Truss Design Optimization using Neutrosophic Optimization Technique: A Comparative Study

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Abstract
In this paper, we have developed a neutrosophic optimization (NSO) approach for optimizing the design of truss with single objective, subject to a specified set of constraints. In this optimum design formulation, weight of truss and deflection of loaded joint are the objective functions. The design variables in the constraints are the cross-sectional areas and the constraints are stresses in members respectively. A classical truss optimization example is presented here in to demonstrate the efficiency of the neutrosophic optimization approach. The test problem includes a two-bar truss subjected to a single load condition. This single-objective structural optimization model is solved by fuzzy as well as intuitionistic fuzzy and neutrosophic optimization approach with consideration of both linear and nonlinear membership Function. Numerical example is given to illustrate our NSO approach. The result shows that the NSO technique plays a significant role in finding the best ever optimal solutions.

1. INTRODUCTION

The research area of optimal structural design has been receiving increasing attention from both academia and industry over the past four decades in order to improve structural performance and to reduce design costs. However, in the real world, uncertainty or vagueness is prevalent in the Engineering Computations. In the context of structural design the uncertainty is connected with lack of accurate data of design factors. This problem has been solving by use of fuzzy mathematical algorithm for dealing with this class of problems. Fuzzy set (FS) theory has long been introduced to deal with inexact and imprecise resources by Zadeh [1], as an application Bellman and Zadeh [2] used the fuzzy set theory to the decision making problem. In such extension, Atanassov [3] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory and is characterized by a membership function, a non membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a degree of acceptance and degree of rejection so that their sum is less than one. As a generalization of fuzzy set and intuitionistic fuzzy set F. Smrandache [4] introduced a new notion which is known as neutrosophic set (NS in short) in 1995. NS is characterized by degree of truth membership, degree of indeterminacy membership and degree of falsity membership. The concept of NS generates the theory of neutrosophic sets by expressing indeterminacy of imprecise information. This theory is considered as complete representation of structural design problems like other decision making problems. Therefore, if uncertainty is involved in a structural model, we use fuzzy theory while dealing indeterminacy, we need neutrosophic theory.

This is the first time neutrosophic optimization technique is applied in structural design. Several researchers like Wang et al. [5] first applied $\alpha$-cut method to structural designs where the non-linear problems were solved with various design levels $\alpha$, and then a sequence of solutions were obtained by setting different level-cut value of $\alpha$. To design a four-bar mechanism for function generating problem, Rao [6] used the same $\alpha$-cut method. Structural optimization with fuzzy parameters was developed by Yeh et al. [7]. Xu [8] used two-phase method for fuzzy optimization of structures. A level-cut of the first and second kind approach used by Shih et al. [9] for structural design optimization problems with fuzzy resources. Shih et al. [10] developed an alternative $\alpha$-level-cuts methods for optimum structural design with fuzzy resources. Dey et al. [11] used generalized fuzzy number in context of a structural design. Dey et al. [12] developed parameterized t-norm based fuzzy optimization method for optimum structural design. Also, a parametric geometric programming is introduced by Dey et al. [13] to Optimize shape design of structural model with imprecise coefficient.

A transportation model was solved by Jana et al. [14] using multi-objective intuitionistic fuzzy linear programming. Dey et al. [15] solved two bar truss non linear problem by using intuitionistic fuzzy optimization problem. Dey et al. [16] used intuitionistic fuzzy optimization technique for multi objective optimum structural design. R-x Liang et al. [17] applied interdependent inputs of single valued trapezoidal neutrosophic information on Multi-criteria group decision making problem. P Ji et al. [18], S Yu et al. [19] did so many research study on application based neutrosophic sets and

The present study investigates computational algorithm for solving single-objective structural problem by single valued NSO approach. The impact of linear and nonlinear truth, indeterminacy and falsity membership functions in such optimization process also have been studied here. A comparison are made numerically among fuzzy optimization technique , Intuitionistic fuzzy and neutrosophic optimization technique. From our numerical result, it is clear that neutrosophic optimization technique provides better results than fuzzy as well as intuitionistic fuzzy optimization.

2. SINGLE-OBJECTIVE STRUCTURAL MODEL

In sizing optimization problems, the aim is to minimize single objective function, usually the weight of the structure under certain behavioural constraints which are displacement or stresses. The design variables are most frequently chosen to be dimensions of the cross sectional areas of the members of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness of cross-sections to a certain set. A discrete structural optimization problem can be formulated in the following form

Minimize \( WT(A) \)

subject to \( \sigma_i(A) \leq [\sigma_i(A)], i = 1,2,\ldots,m \)

\( A_j \in R^d, \quad j = 1,2,\ldots,n \)

where \( WT(A) \) represents objective function, \( \sigma_i(A) \) is the behavioural constraints and \( [\sigma_i(A)] \) denotes the maximum allowable value , m and n are the number of constraints and design variables respectively. A given set of discrete value is expressed by \( R^d \) and in this paper objective function is taken as

\[
WT(A) = \sum_{i=1}^{m} \rho_j A_j
\]

and constraint are chosen to be stress of structures as follows

\( \sigma_i(A) \leq \sigma_i^0 \) with allowable tolerance \( \sigma_i^0 \) for \( i = 1,2,\ldots,m \)
Where $\rho_i$ and $l_i$ are weight of unit volume and length of $i^{th}$ element respectively, $m$ is the number of structural element, $\sigma_i$ and $\sigma_i^0$ are the $i^{th}$ stress, allowable stress respectively.

3. MATHEMATICAL PRELIMINARIES

3.1. Fuzzy Set

Let $X$ be a fixed set. A fuzzy set $A$ set of $X$ is an object having the form $\tilde{A} = \{ (x, T_A(x)) : x \in X \}$ where the function $T_A : X \rightarrow [0,1]$ defined the truth membership of the element $x \in X$ to the set $A$.

3.2. Intuitionistic Fuzzy Set

Let a set $X$ be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}$ in $X$ is an object of the form $\tilde{A} = \{ (x, T_A(x), F_A(x) > |x \in X \}$ where $T_A : X \rightarrow [0,1]$ and $F_A : X \rightarrow [0,1]$ define the truth membership and falsity membership respectively, for every element of $x \in X$, $0 \leq T_A(x) + F_A(x) \leq 1$.

3.3. Neutrosophic Set

Let a set $X$ be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}$ in $X$ is defined by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$ and having the form $\tilde{A} = \{ (x, T_A(x), I_A(x), F_A(x) > |x \in X \}$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $[0^-, 1^+]$. That is $T_A(x) : X \rightarrow [0^-, 1^+]$ $I_A(x) : X \rightarrow [0^-, 1^+]$ $F_A(x) : X \rightarrow [0^-, 1^+]$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

3.4. Single Valued Neutrosophic Set

Let a set $X$ be the universe of discourse. A single valued neutrosophic set $\tilde{A}$ over $X$ is an object having the form $\tilde{A} = \{ (x, T_A(x), I_A(x), F_A(x) > |x \in X \}$ where
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\[ T_A : X \rightarrow [0,1], I_A : X \rightarrow [0,1] \text{ and } F_A : X \rightarrow [0,1] \text{ with } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \text{ for all } x \in X. \]

### 3.5. Complement of Neutrosophic Set

Complement of a single valued neutrosophic set \( A \) is denoted by \( C_A \) and its truth, indeterminacy and falsity membership functions are denoted by \( T_{C(A)} : X \rightarrow [0,1], I_{C(A)} : X \rightarrow [0,1] \text{ and } F_{C(A)} : X \rightarrow [0,1] \) where

\[
T_{C(A)}(x) = F_A(x), \quad I_{C(A)}(x) = 1 - F_A(x), \quad F_{C(A)}(x) = T_A(x)
\]

### 3.6. Union of Neutrosophic Sets

The union of two single valued neutrosophic sets \( A \text{ and } B \) is a single valued neutrosophic set \( U \), written as \( U = A \cup B \), whose truth membership, indeterminacy membership and falsity membership functions are given by

\[
\begin{align*}
T_{U(A)}(x) &= \max\left(T_A(x), T_B(x)\right), \\
I_{U(A)}(x) &= \max\left(I_A(x), I_B(x)\right) \\
F_{U(A)}(x) &= \min\left(F_A(x), F_B(x)\right) \text{ for all } x \in X
\end{align*}
\]

or

\[
\begin{align*}
T_{U(A)}(x) &= \max\left(T_A(x), T_B(x)\right), \\
I_{U(A)}(x) &= \min\left(I_A(x), I_B(x)\right) \\
F_{U(A)}(x) &= \min\left(F_A(x), F_B(x)\right) \text{ for all } x \in X
\end{align*}
\]

### 3.7. Intersection of Neutrosophic Sets

The intersection of two single valued neutrosophic sets \( A \text{ and } B \) is a single valued neutrosophic set \( E \), written as \( E = A \cap B \), whose truth membership, indeterminacy membership and falsity membership functions are given by

\[
\begin{align*}
T_{E(A)}(x) &= \min\left(T_A(x), T_B(x)\right) \\
I_{E(A)}(x) &= \min\left(I_A(x), I_B(x)\right) \\
F_{E(A)}(x) &= \max\left(F_A(x), F_B(x)\right) \text{ for all } x \in X
\end{align*}
\]

or
\[ T_{E(A)}(x) = \min \left( T_A(x), T_B(x) \right) \]

\[ I_{E(A)}(x) = \max \left( I_A(x), I_B(x) \right) \]

\[ F_{E(A)}(x) = \max \left( F_A(x), F_B(x) \right) \text{ for all } x \in X \]

4. MATHEMATICAL ANALYSIS

4.1. Neutrosophic Optimization Technique to Solve Minimization Type Single-Objective Nonlinear Programming Problem

Let us consider a single-objective nonlinear optimization problem as

\[ \text{Minimize } f(x) \]

\[ g_j(x) \leq b_j \quad j = 1, 2, \ldots, m \]

\[ x \geq 0 \]

Usually constraints goals are considered as fixed quantity. But in real life problem, the constraint goal can not be always exact. So we can consider the constraint goal for less than type constraints at least \( b_j \) and it may possible to extend to \( b_j + b_j^0 \). This fact seems to take the constraint goal as a neutrosophic fuzzy set and which will be more realistic descriptions than others. Then the NLP becomes NSO problem with neutrosophic resources, which can be described as follows

\[ \text{Minimize } f(x) \]

\[ g_j(x) \leq \tilde{b}_j \quad j = 1, 2, \ldots, m \]

\[ x \geq 0 \]

To solve the NSO (3), following Warner’s (1987) and Angelov (1995) we are presenting a solution procedure for single-objective NSO problem (3) as follows

**Step-1:** Following Warner’s approach solve the single objective non-linear programming problem without tolerance in constraints (i.e \( g_j(x) \leq b_j \)), with tolerance of acceptance in constraints (i.e \( g_j(x) \leq b_j + b_j^0 \)) by appropriate non-linear programming technique

Here they are

**Sub-problem-1**

\[ \text{Minimize } f(x) \]

\[ g_j(x) \leq b_j \quad j = 1, 2, \ldots, m \]
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\[ x \geq 0 \]

**Sub-problem-2**

Minimize \( f(x) \)

\[ g_j(x) \leq b_j + b^0_j, \quad j = 1, 2, \ldots, m \]

\[ x \geq 0 \]

we may get optimal solutions \( x^* = x^1, f(x^*) = f(x^1) \) and \( x^* = x^2, f(x^*) = f(x^2) \) for sub-problem 1 and 2 respectively.

**Step-2:** From the result of step 1 we now find the lower bound and upper bound of objective functions. If \( U^T_{f(i)}, U^I_{f(i)}, U^F_{f(i)} \) be the upper bounds of truth, indeterminacy, falsity function for the objective respectively and \( L^T_{f(i)}, L^I_{f(i)}, L^F_{f(i)} \) be the lower bound of truth, indeterminacy, falsity membership functions of objective respectively then

\[
U^T_{f(i)} = \max \left\{ f(x^1), f(x^2) \right\},
\]

\[
U^I_{f(i)} = U^T_{f(i)} - L^I_{f(i)} + \varepsilon_{f(i)} \quad \text{where} \quad 0 < \varepsilon_{f(i)} < \left( U^T_{f(i)} - L^I_{f(i)} \right)
\]

\[
L^T_{f(i)} = L^I_{f(i)} + \xi_{f(i)} \quad \text{where} \quad 0 < \xi_{f(i)} < \left( U^T_{f(i)} - L^I_{f(i)} \right)
\]

**Fig.1.** Rough Sketch of Lower and Upper bounds of Truth, Indeterminacy and Falsity Membership Functions

**Step-3:** In this step we calculate linear membership for truth, indeterminacy and falsity membership functions of objective as follows

\[
T_{f(i)}(f(x)) = \begin{cases} 
1 & \text{if } f(x) \leq L^T_{f(i)}, \\
\frac{U^T_{f(i)} - f(x)}{U^T_{f(i)} - L^T_{f(i)}} & \text{if } L^T_{f(i)} < f(x) \leq U^T_{f(i)}, \\
0 & \text{if } f(x) \geq U^T_{f(i)}
\end{cases}
\]
Step-4: In this step using linear, exponential and hyperbolic function for truth, indeterminacy and falsity membership functions, we may calculate membership function for constraints as follows

\[ I_{f(x)}(f(x)) = \begin{cases} 
1 & \text{if } f(x) \leq L_{f(x)}^I \\
\frac{U_{f(x)}^I - f(x)}{U_{f(x)}^I - L_{f(x)}^I} & \text{if } L_{f(x)}^I \leq f(x) \leq U_{f(x)}^I \\
0 & \text{if } f(x) \geq U_{f(x)}^I 
\end{cases} \]

\[ F_{f(x)}(f(x)) = \begin{cases} 
\frac{f(x) - L_{f(x)}^F}{U_{f(x)}^F - L_{f(x)}^F} & \text{if } L_{f(x)}^F \leq f(x) \leq U_{f(x)}^F \\
1 & \text{if } f(x) \geq U_{f(x)}^F 
\end{cases} \]

and exponential and hyperbolic membership for truth, indeterminacy and falsity membership functions as follows

\[ T_{f(x)}(f(x)) = \begin{cases} 
1 & \text{if } f(x) \leq L_{f(x)}^T \\
1 - \exp\left\{-\varphi \left( \frac{U_{f(x)}^T - f(x)}{U_{f(x)}^T - L_{f(x)}^T} \right) \right\} & \text{if } L_{f(x)}^T \leq f(x) \leq U_{f(x)}^T \\
0 & \text{if } f(x) \geq U_{f(x)}^T 
\end{cases} \]

\[ I_{f(x)}(f(x)) = \begin{cases} 
1 & \text{if } f(x) \leq L_{f(x)}^I \\
\exp\left\{ \frac{U_{f(x)}^I - f(x)}{U_{f(x)}^I - L_{f(x)}^I} \right\} & \text{if } L_{f(x)}^I \leq f(x) \leq U_{f(x)}^I \\
0 & \text{if } f(x) \geq U_{f(x)}^I 
\end{cases} \]

\[ F_{f(x)}(f(x)) = \begin{cases} 
0 & \text{if } f(x) \leq L_{f(x)}^F \\
\frac{1}{2} + \frac{1}{2} \tanh \left\{ \left( f(x) - \frac{U_{f(x)}^F + L_{f(x)}^F}{2} \right) \tau_{f(x)} \right\} & \text{if } L_{f(x)}^F \leq f(x) \leq U_{f(x)}^F \\
1 & \text{if } f(x) \geq U_{f(x)}^F 
\end{cases} \]
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\( T_{g_j(x)}(g_j(x)) = \begin{cases} 
1 & \text{if } g_j(x) \leq b_j \\
\frac{b_j + b_j^0 - g_j(x)}{b_j^0} & \text{if } b_j \leq g_j(x) \leq b_j + b_j^0 \\
0 & \text{if } g_j(x) \geq b_j^0
\end{cases} \)

\( I_{g_j(x)}(g_j(x)) = \begin{cases} 
1 & \text{if } g_j(x) \leq b_j \\
\exp\left\{-\psi\left(\frac{U_{g_j(x)}^T - g_j(x)}{U_{g_j(x)}^T - L_{g_j(x)}}\right)\right\} & \text{if } b_j \leq g_j(x) \leq b_j + b_j^0 \\
0 & \text{if } g_j(x) \geq b_j + b_j^0
\end{cases} \)

\( F_{g_j(x)}(g_j(x)) = \begin{cases} 
\frac{g_j(x) - b_j - \varepsilon_{g_j(x)}}{b_j^0 - \varepsilon_{g_j(x)}} & \text{if } b_j + \varepsilon_{g_j(x)} \leq g_j(x) \leq b_j + b_j^0 \\
1 & \text{if } g_j(x) \geq b_j + b_j^0
\end{cases} \)

where and for \( j = 1, 2, ..., m \), \( 0 < \varepsilon_{g_j(x)}, \xi_{g_j(x)} < b_j^0 \). and

\( T_{g_j(x)}(g_j(x)) = \begin{cases} 
1 & \text{if } g_j(x) \leq b_j \\
1 - \exp\left\{-\psi\left(\frac{U_{g_j(x)}^T - g_j(x)}{U_{g_j(x)}^T - L_{g_j(x)}}\right)\right\} & \text{if } b_j \leq g_j(x) \leq b_j + b_j^0 \\
0 & \text{if } g_j(x) \geq b_j + b_j^0
\end{cases} \)

\( I_{g_j(x)}(g_j(x)) = \begin{cases} 
1 & \text{if } g_j(x) \leq b_j \\
\exp\left(\frac{b_j + \varepsilon_{g_j(x)} - g_j(x)}{\xi_{g_j(x)}}\right) & \text{if } b_j \leq g_j(x) \leq b_j + \varepsilon_{g_j(x)} \\
0 & \text{if } g_j(x) \geq b_j + \varepsilon_{g_j(x)}
\end{cases} \)
\[ F_{g_j(x)}(g_j(x)) = \begin{cases} 0 & \text{if } g_j(x) \leq b_j + \varepsilon_{g_j(x)} \\ \frac{1}{2} \left[ 1 + \frac{1}{2} \tanh \left( \frac{g_j(x) - \frac{2b_j + b_j^0 + \varepsilon_{g_j(x)}}{2}}{\tau_{g_j(x)}} \right) \right] & \text{if } b_j + \varepsilon_{g_j(x)} \leq g_j(x) \leq b_j + b_j^0 \\ 1 & \text{if } g_j(x) \geq b_j + b_j^0 \end{cases} \]

where \( \psi, \tau \) are non-zero parameters prescribed by the decision maker and for \( j = 1, 2, \ldots, m \) \( 0 < \varepsilon_{g_j(x)}, \xi_{g_j(x)} < b_j^0 \).

**Step-5:** Now using NSO for single objective optimization technique the optimization problem (2) can be formulated as

**Model-I**

Maximize \((\alpha + \gamma - \beta)\)

Such that

\[ T_{f(x)}(x) \geq \alpha; \quad T_{g_j(x)}(x) \geq \alpha; \]
\[ I_{r(x)}(x) \geq \gamma; \quad I_{g_j(x)}(x) \geq \gamma; \]
\[ F_{r(x)}(x) \leq \beta; \quad F_{g_j(x)}(x) \leq \beta; \]
\[ \alpha + \beta + \gamma \leq 3; \quad \alpha \geq \beta; \quad \alpha \geq \gamma; \]
\[ \alpha, \beta, \nu \in [0, 1] \]

Where \( \alpha = \mu_{D^r}(x) = \min \left\{ T_{f(x)}(f(x)), T_{g_j(x)}(g_j(x)) \right\} \) for \( j = 1, 2, \ldots, m \)

\( \gamma = I_{D^r}(x) = \min \left\{ I_{f(x)}(f(x)), I_{g_j(x)}(g_j(x)) \right\} \) for \( j = 1, 2, \ldots, m \)

\( \beta = \nu_{D^r}(x) = \max \left\{ F_{f(x)}(f(x)), F_{g_j(x)}(g_j(x)) \right\} \) for \( j = 1, 2, \ldots, m \)

are the truth, indeterminacy and falsity membership function of decision set

\[ \tilde{D}^n = f^n(x) \bigcap_{j=1}^m g^n_j(x) \]

**Model-II**

Maximize \((\alpha - \gamma - \beta)\)

Such that

\[ T_{f(x)}(x) \geq \alpha; \quad T_{g_j(x)}(x) \geq \alpha; \]
\[ I_{r(x)}(x) \leq \gamma; \quad I_{g_j(x)}(x) \leq \gamma; \]
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\[ F_{n_j}(x) \leq \beta; \quad F_{g_j}(x) \leq \beta; \]
\[ \alpha + \beta + \gamma \leq 3; \quad \alpha \geq \beta; \quad \alpha \geq \gamma; \]
\[ \alpha, \beta, \nu \in [0,1] \]

Where \( \alpha = \mu_{F_j}(x) = \min \{ T_{f_j}(f(x)), T_{g_j}(g_j(x)) \} \) for \( j = 1, 2, \ldots, m \)
\[ \gamma = I_{F_j}(x) = \max \{ I_{f_j}(f(x)), I_{g_j}(g_j(x)) \} \] for \( j = 1, 2, \ldots, m \) and
\[ \beta = \nu_{F_j}(x) = \max \{ F_{f_j}(f(x)), F_{g_j}(g_j(x)) \} \] for \( j = 1, 2, \ldots, m \)

are the truth, indeterminacy and falsity membership function of decision set
\[ \tilde{D}^n = f^n(x) \bigcap_{j=1}^{m} g^n_j(x). \]

Now the above problem (Model-I) (6) can be simplified to following crisp linear programming problem for linear membership function as

Maximize \( (\alpha + \gamma - \beta) \) \hspace{1cm} (8)

such that \( f(x) + (U^T - L^T) \alpha \leq U^T; \)
\[ f(x) + U_{f_j}^T - L_{f_j}^T, \gamma \leq U_{f_j}^T; \]
\[ f(x) + U_{f_j}^T - L_{f_j}^T, \beta \leq L_{f_j}^T; \] for \( k = 1, 2, \ldots, p \)
\[ \alpha + \beta + \gamma \leq 3; \quad \alpha \geq \beta; \quad \alpha \geq \gamma; \quad \alpha, \beta, \gamma \in [0,1]; \]
\[ g_j(x) \leq b_j; \quad x \geq 0, \]

and for non linear membership function as

Maximize \( (\theta + \kappa - \eta) \) \hspace{1cm} (9)

Such that
\[ f(x) + \theta \frac{U_{f_j}^T - L_{f_j}^T}{\psi} \leq U_{f_j}^T; \quad f(x) + \kappa \xi_{f_j(x)} \leq U_{f_j}^T; \]
\[ f(x) + \frac{\eta}{\tau_{f_j(x)}} \leq \frac{U_{f_j}^T + L_{f_j}^T + \varepsilon_{f_j(x)}}{2}; \]
\[ g_j(x) + \theta \frac{b_j^0}{\psi} \leq b_j + b_j^0; \]
\[ g_j(x) + \kappa \xi_{g_j(x)} \leq b_j^0 + \xi_{g_j(x)}; \]
\[ g_j(x) + \frac{\eta}{\tau_{s(x)}} \leq \frac{2b_j + b_j^0 + \varepsilon_{s_j(x)}}{2}; \]
\[ \theta + \kappa + \eta \leq 3; \]
\[ \theta \geq \kappa; \theta \geq \eta; \]
\[ \theta, \kappa, \eta \in [0,1] \]

where \( \theta = -\ln(1 - \alpha); \psi = 4; \tau_{f(x)} = \frac{6}{(U_{f(x)}^E - L_{f(x)}^E)}; \)
\[ \tau_{s_j(x)} = \frac{6}{(b_j^0 - \varepsilon_j)} \]

for \( j = 1, 2, \ldots, m = \ln \gamma; \eta = -\tanh^{-1}(2\beta - 1) \) for linear and nonlinear membership function respectively.

Again the problem (Model-II) (7) can be reduced to following crisp linear programming problem for linear membership function as

Maximize \( (\alpha - \beta - \gamma) \) (10)

such that \( f(x) + (U^T - L^T) \alpha \leq U^T; \)
\( f(x) + (U_{f(x)}^I - L_{f(x)}^I) \gamma \geq U_{f(x)}^I; \)
\( f(x) + (U_{f(x)}^E - L_{f(x)}^E) \beta \leq L_{f(x)}^E; \) for \( k = 1, 2, \ldots, p \)
\( \alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma; \alpha, \beta, \gamma \in [0,1]; \)
\( g_j(x) \leq b_j \quad x \geq 0, \)

and for non linear membership function as

Maximize \( (\theta - \kappa - \eta) \) (11)

Such that
\( f(x) + \theta \frac{(U_{f(x)}^T - L_{f(x)}^T)}{\psi} \leq U_{f(x)}^T; \quad f(x) + \kappa \xi_{f(x)} \geq U_{f(x)}^T; \)
\( f(x) + \frac{\eta}{\tau_{f(x)}} \leq \frac{U_{f(x)}^T + L_{f(x)}^T + \varepsilon_{f(x)}}{2}; \)
\( g_j(x) + \frac{\theta b_j^0}{\psi} \leq b_j + b_j^0; \quad g_j(x) + \kappa \xi_{s_j(x)} \geq b_j^0 + \xi_{s_j(x)}; \quad g_j(x) + \frac{\eta}{\tau_{s_j(x)}} \leq \frac{2b_j + b_j^0 + \varepsilon_{s_j(x)}}{2}; \)
\[ \theta + \kappa + \eta \leq 3; \ \theta \geq \kappa; \ \theta \geq \eta; \ \theta, \kappa, \eta \in [0,1] \]

where \( \theta = -\ln(1-\alpha); \ \psi = 4; \ \tau_{f(s)} = \frac{6}{(U^{f}_{f(s)} - L^{f}_{f(s)})}; \)

\[ \tau_{\xi_{j}(s)} = \frac{6}{(b^{0}_{j} - \varepsilon_{j})}, \text{ for } j = 1, 2, \ldots, m \ \kappa = \ln \gamma; \ \eta = -\tanh^{-1}(2\beta - 1). \]

for linear and nonlinear membership function respectively.

All these crisp nonlinear programming problems (8),(9),(10),(11) can be solved by appropriate mathematical algorithm.

5. SOLUTION OF SINGLE-OBJECTIVE STRUCTURAL OPTIMIZATION PROBLEM (SOSOP) BY NEUTROSOPHIC OPTIMIZATION TECHNIQUE

To solve the SOSOP (1), step 1 of 4 is used and we will get optimum solutions of two sub problem as \( A^1 \) and \( A^2 \). After that according to step 2 we find upper and lower bound of membership function of objective function as \( U^{T}_{WT(A)}, U^{L}_{WT(A)}, U^{F}_{WT(A)} \) and \( L^{T}_{WT(A)}, L^{L}_{WT(A)}, L^{F}_{WT(A)} \) where \( U^{T}_{WT(A)} = \max \{WT(A^1), WT(A^2)\} \), \( L^{T}_{WT(A)} = \min \{WT(A^1), WT(A^2)\} \), \( U^{F}_{WT(A)} = U^{T}_{WT(A)}, L^{F}_{WT(A)} = L^{T}_{WT(A)} + \varepsilon_{WT(A)} \)

where \( 0 < \varepsilon_{WT(A)} < (U^{T}_{WT(A)} - L^{T}_{WT(A)}) \)

\( L^{T}_{WT(A)} = L^{F}_{WT(A)}, \ U^{T}_{WT(A)} = L^{T}_{WT(A)} + \xi_{WT(A)} \)

where \( 0 < \xi_{WT(A)} < (U^{T}_{WT(A)} - L^{T}_{WT(A)}) \)

Let the linear membership function for objective be

\[
T_{WT(A)}(WT(A)) = \begin{cases} 
1 & \text{if } WT(A) \leq L^{T}_{WT(A)} \\
\frac{U^{T}_{WT(A)} - WT(A)}{U^{T}_{WT(A)} - L^{T}_{WT(A)}} & \text{if } L^{T}_{WT(A)} \leq WT(A) \leq U^{T}_{WT(A)} \\
0 & \text{if } WT(A) \geq U^{T}_{WT(A)} 
\end{cases}
\]

\[
I_{WT(A)}(WT(A)) = \begin{cases} 
1 & \text{if } WT(A) \leq L^{T}_{WT(A)} \\
\frac{(L^{T}_{WT(A)} + \xi_{WT(A)}) - WT(A)}{L^{T}_{WT(A)} - \xi_{WT(A)}} & \text{if } L^{T}_{WT(A)} \leq WT(A) \leq L^{T}_{WT(A)} + \xi_{WT(A)} \\
0 & \text{if } WT(A) \geq L^{T}_{WT(A)} + \xi_{WT(A)} 
\end{cases}
\]
\[ \nu_{\text{WT}(A)}(\text{WT}(A))= \begin{cases} 
0 & \text{if } \text{WT}(A) \leq L_{\text{WT}(A)}^T + \varepsilon_{\text{WT}(A)} \\
\frac{\text{WT}(A) - (L_{\text{WT}(A)}^T + \varepsilon_{\text{WT}(A)})}{U_{\text{WT}(A)}^T - L_{\text{WT}(A)}^T - \varepsilon_{\text{WT}(A)}} & \text{if } L_{\text{WT}(A)}^T + \varepsilon_{\text{WT}(A)} \leq \text{WT}(A) \leq U_{\text{WT}(A)}^T \\
1 & \text{if } \text{WT}(A) \geq U_{\text{WT}(A)}^T 
\end{cases} \]

and constraints be

\[ T_{\sigma_i}(\sigma_i(A))= \begin{cases} 
1 & \text{if } \sigma_i(A) \leq \sigma_i \\
\frac{(\sigma_i + \sigma_0^{i_0}) - \sigma_i(A)}{\sigma_i} & \text{if } \sigma_i \leq \sigma_i(A) \leq \sigma_i + \sigma_0^{i_0} \\
0 & \text{if } \sigma_i(A) \geq \sigma_i + \sigma_0^{i_0} 
\end{cases} \]

\[ I_{\sigma_i}(\sigma_i(A))= \begin{cases} 
1 & \text{if } \sigma_i(A) \leq \sigma_i + \xi_{\sigma_i(A)} \\
\frac{\sigma_i(A) - \sigma_i - \varepsilon_{\sigma_i(A)}}{\xi_{\sigma_i(A)}} & \text{if } \sigma_i \leq \sigma_i(A) \leq \sigma_i + \xi_{\sigma_i(A)} \\
0 & \text{if } \sigma_i(A) \geq \sigma_i + \xi_{\sigma_i(A)} 
\end{cases} \]

\[ F_{\sigma_i}(\sigma_i(A))= \begin{cases} 
0 & \text{if } \sigma_i + \varepsilon_{\sigma_i(A)} \leq \sigma_i(A) \leq \sigma_i + \sigma_0^{i_0} \\
1 & \text{if } \sigma_i(A) \geq \sigma_i + \sigma_0^{i_0} 
\end{cases} \]

where for \( j = 1, 2, \ldots, m \) \( 0 < \varepsilon_{\sigma_i(A)}, \xi_{\sigma_i(A)} < \sigma_0^{i_0} \)

and if non-linear membership function be considered for objective function \( \text{WT}(A) \) then

\[ T_{\text{WT}(A)}(\text{WT}(A))= \begin{cases} 
1 & \text{if } \text{WT}(A) \leq L_{\text{WT}(A)}^T \\
1 - \exp \left( -\psi \left( \frac{U_{\text{WT}(A)}^T - \text{WT}(A)}{U_{\text{WT}(A)}^T - L_{\text{WT}(A)}^T} \right) \right) & \text{if } L_{\text{WT}(A)}^T \leq \text{WT}(A) \leq U_{\text{WT}(A)}^T \\
0 & \text{if } \text{WT}(A) \geq U_{\text{WT}(A)}^T 
\end{cases} \]
Truss Design Optimization using Neutrosophic Optimization Technique

\[ I_{WT(A)}(WT(A)) = \begin{cases} 
1 & \text{if } WT(A) \leq L_{WT(A)}^T \\
\exp \left( \frac{L_{WT(A)}^T + \xi_{WT}}{\xi_{WT}} - WT(A) \right) & \text{if } L_{WT(A)}^T \leq WT(A) \leq L_{WT(A)}^T + \xi_{WT} \\
0 & \text{if } WT(A) \geq L_{WT(A)}^T + \xi_{WT} 
\end{cases} \]

\[ F_{WT(A)}(WT(A)) = \begin{cases} 
0 & \text{if } WT(A) \leq L_{WT(A)}^T + \epsilon_{WT} \\
\frac{1}{2} + \frac{1}{2} \tanh \left( \left( \frac{WT(A) - U_{WT(A)}^T + L_{WT(A)}^T + \epsilon_{WT}}{2} \right) \tau_{WT} \right) & \text{if } L_{WT(A)}^T + \epsilon_{WT} \leq WT(A) \leq U_{WT(A)}^T \\
1 & \text{if } WT(A) \geq U_{WT(A)}^T 
\end{cases} \]

where \( 0 < \epsilon_{WT}, \xi_{WT} < (U_{WT}^T - L_{WT}^T) \) and if nonlinear truth, indeterminacy and falsity membership functions be considered for constraints then

\[ T_{\sigma_i(A)}(\sigma_i(A)) = \begin{cases} 
1 & \text{if } \sigma_i(A) \leq L_{\sigma_i}^T \\
1 - \exp \left( -\psi \left( \frac{U_{\sigma_i}^T - \delta(A)}{U_{\sigma_i}^T - L_{\sigma_i}^T} \right) \right) & \text{if } L_{\sigma_i}^T \leq \sigma_i(A) \leq U_{\sigma_i}^T \\
0 & \text{if } \sigma_i(A) \geq U_{\sigma_i}^T 
\end{cases} \]

\[ I_{\sigma_i(A)}(\sigma_i(A)) = \begin{cases} 
\exp \left( \frac{L_{\sigma_i}^T + \xi_{\sigma_i}}{\xi_{\sigma_i}} - \sigma_i(A) \right) & \text{if } L_{\sigma_i}^T \leq \sigma_i(A) \leq L_{\sigma_i}^T + \xi_{\sigma_i} \\
0 & \text{if } \sigma_i(A) \geq L_{\sigma_i}^T + \xi_{\sigma_i} 
\end{cases} \]

\[ F_{\sigma_i(A)}(\sigma_i(A)) = \begin{cases} 
\frac{1}{2} + \frac{1}{2} \tanh \left( \left( \frac{\sigma_i(A) - (U_{\sigma_i}^T + L_{\sigma_i}^T) + \epsilon_{\sigma_i}}{2} \right) \tau_{\sigma_i} \right) & \text{if } L_{\sigma_i}^T + \epsilon_{\sigma_i} \leq \sigma_i(A) \leq U_{\sigma_i}^T \\
1 & \text{if } \sigma_i(A) \geq U_{\sigma_i}^T 
\end{cases} \]

for objectives where \( \psi, \tau \) are non-zero parameters prescribed by the decision maker and for where \( 0 < \epsilon_{\sigma_i}, \xi_{\sigma_i} < (U_{\sigma_i}^T - L_{\sigma_i}^T) \)
then neutrosophic optimization problem can be formulated as

**Model-I**

Maximize \((\alpha + \gamma - \beta)\)

such that

\[
T_{\text{WT}(A)}(WT(A)) \geq \alpha; \quad T_{\sigma_i(A)}(\sigma_i(A)) \geq \alpha;
\]

\[
I_{\text{wt}(A)}(WT(A)) \geq \gamma; \quad I_{\sigma_i(A)}(\sigma_i(A)) \geq \gamma;
\]

\[
F_{\text{wt}(A)}(WT(A)) \leq \beta; \quad F_{\sigma_i(A)}(\sigma_i(A)) \leq \beta
\]

\[
\sigma_i(x) \leq [\sigma_i]; \quad \alpha + \beta + \gamma \leq 3; \quad \alpha \geq \beta; \quad \alpha \geq \gamma; \quad \alpha, \beta, \gamma \in [0,1]
\]

and

**Model-II**

Maximize \((\alpha - \beta - \gamma)\)

such that

\[
T_{\text{WT}(A)}(WT(A)) \geq \alpha; \quad T_{\sigma_i(A)}(\sigma_i(A)) \geq \alpha;
\]

\[
I_{\text{wt}(A)}(WT(A)) \leq \gamma; \quad I_{\sigma_i(A)}(\sigma_i(A)) \leq \gamma;
\]

\[
F_{\text{wt}(A)}(WT(A)) \leq \beta; \quad F_{\sigma_i(A)}(\sigma_i(A)) \leq \beta
\]

\[
\sigma_i(x) \leq [\sigma_i]; \quad \alpha + \beta + \gamma \leq 3; \quad \alpha \geq \beta; \quad \alpha \geq \gamma; \quad \alpha, \beta, \gamma \in [0,1]
\]

Now the above problem can be simplified to following crisp linear programming problem, whenever linear membership are considered, as

**Model-IA**

Maximize \((\alpha - \beta + \gamma)\) \hspace{1cm} (12)

Such that

\[
WT(A) + \alpha \left( U_{\text{WT}(A)}^T - L_{\text{WT}(A)}^T \right) \leq U_{\text{WT}(A)}^T;
\]

\[
WT(A) + \gamma \left( U_{\text{WT}(A)}^T - L_{\text{WT}(A)}^T - \tilde{\varepsilon}_{\text{WT}(A)} \right) \leq L_{\text{WT}(A)}^T + \tilde{\varepsilon}_{\text{WT}(A)};
\]

\[
WT(A) - \beta \left( U_{\text{WT}(A)}^T - L_{\text{WT}(A)}^T - \tilde{\varepsilon}_{\text{WT}(A)} \right) \leq L_{\text{WT}(A)}^T + \tilde{\varepsilon}_{\text{WT}(A)};
\]

\[
\sigma_i(A) + \alpha \left( U_{\sigma_i(A)}^T - L_{\sigma_i(A)}^T \right) \leq U_{\sigma_i(A)}^T;
\]
Truss Design Optimization using Neutrosophic Optimization Technique

\[ \sigma_T(A) + \gamma \left( U_{\sigma_T(A)}^T \right) \leq U_{\sigma_T(A)}^T + \xi_{\sigma_T(A)}; \]
\[ \sigma_T(A) - \beta \left( U_{\sigma_T(A)}^T \right) \leq L_{\sigma_T(A)}^T + \epsilon_{\sigma_T(A)}; \]
\[ \sigma_c(A) - \alpha \left( U_{\sigma_c(A)}^T \right) \leq U_{\sigma_c(A)}^T; \]
\[ \sigma_c(A) + \gamma \left( U_{\sigma_c(A)}^T \right) \leq U_{\sigma_c(A)}^T + \xi_{\sigma_c(A)}; \]
\[ \alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma; \alpha, \beta, \gamma \in [0,1] \]

And crisp linear programming problem whenever non-linear membership function is considered as

Model-IB

Maximize \( (\theta + \kappa - \eta) \) \hspace{1cm} (13)

such that

\[ WT(A) + \theta \left( \frac{U_{WT(A)}^T}{\psi} - L_{WT(A)}^T \right) \leq U_{WT(A)}^T; \]
\[ WT(A) + \kappa \xi_{WT(A)}^T \leq U_{WT(A)}^T; \]
\[ \sigma_i(A) + \theta \frac{\sigma_i^0}{\psi} \leq \sigma_i + \sigma_i^0; \]
\[ \sigma_i(A) + \kappa \xi_{\sigma_i(A)}^0 \leq \sigma_i^0 + \xi_{\sigma_i(A)}; \]
\[ \sigma_i(A) + \eta \frac{\sigma_i^0}{\tau_{\sigma_i(A)}} \leq \frac{2 \sigma_i + \sigma_i^0 + \epsilon_{\sigma_i(A)}^e}{2}; \]
\[ \theta + \kappa - \eta \leq 3; \theta \geq \kappa; \theta \geq \eta; \theta, \kappa, \eta \in [0,1] \]

Where \( \theta = -\ln(1-\alpha); \psi = 4; \tau_{WT(A)} = \frac{6}{U_{WT(A)}^F - L_{WT(A)}^F}; \kappa = \ln \gamma; \eta = -\tanh^{-1}(2\beta - 1). \)

And \( \tau_{\sigma_i(A)} = \frac{6}{U_{\sigma_i(A)}^F - L_{\sigma_i(A)}^F}; \)

Using linear and nonlinear truth, indeterminacy, and falsity membership function Model-II can be simplified as

Model-IIA

Maximize \( (\alpha - \beta - \gamma) \) \hspace{1cm} (14)

Such that
\[ WT(A) + \alpha \left( U_{WT(A)}^T - L_{WT(A)}^T \right) \leq U_{WT(A)}^T; \]
\[ WT(A) + \gamma \left( U_{WT(A)}^T - L_{WT(A)}^T - \xi_{WT(A)} \right) \geq L_{WT(A)}^T + \xi_{WT(A)}; \]
\[ WT(A) - \beta \left( U_{WT(A)}^T - L_{WT(A)}^T - \varepsilon_{WT(A)} \right) \leq L_{WT(A)}^T + \varepsilon_{WT(A)}; \]
\[ \sigma_t(A) + \alpha \left( U_{\sigma_t(A)}^T - L_{\sigma_t(A)}^T \right) \leq U_{\sigma_t(A)}^T; \]
\[ \sigma_t(A) + \gamma \left( U_{\sigma_t(A)}^T - L_{\sigma_t(A)}^T - \xi_{\sigma_t(A)} \right) \geq U_{\sigma_t(A)}^T + \xi_{\sigma_t(A)}; \]
\[ \sigma_t(A) - \beta \left( U_{\sigma_t(A)}^T - L_{\sigma_t(A)}^T - \varepsilon_{\sigma_t(A)} \right) \leq L_{\sigma_t(A)}^T + \varepsilon_{\sigma_t(A)}; \]
\[ \sigma_e(A) + \alpha \left( U_{\sigma_e(A)}^T - L_{\sigma_e(A)}^T \right) \leq U_{\sigma_e(A)}^T; \]
\[ \sigma_e(A) + \gamma \left( U_{\sigma_e(A)}^T - L_{\sigma_e(A)}^T - \xi_{\sigma_e(A)} \right) \leq U_{\sigma_e(A)}^T + \xi_{\sigma_e(A)}; \]
\[ \alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma; \alpha, \beta, \gamma \in [0,1] \]
and

**Model-IIB**

\[ \text{Maximize } (\theta - \kappa - \eta) \] (15)
such that
\[ WT(A) + \theta \alpha \left( U_{WT(A)}^T - L_{WT(A)}^T \right) / \psi \leq U_{WT(A)}^T; \]
\[ WT(A) + \eta \alpha / \tau_{WT(A)} \leq \frac{U_{WT(A)}^T + L_{WT(A)}^T + \varepsilon_{WT(A)}}{2}; \]
\[ WT(A) + \kappa \xi_{WT(A)} \geq U_{WT(A)}^T; \]
\[ \sigma_i^{(A)} + \theta \sigma_i^0 / \psi \leq \sigma_i + \sigma_i^0; \sigma_i^{(A)} + \kappa \xi_{\sigma_i(A)} \geq \sigma_i + \xi_{\sigma_i(A)}; \]
\[ \sigma_i^{(A)} + \eta / \tau_{\sigma_i(A)} \leq \frac{2 \sigma_i + \sigma_i^0 + \varepsilon_{\sigma_i(A)}}{2}; \]
\[ \theta + \kappa - \eta \leq 3; \theta \geq \kappa; \theta \geq \eta; \]
\[ \theta, \kappa, \eta \in [0,1] \]

where
\[ \theta = -\ln(1-\alpha); \psi = 4; \tau_{WT(A)} = \frac{6}{U_{WT(A)}^F - L_{WT(A)}^F}; \kappa = \ln \gamma; \eta = -\tanh^{-1}(2\beta - 1). \]

and \[ \tau_{\delta(A)} = \frac{6}{U_{\delta(A)}^F - L_{\delta(A)}^F}; \]

All these crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

6. NUMERICAL ILLUSTRATION

A well-known two-bar planar truss structure (Fig.1.) is considered and the detail formulation is given in appendix. The design objective is to minimize weight of the structural \( WT(A_1, A_2, y_B) \) of a statistically loaded two-bar truss subjected to stress \( \sigma_i(A_1, A_2, y_B) \) constraints on each of the truss members \( i = 1, 2 \).

![Design of the two-bar truss](image)

The single-objective optimization problem can be stated as follows

\[
\text{Minimize } WT(A_1, A_2, y_B) = \rho \left( A_1 \sqrt{x_B^2 + (l - y_B)^2} + A_2 \sqrt{x_B^2 + y_B^2} \right)
\]

Such that

\[
\sigma_{AB}(A_1, A_2, y_B) = \frac{P x_B^2 + (l - y_B)^2}{l A_i} \leq [\sigma_{AB}^T];
\]
\[ \sigma_{BC}(A_1, A_2, y_B) = \frac{P \sqrt{x_B^2 + y_B^2}}{l A_2} \leq [\sigma_{BC}^C]; \]

\[ 0.5 \leq y_B \leq 1.5 \]

\[ A_1 > 0, A_2 > 0; \]

where \( P \) = nodal load; \( \rho \) = volume density; \( l \) = length of \( AC \); \( x_B \) = perpendicular distance from \( AC \) to point \( B \). \( A_1 \) = Cross section of bar- \( AB \); \( A_2 \) = Cross section of bar- \( BC \). \([\sigma_T]\) = maximum allowable tensile stress, \( [\sigma_C] \) = maximum allowable compressive stress and \( y_B \) = \( y \)-co-ordinate of node \( B \). Input data are given in table 1.

**Table 1: Input data for crisp model (16)**

<table>
<thead>
<tr>
<th>Applied load ( P ) (KN)</th>
<th>Volume density ( \rho ) (KN/m(^3))</th>
<th>Length ( l ) (m)</th>
<th>Maximum allowable tensile stress ([\sigma_T]) (MPa)</th>
<th>Maximum allowable compressive stress ([\sigma_C]) (MPa)</th>
<th>Distance of ( x_B ) from ( AC ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>7.7</td>
<td>2</td>
<td>130</td>
<td>90</td>
<td>1</td>
</tr>
</tbody>
</table>

| with fuzzy region 20    | with fuzzy region 10            |

Solution: According to step 2 of 4, we find upper and lower bound of membership function of objective function as \( U^T_{WT(A)} \), \( U^F_{WT(A)} \), \( U^T_{WT(A)} \), \( U^F_{WT(A)} \) where

\[ U^T_{WT(A)} = 14.23932 = U^F_{WT(A)} \]

\[ L^T_{WT(A)} = 12.57667 = L^F_{WT(A)} \]

\[ L^T_{WT(A)} = 12.57667 + \frac{1}{2} \]

\[ U_{WT(A)} < 1.66265 \]

\[ \text{and} \]

\[ U_{WT(A)} = U^T_{WT(A)} + U^F_{WT(A)} \]

\[ \text{where} \]

\[ 0 < \frac{1}{2} \]

\[ \text{and} \]

\[ U_{WT(A)} < 1.66265 \]

Now using the bounds we calculate the membership functions for objective as follows

\[ T_{WT(A, A_2, y_B)}(WT(A, A_2, y_B)) = \]

\[ \begin{cases} 
1 & \text{if } WT(A, A_2, y_B) \leq 12.57667 \\
\frac{14.23932 - WT(A, A_2, y_B)}{1.66265} & \text{if } 12.57667 \leq WT(A, A_2, y_B) \leq 14.23932 \\
0 & \text{if } WT(A, A_2, y_B) \geq 14.23932 
\end{cases} \]
Truss Design Optimization using Neutrosophic Optimization Technique

\[ I_{WT(A,A_2,y_B)}(WT(A_1,A_2,y_B)) = \begin{cases} 
1 & \text{if } WT(A_1,A_2,y_B) \leq 12.57667 \\
\frac{(12.57667 + \xi_{WT(A)}) - WT(A_1,A_2,y_B)}{\xi_{WT(A)}} & \text{if } 12.57667 \leq WT(A_1,A_2,y_B) \leq 12.57667 + \xi_{WT(A)} \\
0 & \text{if } WT(A_1,A_2,y_B) \geq 12.57667 + \xi_{WT(A)} 
\end{cases} \]

\[ F_{WT(A,A_2,y_B)}(WT(A_1,A_2,y_B)) = \begin{cases} 
0 & \text{if } WT(A_1,A_2,y_B) \leq 12.57667 + \xi_{WT(A)} \\
\frac{WT(A_1,A_2,y_B) - 12.57667 - \varepsilon_{WT(A)}}{1.66265 - \varepsilon_{WT(A)}} & \text{if } 12.57667 + \varepsilon_{WT(A)} \leq WT(A_1,A_2,y_B) \leq 14.23932 \\
1 & \text{if } WT(A_1,A_2,y_B) \geq 14.23932 
\end{cases} \]

Similarly the membership functions for tensile stress are

\[ T_{\sigma_T(A,A_2,y_B)}(\sigma_T(A_1,A_2,y_B)) = \begin{cases} 
1 & \text{if } \sigma_T(A_1,A_2,y_B) \leq 130 \\
\frac{150 - \sigma_T(A_1,A_2,y_B)}{20} & \text{if } 130 \leq \sigma_T(A_1,A_2,y_B) \leq 150 \\
0 & \text{if } \sigma_T(A_1,A_2,y_B) \geq 150 
\end{cases} \]

\[ I_{\sigma_T(A,A_2,y_B)}(\sigma_T(A_1,A_2,y_B)) = \begin{cases} 
1 & \text{if } \sigma_T(A_1,A_2,y_B) \leq 130 \\
\frac{(130 + \xi_{\sigma_T}) - \sigma_T(A_1,A_2,y_B)}{\xi_{\sigma_T}} & \text{if } 130 \leq \sigma_T(A_1,A_2,y_B) \leq 130 + \xi_{\sigma_T} \\
0 & \text{if } \sigma_T(A_1,A_2,y_B) \geq 130 + \xi_{\sigma_T} 
\end{cases} \]

\[ F_{\sigma_T(A,A_2,y_B)}(\sigma_T(A_1,A_2,y_B)) = \begin{cases} 
0 & \text{if } \sigma_T(A_1,A_2,y_B) \leq 130 + \varepsilon_{\sigma_T} \\
\frac{\sigma_T(A_1,A_2,y_B) - 130 - \varepsilon_{\sigma_T}}{20 - \varepsilon_{\sigma_T}} & \text{if } 130 + \varepsilon_{\sigma_T} \leq \sigma_T(A_1,A_2,y_B) \leq 150 \\
1 & \text{if } \sigma_T(A_1,A_2,y_B) \geq 150 
\end{cases} \]
where \(0 < \varepsilon_{\sigma_c}, \xi_{\sigma_c} < 20\)

and the membership functions for compressive stress constraint are

\[
T_{\sigma_c(A_1, A_2, y_B)}(\sigma_c(A_1, A_2, y_B)) = \begin{cases} 
1 & \text{if } \sigma_c(A_1, A_2, y_B) \leq 90 \\
100 - \sigma_c(A_1, A_2, y_B) & \text{if } 90 \leq \sigma_c(A_1, A_2, y_B) \leq 100 \\
0 & \text{if } \sigma_c(A_1, A_2, y_B) \geq 100 
\end{cases}
\]

\[
I_{\sigma_c(A_1, A_2, y_B)}(\sigma_c(A_1, A_2, y_B)) = \begin{cases} 
1 & \text{if } \sigma_c(A_1, A_2, y_B) \leq 90 \\
\frac{(90 + \xi_{\sigma_c}) - \sigma_c(A_1, A_2, y_B)}{\xi_{\sigma_c}} & \text{if } 90 \leq \sigma_c(A_1, A_2, y_B) \leq 90 + \xi_{\sigma_c} \\
0 & \text{if } \sigma_c(A_1, A_2, y_B) \geq 90 + \xi_{\sigma_c} 
\end{cases}
\]

\[
F_{\sigma_c(A_1, A_2, y_B)}(\sigma_c(A_1, A_2, y_B)) = \begin{cases} 
0 & \text{if } \sigma_c(A_1, A_2, y_B) \leq 90 + \varepsilon_{\sigma_c} \\
\frac{\sigma_c(A_1, A_2, y_B) - 90 - \varepsilon_{\sigma_c}}{10 - \varepsilon_{\sigma_c}} & \text{if } 90 + \varepsilon_{\sigma_c} \leq \sigma_c(A_1, A_2, y_B) \leq 100 \\
1 & \text{if } \sigma_c(A_1, A_2, y_B) \geq 100 
\end{cases}
\]

where \(0 < \varepsilon_{\sigma_c}, \xi_{\sigma_c} < 10\)

Again the nonlinear truth, indeterminacy and falsity membership functions for objectives and constraints can be formulated as

\[
T_{WT(A_1, A_2, y_B)}(WT(A_1, A_2, y_B)) = \begin{cases} 
1 & \text{if } WT(A_1, A_2, y_B) \leq 12.57667 \\
1 - \exp \left\{-4 \left( \frac{14.23932 - WT(A_1, A_2, y_B)}{1.66265} \right) \right\} & \text{if } 12.57667 \leq WT(A_1, A_2, y_B) \leq 14.23932 \\
0 & \text{if } WT(A_1, A_2, y_B) \geq 14.23932 
\end{cases}
\]

\[
I_{WT(A_1, A_2, y_B)}(WT(A_1, A_2, y_B)) = \begin{cases} 
1 & \text{if } WT(A_1, A_2, y_B) \leq 12.57667 \\
\exp \left\{ \frac{(12.57667 + \xi_{WT}) - WT(A_1, A_2, y_B)}{\xi_{WT}} \right\} & \text{if } 12.57667 \leq WT(A_1, A_2, y_B) \leq 12.57667 + \xi_{WT} \\
0 & \text{if } WT(A_1, A_2, y_B) \geq 12.57667 + \xi_{WT} 
\end{cases}
\]
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\[ F_{r(a \cdot A', y_b)}(\mathbf{w}, T(A_1, A_2, y_b)) = \begin{cases} 
0 & \text{if } T(A_1, A_2, y_b) \leq 12.57667 + \epsilon_{WT} \\
\frac{1}{2} + \frac{1}{2} \tanh \left( \frac{\mathbf{w}(A_1, A_2, y_b) - (26.81599 + \epsilon_{WT})}{2} \right) & \frac{6}{1.66265 - \epsilon_{WT}} \right) \right) \text{if } 12.57667 + \epsilon_{WT} \leq T(A_1, A_2, y_b) \leq 14.23932 \\
0 & \text{if } T(A_1, A_2, y_b) \geq 14.23932 
\end{cases} \]

Similarly the membership functions for tensile stress are

\[ T_{\sigma_{t}(A_1, A_2, y_b)}(\sigma_{t}(A_1, A_2, y_b)) = \begin{cases} 
1 & \text{if } \sigma_{t}(A_1, A_2, y_b) \leq 130 \\
1 - \exp \left( -4 \left( \frac{150 - \sigma_{t}(A_1, A_2, y_b)}{20} \right) \right) & \text{if } 130 \leq \sigma_{t}(A_1, A_2, y_b) \leq 150 \\
0 & \text{if } \sigma_{t}(A_1, A_2, y_b) \geq 150 
\end{cases} \]

\[ I_{\sigma_{t}(A_1, A_2, y_b)}(\sigma_{t}(A_1, A_2, y_b)) = \begin{cases} 
1 & \text{if } \sigma_{t}(A_1, A_2, y_b) \leq 130 \\
\exp \left( \frac{130 + \xi_{\sigma_{t}} - \sigma_{t}(A_1, A_2, y_b)}{\xi_{\sigma_{t}}} \right) & \text{if } 130 \leq \sigma_{t}(A_1, A_2, y_b) \leq 130 + \xi_{\sigma_{t}} \\
0 & \text{if } \sigma_{t}(A_1, A_2, y_b) \geq 130 + \xi_{\sigma_{t}} 
\end{cases} \]

\[ F_{\sigma_{t}(A_1, A_2, y_b)}(\sigma_{t}(A_1, A_2, y_b)) = \begin{cases} 
0 & \text{if } \sigma_{t}(A_1, A_2, y_b) \leq 130 + \epsilon_{\sigma_{t}} \\
\frac{1}{2} + \frac{1}{2} \tanh \left( \sigma_{t}(A_1, A_2, y_b) - \frac{280 + \epsilon_{\sigma_{t}}}{2} \right) \right) \text{if } 130 + \epsilon_{\sigma_{t}} \leq \sigma_{t}(A_1, A_2, y_b) \leq 150 \\
0 & \text{if } \sigma_{t}(A_1, A_2, y_b) \geq 150 
\end{cases} \]

where \( 0 < \epsilon_{\sigma_{t}}, \xi_{\sigma_{t}} < 20 \)

and the membership functions for compressive stress constraint are

\[ T_{\sigma_{c}(A_1, A_2, y_b)}(\sigma_{c}(A_1, A_2, y_b)) = \begin{cases} 
1 & \text{if } \sigma_{c}(A_1, A_2, y_b) \leq 90 \\
1 - \exp \left( -4 \left( \frac{100 - \sigma_{c}(A_1, A_2, y_b)}{10} \right) \right) & \text{if } 90 \leq \sigma_{c}(A_1, A_2, y_b) \leq 100 \\
0 & \text{if } \sigma_{c}(A_1, A_2, y_b) \geq 100 
\end{cases} \]
\[
I_{\sigma_c(A_1,A_2,y_B)}(\sigma_c(A_1,A_2,y_B)) = \begin{cases} 
1 & \text{if } \sigma_c(A_1,A_2,y_B) \leq 90 \\
\exp \left( \frac{(90+\xi_{\sigma_c})-\sigma_c(A_1,A_2,y_B)}{\xi_{\sigma_c}} \right) & \text{if } 90 \leq \sigma_c(A_1,A_2,y_B) \leq 90+\xi_{\sigma_c} \\
0 & \text{if } \sigma_c(A_1,A_2,y_B) \geq 90+\xi_{\sigma_c}
\end{cases}
\]

\[
F_{\sigma_c(A_1,A_2,y_B)}(\sigma_c(A_1,A_2,y_B)) = \begin{cases} 
0 & \text{if } \sigma_c(A_1,A_2,y_B) \leq 90+\varepsilon_{\sigma_c} \\
\frac{1}{2} + \frac{1}{2} \tanh \left( \left( \sigma_c(A_1,A_2,y_B) - \left( \frac{190+\varepsilon_{\sigma_c}}{2} \right) \right) \frac{6}{10-\varepsilon_{\sigma_c}} \right) & \text{if } 90+\varepsilon_{\sigma_c} \leq \sigma_c(A_1,A_2,y_B) \leq 100 \\
1 & \text{if } \sigma_c(A_1,A_2,y_B) \geq 100
\end{cases}
\]

where \(0 < \varepsilon_{\sigma_c}, \xi_{\sigma_c} < 10\)

Now, using above mentioned truth, indeterminacy and falsity linear and nonlinear membership function NLP (16) can be solved for Model-IA, Model-IB, Model-IIA, Model-IIB by NSO technique for different values of \(\varepsilon_{WT}, \varepsilon_{\sigma_1}, \varepsilon_{\sigma_c}\) and \(\xi_{WT}, \xi_{\sigma_1}, \xi_{\sigma_c}\). The optimum solution of SOSOP (16) is given in Table 1 and Table 2 and the solution is compared with fuzzy and intuitionistic fuzzy problem.

**Table 2:** Comparison of Optimal solution of SOSOP (16) for Model I based on different methods

<table>
<thead>
<tr>
<th>Methods</th>
<th>Model</th>
<th>(A_i) (m²)</th>
<th>(A_l) (m²)</th>
<th>(WT(A_i,A_l)) (KN)</th>
<th>(y_B) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy single-objective non-linear programming (FSONLP)</td>
<td>IA</td>
<td>.5883491</td>
<td>.7183381</td>
<td><strong>14.23932</strong></td>
<td>1.013955</td>
</tr>
<tr>
<td></td>
<td>IB</td>
<td>.5883491</td>
<td>.7183381</td>
<td><strong>14.23932</strong></td>
<td>1.013955</td>
</tr>
<tr>
<td>Intuitionistic Fuzzy single-objective non-linear programming (FSONLP)</td>
<td>IA</td>
<td>0.33253, 4, 2</td>
<td>0.5482919</td>
<td>0.6692795</td>
<td><strong>13.19429</strong></td>
</tr>
<tr>
<td></td>
<td>IB</td>
<td>0.8, 16, 8</td>
<td>0.6064095</td>
<td>0.6053373</td>
<td><strong>13.59182</strong></td>
</tr>
<tr>
<td>Neutosophic optimization(NSO)</td>
<td>IA</td>
<td>0.33253, 4, 2</td>
<td>0.5954331</td>
<td>0.7178116</td>
<td><strong>13.07546</strong></td>
</tr>
<tr>
<td></td>
<td>IB</td>
<td>0.8, 16, 8</td>
<td>0.5451860</td>
<td>0.677883</td>
<td><strong>13.24173</strong></td>
</tr>
</tbody>
</table>
Table 3: Comparison of Optimal solution of SOSOP (16) for Model II based on different method

<table>
<thead>
<tr>
<th>Methods</th>
<th>Model</th>
<th>$A_i$ ($m^2$)</th>
<th>$A_i$ ($m^2$)</th>
<th>$WT(A_i,A_j)$ (KN)</th>
<th>$y_s$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy single-objective non-linear programming (FSONLP)</td>
<td>II A</td>
<td>.5954331</td>
<td>.7178116</td>
<td>14.23932</td>
<td>0.81818</td>
</tr>
<tr>
<td></td>
<td>II B</td>
<td>1.317107</td>
<td>0.7174615</td>
<td>13.82366</td>
<td>1.399050</td>
</tr>
<tr>
<td>Intuitionistic Fuzzy single-objective non-linear programming (FSONLP)</td>
<td>II A</td>
<td>$\varepsilon_{WT} = 0.33253$, $\varepsilon_{\sigma_c} = 4$, $\varepsilon_{\sigma_c} = 2$</td>
<td>0.5954331</td>
<td>0.7178116</td>
<td>13.50036</td>
</tr>
<tr>
<td></td>
<td>II B</td>
<td>$\varepsilon_{WT} = 0.8$, $\varepsilon_{\sigma_c} = 16$, $\varepsilon_{\sigma_c} = 8$</td>
<td>1.107847</td>
<td>0.2557545</td>
<td>13.78028</td>
</tr>
<tr>
<td>Neutosophic optimization (NSO)</td>
<td>II A</td>
<td>$\varepsilon_{WT} = 0.33253$, $\varepsilon_{\sigma_c} = 4$, $\varepsilon_{\sigma_c} = 2$</td>
<td>$\varepsilon_{WT} = 0.498795$, $\varepsilon_{\sigma_c} = 6$, $\varepsilon_{\sigma_c} = 3$</td>
<td>0.5954331</td>
<td>0.7178116</td>
</tr>
<tr>
<td></td>
<td>II B</td>
<td>$\varepsilon_{WT} = 0.8$, $\varepsilon_{\sigma_c} = 16$, $\varepsilon_{\sigma_c} = 8$</td>
<td>$\varepsilon_{WT} = 0.66506$, $\varepsilon_{\sigma_c} = 8$, $\varepsilon_{\sigma_c} = 4$</td>
<td>0.6494508</td>
<td>0.8336701</td>
</tr>
</tbody>
</table>

Here we get best solutions for the different tolerance $\varepsilon_{WT}$, $\varepsilon_{\sigma_c}$ and $\varepsilon_{\sigma_c}$ for indeterminacy membership function of objective functions whenever indeterminacy is tried to be minimized (i.e in Model I) for this structural optimization problem. From table 2 and 3, it is shown that NSO technique gives better optimal result in the perspective of Structural Optimization.

7. CONCLUSIONS

The main objective of this work is to illustrate how neutrosophic optimization technique using linear and nonlinear membership function can be utilized to solve a single objective-nonlinear structural problem. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; rather, they are independent with degree of indeterminacy. The numerical illustration shows the superiority of neutrosophic optimization over fuzzy optimization and intuitionistic fuzzy optimization. The results of this study may lead to the development of effective neutrosophic technique for solving other models in form of single objective nonlinear programming problem in other field of engineering.

Conflict of interests: The authors declare that there is no conflict of interests.

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REFERENCES


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APPENDIX:

Optimization mode of two-bar truss shown in Fig.1 is designed to support the loading condition. The weight of the structure is \( WT = (A_1 L_1 + A_2 L_2) \), where \( \rho \) is the material density of the bar \( A_1, A_2 \) are the cross sectional area and \( L_1, L_2 \) are the length of bar 1 and bar 2 respectively. Length \( AC = l \), Perpendicular distance from \( AC \) to pint load point \( B \) is \( x_B \), Nodal load = \( P \), Using simple Pythagorean’s theorem we may find the length of the each bars

\[
L_1 = \sqrt{x_B^2 + (l - y_B)^2}, \quad L_2 = \sqrt{x_B^2 + y_B^2}.
\]

Therefore weight of the structure is

\[
WT = \rho \left( A_1 \sqrt{x_B^2 + (l - y_B)^2} + A_2 \sqrt{x_B^2 + y_B^2} \right).
\]

Let \( P_1 \) and \( P_2 \) be the reaction forces along the bar 1 and bar 2 respectively. Considering the equilibrium condition at loading point, the following equations are obtained

\[
P_1 \cos \theta_1 + P_2 \cos \theta_2 = P, \quad P_1 \sin \theta_1 - P_2 \sin \theta_2 = 0.
\]

Solving these two equations we get

the axial force on bar 1 as \( P_1 = \frac{P \sqrt{x_B^2 + (l - y_B)^2}}{l} \), the axial force on bar 2 as \( P_2 = \frac{- P \sqrt{x_B^2 + y_B^2}}{l} \), the stress of bar 1 as \( \sigma_1 = \frac{P_1}{A_1} = \frac{P \sqrt{x_B^2 + (l - y_B)^2}}{l A_1} \), i.e tensile stress.

the stress of bar 2 as \( \sigma_2 = \frac{P_2}{A_2} = \frac{P \sqrt{x_B^2 + y_B^2}}{l A_2} \), i.e compressive stress. As

\[
\cos \theta_1 = \frac{l - y_B}{\sqrt{x_B^2 + (l - y_B)^2}}, \quad \sin \theta_1 = \frac{x_B}{\sqrt{x_B^2 + (l - y_B)^2}}, \quad \cos \theta_2 = \frac{y_B}{\sqrt{x_B^2 + y_B^2}}, \quad \sin \theta_2 = \frac{x_B}{\sqrt{x_B^2 + y_B^2}}.
\]