Weakly β Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy weakly β generalized continuous mappings in intuitionistic fuzzy topological spaces. We investigate some of their properties.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy weakly β generalized continuous mappings.

I. INTRODUCTION

II. PRELIMINARIES

Definition 2.1 [1]: An intuitionistic fuzzy set (IFS for short) $A$ is an object having the form

$$A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in $X$.

An intuitionistic fuzzy set $A$ in $X$ is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}$.

Definition 2.2 [1]: Let $A$ and $B$ be two IFSs of the form

$$A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \}$$

and

$$B = \{ (x, \mu_B(x), \nu_B(x)) : x \in X \}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{ (x, \nu_A(x), \mu_A(x)) : x \in X \}$,

(d) $A \cup B = \{ (x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)) : x \in X \}$,

(e) $A \cap B = \{ (x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)) : x \in X \}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1 \rangle$ and $1_\sim = \langle x, 1, 0 \rangle$ are respectively the empty set and the whole set of $X$.

Definition 2.3 [2]: An intuitionistic fuzzy topology (IFT in short) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\bigcup G_i \in \tau$ for any family $\{ G_i : i \in J \} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS in short) in...
X. The complement $A^c$ of an IFOS $A$ in an IFTS $(X, \tau)$ is called an **intuitionistic fuzzy closed set** (IFCS in short) in $X$.

**Definition 2.4 [8]:** Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the $\beta$ generalized interior and $\beta$ generalized closure of $A$ are defined as

$$\beta\text{gint}(A) = \bigcup \{G / G \text{ is an IF}\beta\text{GOS in } X \text{ and } G \subseteq A\},$$
$$\beta\text{gcl}(A) = \bigcap \{K / K \text{ is an IF}\beta\text{GCS in } X \text{ and } A \subseteq K\}.$$  

Note that for any IFS $A$ in $(X, \tau)$, we have $\beta\text{gcl}(A^c) = (\beta\text{gint}(A))^c$ and $\beta\text{gint}(A^c) = (\beta\text{gcl}(A))^c$.

**Remark 2.5:** If an IFS $A$ in an IFTS $(X, \tau)$ is an IF$\beta$GCS in $X$, then $\beta\text{gcl}(A) = A$. But the converse may not be true in general, since intersection does not exist in IF$\beta$GCS [5].

**Remark 2.6:** If an IFS $A$ in an IFTS $(X, \tau)$ is an IF$\beta$GOS in $X$, then $\beta\text{gint}(A) = A$. But the converse may not be true in general, since union does not exist in IF$\beta$GOS [6].

**Definition 2.7 [5]:** An IFS $A$ in an IFTS $(X, \tau)$ is said to be an **intuitionistic fuzzy $\beta$ generalized closed set** (IF$\beta$GCS for short) if $\beta\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IF$\beta$OS in $(X, \tau)$.

**Definition 2.8 [7]:** A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy $\beta$ generalized continuous** (IF$\beta$G continuous for short) **mapping** if $f^{-1}(V)$ is an IF$\beta$GCS in $(X, \tau)$ for every IFCS $V$ of $(Y, \sigma)$.

**Definition 2.9 [9]:** A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy completely $\beta$ generalized continuous** (IF completely $\beta$G continuous for short) **mapping** if $f^{-1}(V)$ is an IFRCS in $X$ for every IF$\beta$GCS $V$ in $Y$. 
III. WEAKLY $\beta$ GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section we have introduced intuitionistic fuzzy weakly $\beta$ generalized continuous mappings and studied some of their properties.

**Definition 3.1:** A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy weakly $\beta$ generalized continuous* (IFW$\beta$G continuous for short) **mapping** if $f^{-1}(V) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(V)))$ for each IFOS $V$ in $Y$.

**Example 3.2:** Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5a, 0.4b), (0.5a, 0.6b) \rangle$, $G_2 = \langle y, (0.4u, 0.3v), (0.6u, 0.7v) \rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$.

Then, $\text{IF}_{\beta\text{C}}(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$,

$\text{IF}_{\beta\text{O}}(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$,

$\text{IF}_{\beta\text{C}}(Y) = \{0\sim, 1\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / 0 \leq \mu_u + \nu_u \leq 1 \text{ and } 0 \leq \mu_v + \nu_v \leq 1\}$,

$\text{IF}_{\beta\text{O}}(Y) = \{0\sim, 1\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / 0 \leq \mu_u + \nu_u \leq 1 \text{ and } 0 \leq \mu_v + \nu_v \leq 1\}$.

The IFS $G_2 = \langle y, (0.4u, 0.3v), (0.6u, 0.7v) \rangle$ is an IFOS in $Y$. Now $G_2^c = \langle y, (0.6u, 0.7v), (0.4u, 0.3v) \rangle$ is an IFCS in $Y$.

We have $\beta\text{gint}(f^{-1}(\text{cl}(G_2))) = \langle x, (0.6a, 0.7b), (0.4a, 0.3b) \rangle$. Hence $f^{-1}(G_2) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(G_2)))$. Therefore $f$ is an IFW$\beta$G continuous mapping.

**Theorem 3.3:** Every IF$\beta$G continuous mapping is an IFW$\beta$G continuous mapping but not conversely.

**Proof:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF$\beta$G continuous mapping. Let $V$ be any IFOS in $Y$. Then by hypothesis, $f^{-1}(V)$ is an IF$\beta$GOS in $X$. Therefore $\beta\text{gint}(f^{-1}(V)) = f^{-1}(V)$. Now $f^{-1}(V) = \beta\text{gint}(f^{-1}(V)) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(V)))$. Hence $f$ is an IFW$\beta$G continuous mapping.
Example 3.4: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \langle x, (0.5a, 0.6b), (0.2a, 0.1b) \rangle \), \( G_2 = \langle x, (0.4a, 0.1b), (0.2a, 0.1b) \rangle \) and \( G_3 = \langle y, (0.2u, 0.1v), (0.4u, 0.4v) \rangle \). Then \( \tau = \{0_-, G_1, G_2, 1_-\} \) and \( \sigma = \{0_-, G_3, 1_-\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \).

Then, \( \text{IF}^{\beta}C(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \text{either} \mu_a < 0.4 \text{ or } \mu_b < 0.1, \text{ and } 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\} \),

\( \text{IF}^{\beta}O(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \text{either} \mu_a > 0.2 \text{ or } \mu_b > 0.1, \text{ and } 0 \leq \mu_a + \nu_a \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\} \),

\( \text{IF}^{\beta}C(Y) = \{0\sim, 1\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / 0 \leq \mu_u + \nu_u \leq 1 \text{ and } 0 \leq \mu_v + \nu_v \leq 1\} \),

\( \text{IF}^{\beta}O(Y) = \{0\sim, 1\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / 0 \leq \mu_u + \nu_u \leq 1 \text{ and } 0 \leq \mu_v + \nu_v \leq 1\} \).

The IFS \( G_3 = \langle y, (0.2u, 0.1v), (0.4u, 0.4v) \rangle \) is an IFOS in \( Y \). Now \( G_3^c = \langle y, (0.4u, 0.4v), (0.2u, 0.1v) \rangle \) is an IFCS in \( Y \).

We have \( \beta g \text{int}(f^{-1}(cl(G_3))) = \langle x, (0.4a, 0.4b), (0.2a, 0.1b) \rangle \). Hence \( f^{-1}(G_3) \subseteq \beta g \text{int}(f^{-1}(cl(G_3))) \). Therefore \( f \) is an IFW\( \beta \)G continuous mapping but not an IF\( \beta \)G continuous mapping, since \( G_2 = \langle y, (0.4u, 0.3v), (0.6u, 0.7v) \rangle \) is an IF\( \beta \)GCS in \( Y \) but \( f^{-1}(G_3^c) = \langle x, (0.4a, 0.4b), (0.2a, 0.1b) \rangle \) is not an IF\( \beta \)GCS, since \( f^{-1}(G_3^c) \subseteq G_1 \) but \( \beta cl(f^{-1}(G_3^c)) = 1_\sim \not\subseteq G_1 \).

Theorem 3.5: Every IF completely \( \beta \)G continuous mapping is an IFW\( \beta \)G continuous mapping but not conversely.

Proof: Let \( f: (X, \tau) \to (Y, \sigma) \) be an IF completely \( \beta \)G continuous mapping. Let \( V \) be any IFOS in \( Y \). Since every IFOS is an IF\( \beta \)GOS, \( V \) is an IF\( \beta \)GOS. Then by hypothesis, \( f^{-1}(V) \) is an IFROS in \( X \) and hence \( f^{-1}(V) \) is an IFOS in \( X \). Now as every IFOS is an IF\( \beta \)GOS, \( \beta g \text{int}(f^{-1}(V)) = f^{-1}(V) \). Therefore \( f^{-1}(V) = \beta g \text{int}(f^{-1}(V)) \subseteq \beta g \text{int}(f^{-1}(cl(V))) \).

Example 3.6: In Example 3.2, \( f \) is an IFW\( \beta \)G continuous mapping but not an IF completely \( \beta \)G continuous mapping, since \( G_2 = \langle y, (0.4u, 0.3v), (0.6u, 0.7v) \rangle \) is an IF\( \beta \)GCS in \( Y \) but not an IFRCS in \( X \), since \( cl(int(f^{-1}(G_2^c))) = cl(G_1) = G_1^c \neq f^{-1}(G_2^c) \).
Theorem 3.7: Every IFβ continuous mapping [2] is an IFWβG continuous mapping but not conversely.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFβ continuous mapping. Since every IFβ continuous mapping is an IFβG continuous mapping [7], by Theorem 3.3, \( f \) is an IFWβG continuous mapping.

Example 3.8: In Example 3.4, \( f \) is an IFWβGCts.M but not an IFβCts.M, since \( G^c_3 = (y, (0.4_u, 0.4_v), (0.2_u, 0.1_v)) \) is an IFCS in \( Y \) but \( f^{-1}(G^c_3) = (x, (0.4_u, 0.4_b), (0.2_u, 0.1_b)) \) is not an IFβCS in \( X \), since \( \text{int}(\text{cl}(f^{-1}(G^c_3))) = \text{int}(\text{cl}(G_2)) = 1 \sim \not\subset f^{-1}(G^c_3) \).

The relation between various types of intuitionistic fuzzy continuity is given in the following examples. In this diagram ‘Cts.M’ means continuous mapping.

\[
\begin{array}{ccc}
\text{IFβCts.M} & \text{IFβG Cts. M} & \text{IFComβG Cts.M} \\
\downarrow & \downarrow & \downarrow \\
\text{IFWβGCts.M} & & \\
\end{array}
\]

The reverse implications are not true in general in the above diagram.

Theorem 3.9: For a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \), the following are equivalent:

(i) \( f \) is an IFWβG continuous mapping

(ii) \( f^{-1}(\text{int}(A)) \subseteq \beta\text{gint}(f^{-1}(A)) \) for each IFCS \( A \) in \( Y \)

(iii) \( f^{-1}(\text{cl}(A)) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A))) \) for each IFOS \( A \) in \( Y \)

(iv) \( f^{-1}(A) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A))) \) for each IFPOS \( A \) in \( Y \)

(v) \( f^{-1}(A) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A))) \) for each IFcOS \( A \) in \( Y \)
**Proof:** (i) ⇒ (ii) Let $A \subseteq Y$ be an IFCS. Then $\text{int}(A) = \text{int}(\text{cl}(A))$, which is an IFROS. Hence $\text{int}(A)$ is an IFOS in $Y$. Therefore by (i) $f^{-1}(\text{int}(A)) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A)))) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A)))$, as $\text{cl}(A) = A$. Hence $f^{-1}(\text{int}(A)) \subseteq \beta\text{gint}(f^{-1}(A))$.

(ii) ⇒ (iii) Let $A \subseteq Y$ be an IFOS. Then $\text{int}(A) = A$. Now $\text{cl}(A) = \text{cl}(\text{int}(A))$, which is an IFRCS. Therefore $\text{cl}(A)$ is an IFCS in $Y$. By (ii) $f^{-1}(\text{int}(\text{cl}(A))) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A))))$.

(iii) ⇒ (iv) Let $A$ be an IFPOS in $Y$. Then $A \subseteq \text{int}(\text{cl}(A))$. Now $f^{-1}(A) \subseteq f^{-1}(\text{int}(\text{cl}(A)))$. Since $\text{int}(\text{cl}(A))$ is an IFOS, it is an IFOS. Hence (iii) implies $f^{-1}(\text{int}(\text{cl}(A))) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(\text{cl}(A)))) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A)))$. That is $f^{-1}(\text{int}(\text{cl}(A))) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A)))$. Thus $f^{-1}(A) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A)))$.

(iv) ⇒ (v) is obvious, since every IFαOS is an IFPOS.

(v) ⇒ (i) Let $A \subseteq Y$ be an IFOS. Since every IFOS is an IFαOS, $A$ is an IFαOS. Hence by (v), $f^{-1}(A) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A)))$. Therefore $f$ is an IFWβGCts.M.

**Theorem 3.10:** Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then the following are equivalent:

(i) $f$ is an IFWβG continuous mapping

(ii) $\beta\text{gcl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$ for each IFOS $A$ in $Y$

(iii) $\beta\text{gcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(A)$ for each IFCS $A$ in $Y$

**Proof:** (i) ⇒ (iii) Let $A \subseteq Y$ be an IFCS. $A^c$ is an IFOS in $Y$. By (i) $f^{-1}(A^c) \subseteq \beta\text{gint}(f^{-1}(\text{cl}(A^c)))$. This implies $(f^{-1}(A))^c \subseteq (\beta\text{gcl}(f^{-1}(\text{int}(A))))^c$. Thus $\beta\text{gcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(A)$.

(iii) ⇒ (i) is obvious.

(ii) ⇒ (iii) Let $A \subseteq Y$ be an IFCS. Then $\text{int}(A)$ is an IFOS in $Y$. By (ii), $\beta\text{gcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(\text{cl}(\text{int}(A))) \subseteq f^{-1}(\text{cl}(A)) = f^{-1}(A)$, since $\text{cl}(A) = A$. Hence $\beta\text{gcl}(f^{-1}(\text{int}(A))) \subseteq f^{-1}(A)$.

(iii) ⇒ (ii) Let $A \subseteq Y$ be an IFOS, then $\text{int}(A) = A$ and $\text{cl}(A)$ is an IFCS. By (iii), $\beta\text{gcl}(f^{-1}(\text{int}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$. Now $\beta\text{gcl}(f^{-1}(A)) = \beta\text{gcl}(f^{-1}(\text{int}(A))) \subseteq \beta\text{gcl}(f^{-1}(\text{int}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$. Hence $\beta\text{gcl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$. 


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