)), that satisfies the following conditions:

(i) \( \tau(0) = \tau(1) = 1 \);

(ii) if \( A, B \in I^X \), then \( \tau(A \cap B) \geq \tau(A) \land \tau(B) \);

(iii) if \( \{ A_i : i \in J \} \subseteq I^X \), then \( \tau(\bigcup_{i \in J} A_i) \geq \bigcap_{i \in J} \tau(A_i) \).
Then the pair \((X, \tau)\) is called the fuzzy topological space and the real number \(\tau(A)\) is called the degree of openness of a fuzzy set \(A\).


In this paper, we will generalize the intuitionistic fuzzy semi-pre open sets and intuitionistic fuzzy semi-pre continuous mappings using the concept of gradation of openness in I-fuzzy topological spaces and also investigate their characteristic properties.

2. Preliminaries

Let \(X\) be a universal set and \(I \equiv [0, 1]\) be closed unit interval of real line. Let \(\xi^X\) denote the set of all intuitionistic fuzzy sets on \(X\). For the sake of completeness first we define the concept of intuitionistic fuzzy sets (see [1]).

**Definition 2.1.** [1] Let \(X\) be a non-empty set. An intuitionistic fuzzy set (IF-set in short) \(A\) on \(X\) is an object having the form

\[
A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}
\]

where the functions \(\mu_A : X \to I\) and \(\nu_A : X \to I\) define the degree of membership (namely \(\mu_A(x)\)) and the degree of non-membership (namely \(\nu_A(x)\)) respectively of an element \(x\) in the IF-set \(A\) with \(0 \leq \mu_A(x) + \nu_A(x) \leq 1\) for each \(x \in X\). We shall denote for simplicity, the IF-set \(A\) given in (i) by \(A = \{(x, \mu_A, \nu_A)\}\).

The IF-sets \(\{< x, 0, 1 >\}\) and \(\{< x, 1, 0 >\}\) are called the null IF-set denoted as 0 and whole IF-set denoted as 1 respectively on \(X\).

**Definition 2.2.** Let \(X\) be a non-empty set and \(A_i = \{< x, \mu_{A_i}, \nu_{A_i} >\}, \forall i \in J\) be a family of IF-sets on \(X\). Then for each \(x \in X\) and \(i \in j\), we define the union, intersection and complement of IF-sets as follows:

\[
\bigcup_i A_i = \{ < x, \bigvee_i \mu_{A_i}(x), \bigwedge_i \nu_{A_i}(x) > \},
\]

\[
\bigcap_i A_i = \{ < x, \bigwedge_i \mu_{A_i}(x), \bigvee_i \nu_{A_i}(x) > \},
\]

\[
A^c = \{ < x, \nu_A, \mu_A > \}
\]

In [3], Chattopadhyay et.al. introduced the concept of gradation of openness of fuzzy sets in fuzzy topological space. We shall extend this concept in case of intuitionistic fuzzy sets as follows:
Definition 2.3. Gradation of Openness of IF-sets Let $X$ be a non-empty set. A function $\tau : \mathcal{X}^X \to I$ satisfying the following axioms:

1. $\tau(0) = \tau(1) = 1$;
2. if $A, B \in \mathcal{X}^X$, then $\tau(A \cap B) \geq \tau(A) \land \tau(B)$;
3. if $\{A_i : i \in J\} \subseteq \mathcal{X}^X$, then $\tau(\bigcup_{i \in J} A_i) \geq \bigcap_{i \in J} \tau(A_i)$.

Here we call $\tau$ is the gradation of openness on $X$ or fuzzy topology of IF-sets on $X$.

The pair $(X, \tau)$ will be called an I-fuzzy topological space and the real number $\tau(A)$ is called the degree of openness of IF-set $A$.

Definition 2.4. Let $(X, \tau)$ be an I-fuzzy topological space. A mapping $\tau^* : \mathcal{X}^X \to I$ defined by $\tau^*(A) = \tau(A^c)$ for every $A \in \mathcal{X}^X$ is called the gradation of closedness. The number $\tau^*(A)$ is called degree of closedness of an IF-set $A$.

Now we consider for any IF-set $A \in \mathcal{X}^X$, the map $\tau : \mathcal{X}^X \to I$ as follows:

$$\tau(A) = \begin{cases} 1, & \text{if } A = 0 \\ \inf\left\{\frac{1}{2}[\mu_A(x) + (1 - \nu_A(x))]: x \in X\right\}, & \text{if } A \neq 0 \end{cases}$$

We observe that $\tau$ is compatible with the gradation of openness on $X$ defined by Chattopadhyay [3]. In fact we have the following proposition.

Proposition 2.5. Let $X$ be a non-empty set. Then the map $\tau : \mathcal{X}^X \to I$ given by $\tau(0) = 1$ and $\tau(A) = \inf\left\{\frac{1}{2}[\mu_A(x) + (1 - \nu_A(x))]: x \in X\right\}$ if $A \neq 0$, satisfies the axioms of gradation of openness.

Definition 2.6. Let $(X, \tau)$ be an I-fuzzy topological space, where $\tau$ is the gradation of openness on $X$. Then for each $\rho \in [0, 1]$, $\tau_\rho = \{A \in \mathcal{X}^X : \tau(A) \geq \rho\}$ is actually an intuitionistic fuzzy topology in sense of Coker [4] and it will be called the $\rho$-level I-fuzzy topology on $X$ with respect to the gradation of openness $\tau$.

All IF-sets belong to $\tau_\rho$ are called IF-$\rho$-open sets and their complements are called IF-$\rho$-closed sets. We denote the interior and closure of any IF-set with respect to $\tau_\rho$ as $Int_\rho$ and $Cl_\rho$ respectively. Thus we have following.

Definition 2.7. Let $(X, \tau)$ be an I-fuzzy topological space and $A$ be an IF-set on $X$, then the $\rho$-interior and $\rho$-closure of $A$ are defined as follows:

$$Int_\rho(A) = \bigcup\{G \in \mathcal{X}^X : G \subseteq A \text{ and } G \in \tau_\rho\}$$

$$Cl_\rho(A) = \bigcap\{K \in \mathcal{X}^X : A \subseteq K \text{ and } K^c \in \tau_\rho\}$$

Proposition 2.8. The $\rho$-closure and $\rho$-interior operators satisfy the following properties:
\( (i) \) \( \text{Int}_\rho(A) \subseteq A \) ; \( (ii) \) \( A \subseteq \text{Cl}_\rho(A) \)

\( (iii) \) \( \text{Int}_\rho(\text{Int}_\rho(A)) = \text{Int}_\rho(A) \)

\( (iv) \) \( \text{Cl}_\rho(\text{Cl}_\rho(A)) = \text{Cl}_\rho(A) \)

\( (v) \) \( \text{Int}_\rho(A \cap B) = \text{Int}_\rho(A) \cap \text{Int}_\rho(B) \)

\( (vi) \) \( \text{Cl}_\rho(A \cup B) = \text{Cl}_\rho(A) \cup \text{Cl}_\rho(B) \)

\( (vii) \) \( (\text{Int}_\rho(A))^c = \text{Cl}_\rho(A^c) \)

\( (viii) \) \( (\text{Cl}_\rho(A))^c = \text{Int}_\rho(A^c) \).

3. Intuitionistic Fuzzy \( \rho \)-Semi-Pre Open (Closed) Sets

**Definition 3.1.** Let \( (X, \tau) \) be an I-fuzzy topological space and \( A \) be an IF-set on \( X \). Then for each \( \rho \in I \), IF-set \( A \) is called an

(i) IF-\( \rho \)-semi open set if \( A \subseteq \text{Cl}_\rho(\text{Int}_\rho(A)) \)

(ii) IF-\( \rho \)-alpha open set if \( A \subseteq \text{Int}_\rho(\text{Cl}_\rho(\text{Int}_\rho(A))) \)

(iii) IF-\( \rho \)-pre open set if \( A \subseteq \text{Int}_\rho(\text{Cl}_\rho(A)) \).

**Remark 3.2.** It is clear that for each \( \rho \in I \equiv [0, 1] \)

(i) Every IF-\( \rho \)-open (resp. IF-\( \rho \)-closed) set is IF-\( \rho \)-alpha open (resp. IF-\( \rho \)-alpha closed) set.

(ii) Every IF-\( \rho \)-alpha open (resp. IF-\( \rho \)-alpha closed) set is IF-\( \rho \)-semi open (resp. IF-\( \rho \)-semi closed) set.

(iii) Every IF-\( \rho \)-alpha open (resp. IF-\( \rho \)-alpha closed) set is IF-\( \rho \)-pre open (resp. IF-\( \rho \)-pre closed) set.

But converse of (i), (ii), (iii) may not be true in general. We have the following:

**Example 3.3.** Let \( X = \{a, b\} \) and \( A, B, C, D \in \xi^X \) are IF-sets defined as

\[ A = \{<a, 0.3, 0.5>, <b, 0.6, 0.4>\} \]

\[ B = \{<a, 0.4, 0.2>, <b, 0.3, 0.6>\} \]

\[ C = \{<a, 0.4, 0.2>, <b, 0.6, 0.4>\} \]

\[ D = \{<a, 0.3, 0.5>, <b, 0.3, 0.6>\} \]

We define an I-fuzzy topology \( \tau : \xi^X \rightarrow I \) as follows:

\[ \tau(F) = \begin{cases} 
1, & \text{if } F = 0, 1 \\
0.4, & \text{if } F = A, \\
0.35, & \text{if } F = B, D, \\
0.6, & \text{if } F = C, \\
0, & \text{otherwise}
\end{cases} \]
Let $\rho = 0.2$ and $E \in \xi^X$ be an IF-set defined as $E = \{<a,0.6,0.2>,<b,0.5,0.4>\}$. Then we see that $E$ is an IF-$\rho$-alpha open set because $E \subseteq Int_\rho(Cl_\rho(\text{Int}_\rho(E))) = 1$. But $E$ is not an IF-$\rho$-open set in $X$.

Now suppose $F = \{<a,0.4,0.4>,<b,0.6,0.3>\} \in \xi^X$ be an IF-set. We observe that $F$ is an IF-$\rho$-semi open set because $F \subseteq Cl_\rho(\text{Int}_\rho(F)) = 0$. But $F$ is not an IF-$\rho$-alpha open set in $X$ because $F \not\subseteq Int_\rho(Cl_\rho(\text{Int}_\rho(F))) = A$.

Again consider an IF-set $G = \{<a,0.3,0.6>,<b,0.2,0.7>\} \in \xi^X$. We see that IF-set $G$ is an IF-$\rho$-pre open set because $G \subseteq Int_\rho(Cl_\rho(G)) = A$. But $G$ is not an IF-$\rho$-alpha open set because $G \not\subseteq Int_\rho(Cl_\rho(\text{Int}_\rho(F))) = 0$.

**Definition 3.4.** Let $A$ be an IF-set in the I-fuzzy topological space $(X,\tau)$. Then for each $\rho \in I$, the IF-set $A$ is called an

(i) **IF-$\rho$-semi pre open set** if there exists an IF-$\rho$-pre open set $B$ such that $B \subseteq A \subseteq Cl_\rho(B)$

(ii) **IF-$\rho$-semi pre closed set** if there exists an IF-$\rho$-pre closed set $B$ such that $\text{Int}_\rho(B) \subseteq A \subseteq B$.

**Example 3.5.** Considering Example 3.3, we see that $E$ is an IF-$\rho$-semi pre open set because there exists an IF-$\rho$-pre open set $B$ such that $B \subseteq E \subseteq Cl_\rho(B) = 1$.

**Example 3.6.** Let $X = \{a,b,c\}$ and $A,B,C \in \xi^X$ be IF-sets defined as

$$A = \{<a,0.3,0.3>,<b,0.1,0.3>,<c,0.4,0.4>\}$$

$$B = \{<a,0.3,0.2>,<b,0.2,0.2>,<c,0.5,0.4>\}$$

$$C = \{<a,0.4,0.3>,<b,0.4,0.3>,<c,0.5,0.4>\}$$

We define an I-fuzzy topology $\tau : \xi^X \rightarrow I$ as follows:

$$\tau(F) = \begin{cases} 
1, & \text{if } F = 0,1 \\
0.4, & \text{if } F = A, \\
0.5, & \text{if } F = B, \\
0, & \text{otherwise}
\end{cases}$$

Let $\rho = 0.2$. We see that IF-set $C$ is an IF-$\rho$-pre open set in $X$ because $C \subseteq \text{Int}_\rho(Cl_\rho(C)) = 1$. Now suppose $M = \{<a,0.5,0.2>,<b,0.6,0.2>,<c,0.7,0.3>\}$ be an IF-set in $X$. Then we observe that there exists an IF-$\rho$-pre open set $C$ such that $C \subseteq M \subseteq Cl_\rho(C) = 1$. Hence $M$ is an IF-$\rho$-semi pre open set in $X$.

**Remark 3.7.** It is obvious that

(i) Every IF-$\rho$-semi open (resp. IF-$\rho$-semi closed) set is an IF-$\rho$-semi pre open (resp. IF-$\rho$-semi pre closed) set.
(ii) Every IF-\(\rho\)-pre open (resp. IF-\(\rho\)-pre closed) set is an IF-\(\rho\)-semi pre open (resp. IF-\(\rho\)-semi pre closed) set.

But converse of (i) and (ii) may not be true in general.

**Example 3.8.** Considering Example 3.6, we observe that IF-set \(M\) is an IF-\(\rho\)-semi pre open set. But \(M\) is not an IF-\(\rho\)-semi open set because \(M \not\subseteq \text{Cl}_{\rho}(\text{Int}_{\rho}(M)) = A^c\).

**Example 3.9.** Let \(X = \{a, b\}\) and \(A, B, C \in \xi^X\) be IF-sets defined as

\[
A = \{<a, 0.3, 0.7 >, <b, 0.4, 0.6 >\}
\]

\[
B = \{<a, 0.2, 0.8 >, <b, 0.3, 0.6 >\}
\]

\[
C = \{<a, 0.6, 0.3 >, <b, 0.5, 0.4 >\}
\]

We define an I-fuzzy topology \(\tau : \xi^X \to I\) as follows:

\[
\tau(F) = \begin{cases} 
1, & \text{if } F = 0, 1 \\
0.3, & \text{if } F = A, \\
0.2, & \text{if } F = B, \\
0, & \text{otherwise}
\end{cases}
\]

Suppose \(\rho = 0.1\). We see that IF-set \(C \in \xi^X\) is an IF-\(\rho\)-semi pre open set because there exists an IF-\(\rho\)-pre open set \(A\) such that \(A \subseteq C \subseteq \text{Cl}_{\rho}(A) = A^c\). But it is not an IF-\(\rho\)-pre open set because \(C \not\subseteq \text{Int}_{\rho}(\text{Cl}_{\rho}(C)) = A\).

**Theorem 3.10.** Let \(A\) be an IF-set in the I-fuzzy topological space \((X, \tau)\). Then for each \(\rho \in I\), following statements are equivalent:

(a) \(A\) is an IF-\(\rho\)-semi pre open set;

(b) \(A^c\) is an IF-\(\rho\)-semi pre closed set;

(c) \(A \subseteq \text{Cl}_{\rho}(\text{Int}_{\rho}(\text{Cl}_{\rho}(A)))\);

(d) \(\text{Int}_{\rho}(\text{Cl}_{\rho}(\text{Int}_{\rho}(A^c))) \subseteq A^c\).

**Proof.** Let \((X, \tau)\) be an I-fuzzy topological space and \(\rho \in I\), then we will prove the theorem in following steps

(1) (a) \(\Rightarrow\) (b): Let \(A\) be an IF-\(\rho\)-semi pre open set in \(X\), then there exists an IF-\(\rho\)-pre open set \(B\) such that \(B \subseteq A \subseteq \text{Cl}_{\rho}(B)\). It follows \(B^c \supseteq A^c \supseteq \text{Int}_{\rho}(B^c)\). Since \(B\) is an IF-\(\rho\)-pre open set, so that \(B^c\) is an IF-\(\rho\)-pre closed set and also \(\text{Int}_{\rho}(B^c) \subseteq A^c \subseteq B^c\). Hence \(A^c\) is an IF-\(\rho\)-semi pre closed set in \(X\). Similarly we can prove (b) \(\Rightarrow\) (a)
(2) (a) $\Rightarrow$ (c): Let $A$ be an IF-$\rho$-semi pre open set in $X$, then there exists an IF-$\rho$-pre open set $B$ such that
\[ B \subseteq A \subseteq Cl_\rho(B) \] (3.10.1)

Since $B$ is an IF-$\rho$-pre open set in $X$, therefore $B \subseteq Int_\rho(Cl_\rho(B))$. From (3.10.1) we have $B \subseteq A$, it follows $B \subseteq Int_\rho(Cl_\rho(B)) \subseteq Int_\rho(Cl_\rho(A))$. Hence $B \subseteq Int_\rho(Cl_\rho(A))$. It implies
\[ Cl_\rho(B) \subseteq Cl_\rho(Int_\rho(Cl_\rho(A))) \] (3.10.2)

Thus from equations (3.10.1) and (3.10.2), we obtain
\[ A \subseteq Cl_\rho(Int_\rho(Cl_\rho(A))) \]

(3) (c) $\Rightarrow$ (a): Suppose $A \in \xi^X$ such that $A \subseteq Cl_\rho(Int_\rho(Cl_\rho(A)))$ and suppose $Int_\rho(Cl_\rho(A)) = B$.

Now being $\rho$-interior, IF-set $B$ is an IF-$\rho$-open set. Since every IF-$\rho$-open set is an IF-$\rho$-alpha open set and every IF-$\rho$-alpha open set is an IF-$\rho$-pre open set, hence $B$ is an IF-$\rho$-pre open set in $X$. Further $Int_\rho(Cl_\rho(A)) \subseteq A$. Therefore $B = Int_\rho(Cl_\rho(A)) \subseteq A \subseteq Cl_\rho(B)$, it implies $B \subseteq A \subseteq Cl_\rho(B)$. Thus $A$ is an IF-$\rho$-semi pre open set in $X$.

(4) (b) $\iff$ (d) can easily be proved.

Theorem 3.11. Let $(X, \tau)$ be an I-fuzzy topological space and $\rho \in I$. Then

(i) Any union of IF-$\rho$-semi pre open sets is an IF-$\rho$-semi pre open set;

(ii) Any intersection of IF-$\rho$-semi pre closed set is an IF-$\rho$-semi pre closed set.

Proof. (i) Let $\{A_i : i \in J\}$ be a collection of IF-$\rho$-semi pre open sets in $(X, \tau)$. Then there exists an IF-$\rho$-pre open set $B_i$ such that
\[ B_i \subseteq A_i \subseteq Cl_\rho(B_i), \forall i \in J \] (3.11.1)

Since $\tau(\cup_{i \in J} B_i) \geq \bigwedge_{i \in J} \tau(B_i) \geq \rho$. Thus $\bigcup_{i \in J} B_i$ is an IF-$\rho$-open set and every IF-$\rho$-open set is an IF-$\rho$-pre open set. Therefore from (3.11.1), we have $\bigcup_{i \in J} B_i \subseteq \bigcup_{i \in J} A_i \subseteq \bigcup_{i \in J} Cl_\rho(B_i) \subseteq Cl_\rho(\bigcup_{i \in J} B_i)$. It follows $\bigcup_{i \in J} B_i \subseteq \bigcup_{i \in J} A_i \subseteq Cl_\rho(\bigcup_{i \in J} B_i)$. Hence $\bigcup_{i \in J} A_i$ is an IF-$\rho$-semi pre open set in $X$.

(ii) We can prove (ii) by taking complement.

Remark 3.12. The intersection of two IF-$\rho$-semi pre open sets is not an IF-$\rho$-semi pre open set in general as seen in following example.
Example 3.13. Let $X = \{a, b\}$ and $A, B \in \xi^X$ be IF-sets defined as

$$A = \{<a, 0.3, 0.6>, <b, 0.7, 0.2>\}$$

$$B = \{<a, 0.7, 0.2>, <b, 0.4, 0.5>\}$$

We define an I-fuzzy topology $\tau : \xi^X \rightarrow I$ as follows:

$$\tau(F) = \begin{cases} 1, & \text{if } F \in \{0, 1\} \\ 0.35, & \text{if } F = A, \\ 0, & \text{otherwise} \end{cases}$$

Let $\rho = 0.2$. We see that IF-set $A$ and $B$ are IF-$\rho$-semi pre open sets in $X$. But $A \cap B$ is not an IF-$\rho$-semi pre open set in $X$.

Theorem 3.14. Let $(X, \tau)$ be an I-fuzzy topological space and $B \in \xi^X$ be an IF-set in $X$.

(i) If $A$ is an IF-$\rho$-semi pre open set such that $A \subseteq B \subseteq Cl_\rho(A)$, then $B$ is an IF-$\rho$-semi pre open set in $X$.

(ii) If $A$ is an IF-$\rho$-semi pre closed set such that $Int_\rho(A) \subseteq B \subseteq A$, then $B$ is an IF-$\rho$-semi pre closed set in $X$.

Proof. We will prove (i) only; and (ii) can similarly be proved.

Let $A$ be an IF-$\rho$-semi pre open set and $B$ be an IF-set in $X$ such that

$$A \subseteq B \subseteq Cl_\rho(A) \quad (3.14.1)$$

Since $A$ is an IF-$\rho$-semi pre open set, then there exists an IF-$\rho$-pre open set $C$ such that

$$C \subseteq A \subseteq Cl_\rho(C) \quad (3.14.2)$$

From equations (3.14.1) and (3.14.2), we observe that

$$C \subseteq A \subseteq B \subseteq Cl_\rho(A) \subseteq Cl_\rho(C)$$

Thus $C \subseteq B \subseteq Cl_\rho(C)$. Hence $B$ is an IF-$\rho$-semi pre open set in $X$. \[\blacksquare\]

4. Intuitionistic Fuzzy-$\rho$-Semi Pre Continuous Mapping

Let $(X, \tau)$ and $(Y, \delta)$ be two I-fuzzy topological spaces. We recall a map $f : X \rightarrow Y$ is said to be IF-$\rho$-continuous if $\tau(f^{-1}(B)) \geq \delta(B)$ for each IF-$\rho$-open set $B \in \xi^Y$ such that $\delta(B) \geq \rho$.

Now we shall proceed to some generalizations of $\rho$-continuity (see [6]) with respect to the gradation of openness in I-fuzzy topological spaces.

Definition 4.1. Let $(X, \tau)$ and $(Y, \delta)$ be two I-fuzzy topological spaces and $f : X \rightarrow Y$ be a map. Then $f$ is said to be an
(i) IF-$\rho$-semi continuous map if $f^{-1}(B)$ is an IF-$\rho$-semi open set in $X$, for each $B \in \xi^Y$ such that $\delta(B) \geq \rho$.

(ii) IF-$\rho$-alpha continuous map if $f^{-1}(B)$ is an IF-$\rho$-alpha open set in $X$, for each $B \in \xi^Y$ such that $\delta(B) \geq \rho$.

(iii) IF-$\rho$-pre continuous map if $f^{-1}(B)$ is an IF-$\rho$-pre open set in $X$, for each $B \in \xi^Y$ such that $\delta(B) \geq \rho$.

Remark 4.2. It is clear that for each $\rho \in I$,

(i) Every IF-$\rho$-continuous mapping is an IF-$\rho$-alpha continuous mapping.

(ii) Every IF-$\rho$-alpha continuous mapping is an IF-$\rho$-semi continuous mapping.

(iii) Every IF-$\rho$-alpha continuous mapping is an IF-$\rho$-pre continuous mapping.

But converse of (i), (ii), (iii) may not be true in general.

Definition 4.3. Let $(X, \tau)$ and $(Y, \delta)$ be two I-fuzzy topological spaces. A mapping $f : X \rightarrow Y$ is said to be IF-$\rho$-semi pre continuous mapping if $f^{-1}(B)$ is an IF-$\rho$-semi pre open set in $X$ for each $B \in \xi^Y$ such that $\delta(B) \geq \rho$.

Remark 4.4. Every IF-$\rho$-semi continuous mapping is an IF-$\rho$-semi pre continuous map, but converse may not be true. This can be easily seen through the following example.

Example 4.5. Let $X = \{a, b\}, Y = \{u, v\}$ and $A, B \in \xi^X, C \in \xi^Y$ be IF-sets defined as follows:

$A = \{< a, 0.3, 0.7 >, < b, 0.4, 0.5 >\}$

$B = \{< a, 0.5, 0.4 >, < b, 0.6, 0.3 >\}$

$C = \{< u, 0.7, 0.3 >, < v, 0.8, 0.2 >\}$

We define I-fuzzy topologies $\tau : \xi^X \rightarrow I$ and $\delta : \xi^Y \rightarrow I$ as follows:

$$\tau(F) = \begin{cases} 1, & \text{if } F = 0, 1 \\ 0.3, & \text{if } F = A, \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(F) = \begin{cases} 1, & \text{if } F = 0, 1 \\ 0.7, & \text{if } F = C, \\ 0, & \text{otherwise} \end{cases}$$

Consider the mapping $f : (X, \tau) \rightarrow (Y, \delta)$ defined by $f(a) = u, f(b) = v$. We see that for each $\rho = 0.1$, IF-sets $f^{-1}(0) \equiv 0$ and $f^{-1}(1) \equiv 1$ and $f^{-1}(C) = \{< a, 0.7, 0.3 >, < b, 0.8, 0.2 >\}$ are IF-$\rho$-semi pre open sets in $X$. Thus $f$ is an IF-$\rho$-semi pre continuous map. But $f$ is not an IF-$\rho$-semi continuous map because we have $Int_{\rho}(f^{-1}(C)) =$
A and $\text{Cl}_\rho(\text{Int}_\rho(F^{-1}(C))) = \text{Cl}_\rho(A) = A^c$. Hence $f^{-1}(C) \not\subseteq \text{Cl}_\rho(\text{Int}_\rho(f^{-1}(C)))$, which shows that $f$ is not IF-$\rho$-semi continuous map.

**Remark 4.6.** Every IF-$\rho$-pre continuous map is an IF-$\rho$-semi pre continuous map, but converse may not true as shown in following example.

**Example 4.7.** Let $X = \{a, b\}, Y = \{u, v\}$ be two I-fuzzy topological spaces. Let $A, B \in \xi^X, C \in \xi^Y$ be IF-sets defined as follows:

- $A = \{(a, 0.3, 0.4), (b, 0.2, 0.5)\}$
- $B = \{(a, 0.4, 0.2), (b, 0.3, 0.2)\}$
- $C = \{(u, 0.3, 0.3), (v, 0.5, 0.3)\}$

We define I-fuzzy topologies $\tau : \xi^X \to I$ and $\delta : \xi^Y \to I$ as follows:

$$
\tau(F) = \begin{cases} 
1, & \text{if } F = 0, 1 \\
0.35, & \text{if } F = A, \\
0.55, & \text{if } F = B, \\
0, & \text{otherwise}
\end{cases}
$$

$$
\delta(F) = \begin{cases} 
1, & \text{if } F = 0, 1 \\
0.6, & \text{if } F = C, \\
0, & \text{otherwise}
\end{cases}
$$

Consider the mapping $f : (X, \tau) \to (Y, \delta)$ defined by $f(a) = u, f(b) = v$. We observe that for $\rho = 0.2$, the map $f$ is an IF-$\rho$-semi pre continuous map, but $f$ is not an IF-$\rho$-pre continuous map. In fact we see that $\text{Cl}_\rho(f^{-1}(C)) = A^c$ and $\text{Int}_\rho(\text{Cl}_\rho(f^{-1}(C))) = \text{Int}_\rho(A^c) = A$. Therefore $f^{-1}(C) \not\subseteq \text{Int}_\rho(\text{Cl}_\rho(f^{-1}(C)))$. Hence $f$ is not an IF-$\rho$-pre continuous map.

**Theorem 4.8.** Every IF-$\rho$-continuous map is an IF-$\rho$-semi pre continuous map.

**Proof.** Let $(X, \tau)$ and $(Y, \delta)$ be two I-fuzzy topological spaces and $f : X \to Y$ be an IF-$\rho$-continuous map. Suppose $B \in \xi^Y$ such that $\delta(B) \geq \rho$, then $\tau(f^{-1}(B)) \geq \delta(B) \geq \rho$. Since $\tau(f^{-1}(B)) \geq \rho$, therefore $f^{-1}(B)$ is an IF-$\rho$-open set in $X$ and every IF-$\rho$-open set is an IF-$\rho$-semi pre open set, hence $f^{-1}(B)$ is an IF-$\rho$-semi pre open set in $X$ for each $B \in \xi^Y$ such that $\delta(B^c) \geq \rho$. Thus $f$ is an IF-$\rho$-semi pre continuous map. But converse may not be true in general. This can be easily seen through the following example. 

**Example 4.9.** Considering Example 4.7, we see that $f$ is an IF-$\rho$-semi pre continuous map, but it is not an IF-$\rho$-continuous map because $f^{-1}(C)$ is not an IF-$\rho$-open set in $(X, \tau)$.

**Theorem 4.10.** Let $(X, \tau)$ and $(Y, \delta)$ be two I-fuzzy topological spaces and $f : X \to Y$ be a map. Then $f$ is an IF-$\rho$-semi pre continuous map iff $f^{-1}(B)$ is an IF-$\rho$-semi pre closed set such that $\tau^+(f^{-1}(B)) \geq \rho$ for each IF-$\rho$-closed set $B$ of $Y$. 

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Proof. Suppose \((X, \tau)\) and \((Y, \delta)\) are two I-fuzzy topological spaces and \(f : X \rightarrow Y\) is an IF-\(\rho\)-semi pre continuous map. Suppose \(B \in \xi^Y\) is an IF-\(\rho\)-closed set such that \(\delta^*(B) = \delta(B^c) \geq \rho\), so that \(B^c\) is an IF \(\rho\)-open set in \(X\). Therefore \(f^{-1}(B^c)\) is an IF-\(\rho\)-semi pre open set in \(X\) and also \(\tau(f^{-1}(B^c)) \geq \rho\). Further \(\tau^*(f^{-1}(B)) = \tau^*[(f^{-1}(B^c))^c] = \tau[f^{-1}(B^c)] \geq \rho\). Hence \(\tau^*(f^{-1}(B)) \geq \rho\). Thus \(f^{-1}(B)\) is an IF-\(\rho\)-semi pre closed set in \(X\).

Conversely; Let \(f : (X, \tau) \rightarrow (Y, \delta)\) be a map and let \(f^{-1}(B)\) is an IF-\(\rho\)-semi pre closed set in \(X\) such that \(\tau^*(f^{-1}(B)) \geq \rho\) for each IF-\(\rho\)-closed \(B\) in \(Y\). Then \(\delta^*(B) = \delta(B^c) \geq \rho\), therefore \(B^c\) is an IF-\(\rho\)-open set in \(Y\). Further we have \(\tau^*(f^{-1}(B)) = \tau(f^{-1}(B))^c = \tau(f^{-1}(B^c)) \geq \rho\). Hence \(f^{-1}(B)\) is an IF-\(\rho\)-semi pre open set in \(X\). Thus \(f\) is an IF-\(\rho\)-semi pre continuous map. \(\blacksquare\)

References