ψ^*α - irresolute maps, quasi ψ^*α -continuous maps and perfectly ψ^*α -continuous maps

N. Balamani
Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore-641043 Tamil Nadu, India.

A. Parvathi
Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore-641043 Tamil Nadu, India.

Abstract

Only a few class of generalized closed sets form a topology. The class of ψ^*α-closed set is one among them. In this paper we introduce new classes of maps called ψ^*α-irresolute maps, quasi ψ^*α-continuous maps and perfectly ψ^*α-continuous maps and study the relationships between the above maps and their properties and characterizations.

Keywords: ψg-closed sets, ψg-open sets, ψ^*α-closed sets and ψ^*α-open sets

1. INTRODUCTION


2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent non-empty topological space on which no separation axioms are assumed, unless otherwise mentioned. The interior
and closure of a subset $A$ of a space $(X, \tau)$ are denoted by $\text{int}(A)$ and $\text{cl}(A)$ respectively.

**Definition 2.1** A subset $A$ of a topological space $(X, \tau)$ is called

1) generalized closed set (briefly g-closed)[8] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

2) semi-generalized closed set (briefly sg-closed)[4] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

3) $\psi$-closed set [13] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is sg-open in $(X, \tau)$.

4) $\psi g$-closed set [12] if $\psi \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

5) $\psi^* \alpha$-closed set [1] if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\psi g$-open in $(X, \tau)$.

6) The $\psi^* \alpha$-closure of a set $A$ is defined as $\psi^* \alpha \text{cl}(A) = \bigcap \{F \subseteq X: A \subseteq F$ and $F$ is $\psi^* \alpha$-closed in $(X, \tau)\}$[1]

**Definition 2.2** A topological space $(X, \tau)$ is said to be a

(i) $\psi^* \alpha T_c$-space if every $\psi^* \alpha$-closed subset of $(X, \tau)$ is closed in $(X, \tau)$.[2]

(ii) $\psi^* \alpha T_\alpha$-space if every $\psi^* \alpha$-closed subset of $(X, \tau)$ is $\alpha$-closed in $(X, \tau)$.[2]

**Definition 2.3** A map $f : (X, \tau) \to (Y, \sigma)$ is called

i. (i)Continuous [8] if $f^{-1}(V)$ is closed in $(X, \tau)$ for each closed set $V$ of $(Y, \sigma)$.

(ii) $\alpha$-continuous [10] if $f^{-1}(V)$ is $\alpha$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

(iii) strongly continuous [7] if $f^{-1}(V)$ is both open and closed in $(X, \tau)$ for every subset $V$ of $(Y, \sigma)$.

(iv) perfectly continuous [11] if $f^{-1}(V)$ is both open and closed in $(X, \tau)$ for every closed subset $V$ of $(Y, \sigma)$.

(v) $\psi^* \alpha$-continuous[3] if $f^{-1}(V)$ is $\psi^* \alpha$-closed in $(X, \tau)$ for each closed set $V$ of $(Y, \sigma)$.

**Definition 2.4** A map $f : (X, \tau) \to (Y, \sigma)$ is called

i. irresolute [5] if $f^{-1}(V)$ is semi closed in $(X, \tau)$ for every semi closed set $V$ of $(Y, \sigma)$.

ii. $\alpha$- irresolute [10] if $f^{-1}(V)$ is $\alpha$-closed in $(X, \tau)$ for every $\alpha$-closed set $V$ of $(Y, \sigma)$.

iii. $\psi g$- irresolute [12] if $f^{-1}(V)$ is $\psi g$-closed in $(X, \tau)$ for every $\psi g$-closed set $V$ of $(Y, \sigma)$.

**Definition 2.5** A map $f : (X, \tau) \to (Y, \sigma)$ is called pre $\alpha$-closed [6] if the image of each $\alpha$-closed(resp. $\alpha$-open) set in $(X, \tau)$ is $\alpha$-closed(resp. $\alpha$-open) in $(Y, \sigma)$.
3. $\psi^*\alpha$ - IRRESOLUTE MAPS

**Definition 3.1** A map $f : (X, \tau) \to (Y, \sigma)$ is called $\psi^*\alpha$ - irresolute if $f^{-1}(V)$ is $\psi^*\alpha$ - closed in $(X, \tau)$ for every $\psi^*\alpha$ - closed set $V$ in $(Y, \sigma)$.

**Example 3.2** Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then $f$ is $\psi^*\alpha$ - irresolute.

**Theorem 3.3** A map $f : (X, \tau) \to (Y, \sigma)$ is $\psi^*\alpha$ - irresolute if and only if $f^{-1}(V)$ is $\psi^*\alpha$ - open in $(X, \tau)$ for every $\psi^*\alpha$ - open set $V$ in $(Y, \sigma)$.

**Proof:** Let $V$ be any $\psi^*\alpha$-open set in $(Y, \sigma)$. Since $f$ is $\psi^*\alpha$ - irresolute, $f^{-1}(V^c)$ is $\psi^*\alpha$-closed in $(X, \tau)$. Since $f^{-1}(V^c) = (f^{-1}(V))^c$, so $f^{-1}(V)$ is $\psi^*\alpha$-open in $(X, \tau)$.

Conversely, let $V$ be $\psi^*\alpha$-closed in $(Y, \sigma)$, then $f^{-1}(V^c)$ is $\psi^*\alpha$ -open in $(X, \tau)$. Since $f^{-1}(V^c) = (f^{-1}(V))^c$, $f^{-1}(V)$ is $\psi^*\alpha$-closed and hence $f$ is $\psi^*\alpha$ - irresolute.

**Theorem 3.4** If a map $f : (X, \tau) \to (Y, \sigma)$ is $\psi^*\alpha$ - irresolute, then it is $\psi^*\alpha$ - continuous but not conversely.

**Proof:** Let $V$ be any closed set in $(Y, \sigma)$. Since every closed set is $\psi^*\alpha$-closed [1] and $f$ is $\psi^*\alpha$ - irresolute, $f^{-1}(V)$ is $\psi^*\alpha$-closed in $(X, \tau)$. Therefore $f$ is $\psi^*\alpha$ - continuous.

**Example 3.5** Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Then $f$ is $\psi^*\alpha$ - continuous but not $\psi^*\alpha$ - irresolute, since $\{a, c\}$ is $\psi^*\alpha$ - closed in $(Y, \sigma)$ but $f^{-1}(\{a, c\}) = \{a, b\}$ is not $\psi^*\alpha$ - closed in $(X, \tau)$.

**Remark 3.6** Every $\psi^*\alpha$-irresolute map is continuous and $\alpha$-continuous, if $(X, \tau)$ is respectively an $\psi^*\alpha T_\alpha$ - space and an $\psi^*\alpha T_\alpha$ - space.

**Theorem 3.7** Let $(X, \tau)$ be any topological space and $(Y, \sigma)$ be an $\psi^*\alpha T_\alpha$ - space and $f : (X, \tau) \to (Y, \sigma)$ be a map. Then the following are equivalent.

(a) $f$ is $\psi^*\alpha$ - irresolute  
(b) $f$ is $\psi^*\alpha$ - continuous

**Proof:** (a) $\Rightarrow$ (b) Let $V$ be a closed set in $(Y, \sigma)$. Since every closed set is $\psi^*\alpha$-closed in $(Y, \sigma)$. Since $f : (X, \tau) \to (Y, \sigma)$ is $\psi^*\alpha$ - irresolute, $f^{-1}(V)$ is $\psi^*\alpha$ - closed in $(X, \tau)$. Hence $f$ is $\psi^*\alpha$ - continuous.

(b) $\Rightarrow$ (a) Let $V$ be a $\psi^*\alpha$ - closed set in $(Y, \sigma)$. Since $(Y, \sigma)$ is an $\psi^*\alpha T_\alpha$ - space, $V$ is closed in $(Y, \sigma)$ and $f : (X, \tau) \to (Y, \sigma)$ is $\psi^*\alpha$ - continuous, $f^{-1}(V)$ is $\psi^*\alpha$ - closed in $(X, \tau)$. Hence $f$ is $\psi^*\alpha$ - irresolute.

**Theorem 3.8** If a map $f : (X, \tau) \to (Y, \sigma)$ is $\psi^*\alpha$ - irresolute then for every subset $A$ of $(X, \tau)$ such that $f(A)$ is $\psi^*\alpha$ - closed in $(Y, \sigma)$, $f(\psi^*\alpha acl(A)) \subseteq \psi^*\alpha acl(f(A))$. 
Proof: For every subset $A \subseteq X$, $\psi^*\alpha\text{cl}(f(A))$ is $\psi^*\alpha$-closed in $(Y, \sigma)$. Since $f$ is $\psi^*\alpha$-irresolute, $f^{-1}(\psi^*\alpha\text{cl}(f(A)))$ is $\psi^*\alpha$-closed in $(X, \tau)$. Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\psi^*\alpha\text{cl}(f(A)))$. Therefore $\psi^*\alpha\text{cl}(A) \subseteq f^{-1}(\psi^*\alpha\text{cl}(f(A)))$ and hence $f((\psi^*\alpha\text{cl}(A)) \subseteq f(f^{-1}(\psi^*\alpha\text{cl}(f(A)))) \subseteq \psi^*\alpha\text{cl}(f(A))$.

**Theorem 3.9** If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\psi^*\alpha$-irresolute then for every $\psi^*\alpha$-closed set $B \subseteq Y$, $\psi^*\alpha\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\psi^*\alpha\text{cl}(B))$.

Proof: Let $B$ be a $\psi^*\alpha$-closed set in $(Y, \sigma)$. Then $\psi^*\alpha\text{cl}(B))$ is $\psi^*\alpha$-closed in $(Y, \sigma)$. Since $f$ is $\psi^*\alpha$-irresolute, $f^{-1}(\psi^*\alpha\text{cl}(B))$ is $\psi^*\alpha$-closed in $(X, \tau)$. Since $B \subseteq \psi^*\alpha\text{cl}(B)$, $f^{-1}(B) \subseteq f^{-1}(\psi^*\alpha\text{cl}(B))$, $\psi^*\alpha\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\psi^*\alpha\text{cl}(B))$.

**Theorem 3.10** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a closed and surjective $\psi^*\alpha$-irresolute map. If $(X, \tau)$ is a $\psi^*\alpha\text{cl}$-space then $(Y, \sigma)$ is a $\psi^*\alpha\text{cl}$-space.

Proof: Let $V$ be any $\psi^*\alpha$-closed set in $(Y, \sigma)$. Since $f$ is a $\psi^*\alpha$-irresolute map, $f^{-1}(V)$ is $\psi^*\alpha$-closed in $(X, \tau)$. Since $(X, \tau)$ is a $\psi^*\alpha\text{cl}$-space, $f^{-1}(V)$ is closed in $(X, \tau)$. Since $f$ is closed and surjective, $f(f^{-1}(V)) = V$ is closed in $(Y, \sigma)$. Hence $(Y, \sigma)$ is a $\psi^*\alpha\text{cl}$-space.

**Definition 3.11** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pre $\psi^*\alpha$-open if the image of each $\psi^*\alpha$-closed (resp. $\psi^*\alpha$-open) set in $(X, \tau)$ is $\psi^*\alpha$-closed (resp. $\psi^*\alpha$-open) in $(Y, \sigma)$.

**Theorem 3.12** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre $\psi^*\alpha$-open and $\psi^*\alpha$-continuous from $(X, \tau)$ to an $\alpha$-space $(Y, \sigma)$, then $f$ is $\psi^*\alpha$-irresolute.

Proof: Let $A$ be a $\psi^*\alpha$-closed set in $(Y, \sigma)$. Let $U$ be any $\psi^*\alpha$-open set in $(X, \tau)$ such that $f^{-1}(A) \subseteq U$. Then $A \subseteq f(U)$. Since $A$ is $\psi^*\alpha$-closed and $f(U)$ is $\psi^*\alpha$-open in $(Y, \sigma)$, $\text{cl}(A) \subseteq f(U)$ holds and $f^{-1}(\text{cl}(A)) \subseteq U$. Since $f$ is $\psi^*\alpha$-continuous and $(Y, \sigma)$ is an $\alpha$-space, $f^{-1}(\text{cl}(A))$ is $\psi^*\alpha$-closed in $(X, \tau)$ and so $\text{cl}(f^{-1}(A)) \subseteq \text{cl}(f^{-1}(\text{cl}(A))) \subseteq U$. Therefore $f^{-1}(A)$ is $\psi^*\alpha$-closed in $(X, \tau)$ and hence $f$ is $\psi^*\alpha$-irresolute.

**Theorem 3.13** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre $\psi^*\alpha$-open and $\alpha$-irresolute, then $f$ is $\psi^*\alpha$-irresolute.

Proof: Let $A$ be a $\psi^*\alpha$-closed set in $(Y, \sigma)$. Let $U$ be any $\psi^*\alpha$-open set in $(X, \tau)$ such that $f^{-1}(A) \subseteq U$. Then $A \subseteq f(U)$. Since $A$ is $\psi^*\alpha$-closed and $f$ is pre $\psi^*\alpha$-open, $\text{cl}(A) \subseteq f(U)$ holds and $f^{-1}(\text{cl}(A)) \subseteq U$. Since $f$ is $\alpha$-irresolute and $\text{cl}(A)$ is $\alpha$-closed in $(Y, \sigma)$, $f^{-1}(\text{cl}(A))$ is $\alpha$-closed in $(X, \tau)$. Thus $\text{cl}(f^{-1}(A)) \subseteq \text{cl}(f^{-1}(\text{cl}(A))) \subseteq U$ and so $f^{-1}(A)$ is $\psi^*\alpha$-closed in $(X, \tau)$ and hence $f$ is $\psi^*\alpha$-irresolute.

**Theorem 3.14** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre $\alpha$-closed and $\psi^*\alpha$-irresolute, then $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is $\psi^*\alpha$-irresolute.
Proof: Let $A$ be $\psi^*\alpha$-closed set in $(X, \tau)$. Let $(f^{-1})(A) = f(A) \subseteq U$, where $U$ is $\psi g$-open set in $(Y, \sigma)$. Then $A \subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is $\psi g$-open in $(X, \tau)$ and $A$ is $\psi^*\alpha$-closed in $(X, \tau)$, $acl(A) \subseteq f^{-1}(U)$ and hence $f(acl(A)) \subseteq U$. Since $f$ is pre $\alpha$-closed and $acl(A)$ is $\alpha$-closed in $(X, \tau)$, therefore $acl(f(acl(A))) \subseteq acl(f(A)) \subseteq U$. Thus $f(A)$ is $\psi^*\alpha$-closed in $(Y, \sigma)$ and so $f^{-1}$ is $\psi^*\alpha$-irresolute.

Remark 3.15 The following examples show that irresolute maps and $\psi^*\alpha$-irresolute maps are independent.

Example 3.16 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then $f$ is irresolute but not $\psi^*\alpha$-irresolute, since for the $\psi^*\alpha$-closed set $\{a\}$ in $(Y, \sigma)$, $f^{-1}(\{a\}) = \{a\}$ is not $\psi^*\alpha$-closed in $(X, \tau)$.

Example 3.17 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then $f$ is $\psi^*\alpha$-irresolute but not irresolute, since for the semi closed sets $\{b\}$ and $\{b, c\}$ in $(Y, \sigma)$, $f^{-1}(\{b\}) = \{b\}$ and $f^{-1}(\{b, c\}) = \{b, c\}$ are not semi closed in $(X, \tau)$.

Remark 3.18 The following examples show that $\alpha$-irresolute maps and $\psi^*\alpha$-irresolute maps are independent.

Example 3.19 Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, X\}$ and $\sigma = \{\phi, \{a, b\}, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = c$. Then $f$ is $\alpha$-irresolute but not $\psi^*\alpha$-irresolute, since for the $\psi^*\alpha$-closed sets $\{a, d\}$, $\{b, d\}$, $\{a, c, d\}$ and $\{b, c, d\}$ in $(Y, \sigma)$, $f^{-1}(\{a, d\}) = \{a, c\}$, $f^{-1}(\{b, d\}) = \{b, c\}$, $f^{-1}(\{a, c, d\}) = \{a, c, d\}$ and $f^{-1}(\{b, c, d\}) = \{b, c, d\}$ are not $\psi^*\alpha$-closed in $(X, \tau)$.

Example 3.20 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then $f$ is $\psi^*\alpha$-irresolute but not $\alpha$-irresolute, since for the $\alpha$-closed sets $\{a, c\}$ and $\{b, c\}$ in $(Y, \sigma)$, $f^{-1}(\{a, c\}) = \{a, c\}$ and $f^{-1}(\{b, c\}) = \{b, c\}$ are not $\alpha$-closed in $(X, \tau)$.

4. QUASI $\psi^*\alpha$-CONTINUOUS AND PERFECTLY $\psi^*\alpha$-CONTINUOUS

Definition 4.1 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called quasi $\psi^*\alpha$-continuous if $f^{-1}(V)$ is closed in $(X, \tau)$ for each $\psi^*\alpha$-closed set $V$ in $(Y, \sigma)$.

Example 4.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then $f$ is quasi $\psi^*\alpha$-continuous.
**Theorem 4.3** A map \( f : (X, \tau) \to (Y, \sigma) \) is quasi \( \psi^*\alpha \)-continuous if and only if the inverse image of every \( \psi^*\alpha \)-open set in \( (Y, \sigma) \) is open in \( (X, \tau) \).

**Proof:** Proof is similar to proposition 3.16[3]

**Definition 4.4** A map \( f : (X, \tau) \to (Y, \sigma) \) is called **perfectly \( \psi^*\alpha \)-continuous** if \( f^{-1}(V) \) is clopen in \( (X, \tau) \) for each \( \psi^*\alpha \)-closed set \( V \) in \( (Y, \sigma) \).

**Example 4.5** Let \( X = Y = \{a, b, c\} \), \( \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b\}, \{c\}, Y\} \). Let \( f : (X, \tau) \to (Y, \sigma) \) be a map defined by \( f(a) = a, f(b) = c, f(c) = b \). Then \( f \) is perfectly \( \psi^*\alpha \)-continuous.

**Theorem 4.6** A map \( f : (X, \tau) \to (Y, \sigma) \) is perfectly \( \psi^*\alpha \)-continuous if and only if the inverse image of every \( \psi^*\alpha \)-open set in \( (Y, \sigma) \) is clopen in \( (X, \tau) \).

**Proof:** Let \( U \) be any \( \psi^*\alpha \)-open set in \( (Y, \sigma) \). Since \( f \) is perfectly \( \psi^*\alpha \)-continuous, \( f^{-1}(U) \) is clopen in \( (X, \tau) \). Conversely, let \( V \) be any \( \psi^*\alpha \)-open set in \( (Y, \sigma) \). Since \( f^{-1}(V) = (f^{-1}(V))^c \) is clopen in \( (X, \tau) \). Thus \( f \) is perfectly \( \psi^*\alpha \)-continuous.

**Theorem 4.7** Every perfectly \( \psi^*\alpha \)-continuous map \( f : (X, \tau) \to (Y, \sigma) \) is quasi \( \psi^*\alpha \)-continuous but not conversely.

**Proof:** Let \( V \) be a \( \psi^*\alpha \)-closed set in \( (Y, \sigma) \). Since \( f \) is perfectly \( \psi^*\alpha \)-continuous, \( f^{-1}(V) \) is clopen in \( (X, \tau) \). Hence \( f \) is quasi \( \psi^*\alpha \)-continuous.

**Example 4.8** Let \( X = Y = \{a, b, c\} \), \( \tau = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, Y\} \). Let \( f : (X, \tau) \to (Y, \sigma) \) be a map defined by \( f(a) = c, f(b) = a, f(c) = b \). Then \( f \) is quasi \( \psi^*\alpha \)-continuous but not perfectly \( \psi^*\alpha \)-continuous, since for the \( \psi^*\alpha \)-closed set \( \{b\} \) in \( (Y, \sigma) \), \( f^{-1}(\{b\}) = \{c\} \) is closed but not open in \( (X, \tau) \).

**Theorem 4.9** Let \( f : (X, \tau) \to (Y, \sigma) \) be a continuous map. If \( (Y, \sigma) \) is a \( \psi^*\alpha T_c \)-space then \( f \) is quasi \( \psi^*\alpha \)-continuous.

**Proof:** Let \( V \) be a \( \psi^*\alpha \)-closed set in \( (Y, \sigma) \). Since \( f \) is continuous, \( f^{-1}(V) \) is open in \( (X, \tau) \). Hence \( f^{-1}(V) \) is clopen in \( (X, \tau) \). Hence \( f \) is perfectly \( \psi^*\alpha \)-continuous.

**Theorem 4.10** Let \( f : (X, \tau) \to (Y, \sigma) \) be a continuous map. If \( (Y, \sigma) \) is a \( \psi^*\alpha T_c \)-space and a discrete space then \( f \) is perfectly \( \psi^*\alpha \)-continuous.

**Proof:** Let \( V \) be a \( \psi^*\alpha \)-closed set in \( (Y, \sigma) \). Since \( f \) is continuous, \( f^{-1}(V) \) is closed in \( (X, \tau) \). Since \( (X, \tau) \) is discrete, \( f^{-1}(V) \) is open in \( (X, \tau) \). Hence \( f^{-1}(V) \) is clopen in \( (X, \tau) \). Hence \( f \) is perfectly \( \psi^*\alpha \)-continuous.

**Theorem 4.11** Let \( f : (X, \tau) \to (Y, \sigma) \) be a continuous map. If \( (Y, \sigma) \) is a \( \psi^*\alpha T_c \)-space and a discrete space then \( f \) is perfectly \( \psi^*\alpha \)-continuous.
Theorem 4.16

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = c, f(b) = a, f(c) = b$. Then $f$ is quasi $\psi'\alpha$-continuous if both $f$ and $g$ are quasi $\psi'\alpha$-continuous (resp. perfectly $\psi'\alpha$-continuous).
(vii) $\psi^* \alpha$ - continuous if $f$ is $\psi^* \alpha$- irresolute and $g$ is continuous (resp. $\alpha$-continuous).

(viii) $\psi^* \alpha$ - irresolute if both $f$ and $g$ are $\psi^* \alpha$- irresolute.

(ix) $\psi^* \alpha$ - irresolute if $g$ is quasi $\psi^* \alpha$- continuous (resp. perfectly $\psi^* \alpha$- continuous) and $f$ is $\psi^* \alpha$ - continuous.

**Proof:**

(i) Let $V$ be any $\psi^* \alpha$ - closed set in $(Z, \eta)$. Since $g$ is strongly continuous, $g^{-1}(V)$ is both open and closed in $(Y, \sigma)$. Since every closed set is $\psi^* \alpha$ - closed, $g^{-1}(V)$ is $\psi^* \alpha$ - closed in $(Y, \sigma)$. Since $f$ is quasi $\psi^* \alpha$- continuous (resp. perfectly $\psi^* \alpha$- continuous), $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed (resp. clopen) in $(X, \tau)$. Hence $g \circ f$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$- continuous).

(ii) Let $V$ be any $\psi^* \alpha$-closed set in $(Z, \eta)$. Since $g$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$ - continuous), $g^{-1}(V)$ is closed (resp. clopen) in $(Y, \sigma)$. Since $f$ is continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed (resp. clopen) in $(X, \tau)$. Hence $g \circ f$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$- continuous).

(iii) Let $V$ be any closed set in $(Z, \eta)$. Since every closed set is $\psi^* \alpha$-closed, $V$ is $\psi^* \alpha$-closed set in $(Z, \eta)$. Since $g$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$ - continuous), $g^{-1}(V)$ is closed (resp. clopen) in $(Y, \sigma)$. Since $f$ is quasi $\psi^* \alpha$- continuous (resp. perfectly $\psi^* \alpha$ - continuous), $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$-closed in $(X, \tau)$. Hence $g \circ f$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$- continuous).

(iv) Since every closed set is $\psi^* \alpha$ - closed, the result follows.

(v) Let $V$ be any $\psi^* \alpha$ - closed set in $(Z, \eta)$. Then $g^{-1}(V)$ is closed (resp. clopen) in $(Y, \sigma)$ as $g$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$ - continuous). Since every closed set is $\psi^* \alpha$ - closed, $g^{-1}(V)$ is $\psi^* \alpha$ - closed in $(Y, \sigma)$. Since $f$ is quasi $\psi^* \alpha$- continuous (resp. perfectly $\psi^* \alpha$ - continuous), $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed (resp. clopen) in $(X, \tau)$ and hence $g \circ f$ is quasi $\psi^* \alpha$ - continuous (resp. perfectly $\psi^* \alpha$- continuous).

(vi) Let $V$ be any closed set in $(Z, \eta)$. Since $g$ is $\psi^* \alpha$ - continuous, $g^{-1}(V)$ is $\psi^* \alpha$ - closed in $(Y, \sigma)$. Since $f$ is $\psi^* \alpha$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ - closed in $X$. Hence $g \circ f$ is $\psi^* \alpha$ - continuous.

(vii) Let $V$ be any closed set in $(Z, \eta)$. Since $g$ is continuous (resp. $\alpha$ - continuous), $g^{-1}(V)$ is closed (resp. $\alpha$ - closed) in $(Y, \sigma)$. Since every closed (resp. $\alpha$ - closed) set is $\psi^* \alpha$ - closed, $g^{-1}(V)$ is $\psi^* \alpha$ - closed. Since $f$ is $\psi^* \alpha$ - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^* \alpha$ - closed in $(X, \tau)$. Therefore $g \circ f$ is $\psi^* \alpha$ - continuous.
(viii) Let \( V \) be any \( \psi^*\alpha \)-closed set in \((Z, \eta)\). Since \( g \) is \( \psi^*\alpha \)-irresolute, \( g\\undi g(V) \) is \( \psi^*\alpha \)-closed in \((Y, \sigma)\). Since \( f \) is \( \psi^*\alpha \)-irresolute, \((g\\undi f)\\undi g(V) = f\\undi (g\\undi g(V))\) is \( \psi^*\alpha \)-closed in \((X, \tau)\). Therefore \( g\\undi f \) is \( \psi^*\alpha \)-irresolute.

(ix) Let \( V \) be any \( \psi^*\alpha \)-closed set in \((Z, \eta)\). Then \( g\\undi g(V) \) is closed (resp. clopen) in \((Y, \sigma)\) as \( g \) is quasi \( \psi^*\alpha \)-continuous (resp. perfectly \( \psi^*\alpha \)-continuous). Since \( f \) is \( \psi^*\alpha \)-continuous, \((g\\undi f)\\undi g(V) = f\\undi (g\\undi g(V))\) is \( \psi^*\alpha \)-closed in \((X, \tau)\). Therefore \( g\\undi f \) is \( \psi^*\alpha \)-irresolute.

REFERENCES


