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Ranking Exponential K+1-Trapezoidal Fuzzy Numbers with Cardinality

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Abstract

Ranking fuzzy numbers plays an important role in many applied models in the world and decision making, optimization, forecasting etc.In this paper, we generalize the ranking method introduced in [16] to rank exponential k+1-trapezoidal fuzzy numbers.

Keywords k+1- Trapezoidal Fuzzy Numbers, Exponential k+1-Trapezoidal Fuzzy Numbers, Cardinality of Fuzzy Numbers.

1. Introduction

Ranking fuzzy numbers is used mainly in data analysis, artificial intelligence and various other fields of operations research. The First method for ranking fuzzy numbers was introduced by Jain in the year 1976 [6]. Since 1976, several methods have been proposed and developed to rank different types of fuzzy numbers [1-6] and [9-19]. In [2], Bortolan & Degani reviewed some of these ranking methods and compared between them.

In [11], we introduced k+1- trapezoidal fuzzy numbers and we developed different methods to rank them [11-13].

In [16], Rezwani used cardinality to rank Exponential Trapezoidal Fuzzy Numbers. In this paper, we generalize the ranking method introduced in [16] to rank Exponential k+1-Trapezoidal Fuzzy Numbers.

In section 2 we give basic concepts. Special types of fuzzy numbers are mentioned in section 3. In section 4 we give the ranking method. Finally, we give numerical examples in section 5.

2. Preliminaries

Definition 1: A fuzzy subset A of some set Ω is defined by its membership function written $A(x)$ which produces values in $[0,1]$ for all x in Ω. That is $A(x)$ is a function mapping Ω into $[0,1]$. We place a bar over a letter to denote a fuzzy set, that is $\overline{A}$. The term crisp means not fuzzy. A crisp set is a regular set.

Definition 2: Let $\Omega = R$. An $\alpha-$cut of $\overline{A}$, written $\overline{A}[\alpha]$, is defined as $\{x : \overline{A}(x) \geq \alpha\}$, for $0 < \alpha \leq 1$. $\overline{A}[0]$, the support of $\overline{A}$ is defined as the closure of the union of all the $\overline{A}[\alpha]$, for $0 < \alpha \leq 1$.

Definition 3: A confidence interval is an interval of real numbers that provides a representation for an imprecise numerical value by means of its sharpest enclosing range.

Definition 4: A presumption level is an estimated truth-value about some knowledge. Presumption levels belong to the [0,1] interval; the maximum of estimated truth-value is at level 1 and the minimum is at level 0.

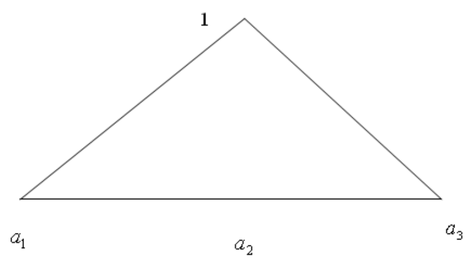
Definition 5: A fuzzy number $\overline{N}$ is a fuzzy subset of the real numbers satisfying:

(1) $\exists x \in R : \overline{N}(x) = 1$ (2) $\overline{N}[\alpha]$ is a closed and bounded interval for $0 \leq \alpha \leq 1$.

More details can be found in [7-8] and [20].

3. Special Types of Fuzzy Numbers

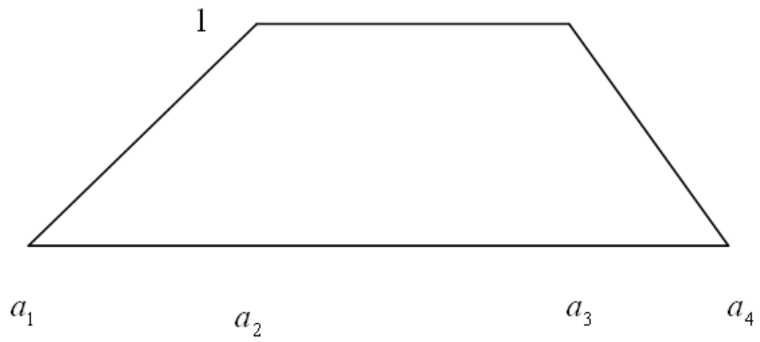
A special type of fuzzy numbers $\overline{M}$ is called a triangular fuzzy number (Figure 1). $\overline{M}$ is defined by three numbers $a_1 < a_2 < a_3$ where (1) $\overline{M}(x) = 1$ at $x = a_2$ (2) The graph of $\overline{M}(x)$ on $[a_1, a_2]$ is a straight line from $(a_1, 0)$ to $(a_2, 1)$ and also on $[a_2, a_3]$ the graph is a straight line from $(a_2, 1)$ to $(a_3, 0)$ (3) $\overline{M}(x) = 0$ for $x \leq a_1$ or $x \geq a_3$. We write $\overline{M} = (a_1 / a_2 / a_3)$ for triangular fuzzy number $\overline{M}$. If at least one of the graphs described above is not a straight line (curve), then $\overline{M}$ is called triangular shaped fuzzy number and we write $\overline{M} \approx (a_1 / a_2 / a_3)$.



(Figure1)

Another special type of fuzzy numbers $\bar{\bar{M}}$ is called a trapezoidal fuzzy number (Figure 2). $\bar{\bar{M}}$ is defined by four numbers $a_1 < a_2 < a_3 < a_4$ where (1) $\bar{\bar{M}}(x) = 1$ on $[a_2, a_3]$ (2) The graph of $\bar{\bar{M}}(x)$ on $[a_1, a_2]$ is a straight line from $(a_1, 0)$ to $(a_2, 1)$ and also on $[a_3, a_4]$ the graph is a straight line from $(a_3, 1)$ to $(a_4, 0)$ (3) $\bar{\bar{M}}(x) = 0$ for $x \le a_1$ or $x \ge a_4$. We write $\bar{\bar{M}} = (a_1, a_2, a_3, a_4)$ for trapezoidal fuzzy number $\bar{\bar{M}}$. If at least one of the graphs described above is not a straight line (curve), then $\bar{\bar{M}}$ is called trapezoidal shaped fuzzy number and we write $\bar{\bar{M}} \approx (a_1, a_2, a_3, a_4)$

If $\bar{\bar{M}}(x) = w < 1$ on $[a_2, a_3]$, then it is called a generalized trapezoidal fuzzy number.



(Figure2)

In [11], we introduced the concept of k+1-trapezoidal fuzzy numbers (Figure 3). A k+1- trapezoidal fuzzy number is a fuzzy subset of the real line R which is determined by n real numbers $a_1, a_2, ..., a_{2k+4}$ and c_i, $0 \le c_i \le 1$, i=1,2,…,k where n=4,7,10,…and $a_1 < a_2 < .... < a_{2k+4}$. Denote it by $\bar{A}_{(c_1,....,c_k)} = (a_1, ..., a_{2k+4})$ and whose membership function is given by

$$\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)=\begin{cases}0, x\le a_1\\ \dfrac{c_1(x-a_1)}{a_2-a_1}, a_1\le x\le a_2\\ .\\ .\\ 1, a_{k+2}\le x\le a_{k+3}\\ .\\ .\\ \dfrac{-c_1(x-a_{2k+4})}{a_{2k+4}-a_{2k+3}}, a_{2k+3}\le x\le a_{2k+4}\\ 0, x\ge a_{2k+4}\end{cases}$$

1- $\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)$ is a line from $(a_1,0)$ to (a_2,c_1) and from (a_{2k+3},c_1) to $(a_{2k+4},0)$

2- $\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)$ is a line from (a_{i+1},c_i) to (a_{i+2},c_{i+1}) ,i=1,2,….,k-1.

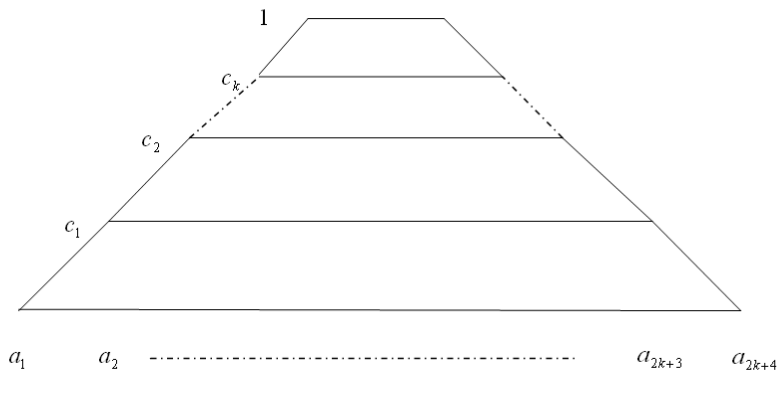
3- $\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)$ is a line from (a_{k+4+i},c_{k-i}) to (a_{k+5+i},c_{k-i-1}) ,i=0,1,2,….,k-2.

4- $\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)$ is a line from (a_{k+1},c_k) to $(a_{k+2},1)$ and from $(a_{k+3},1)$ to (a_{k+4},c_k)

5- $\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)=1$ if $a_{k+2}\le x\le a_{k+3}$.

6- $\mu_{\bar{A}_{(c_1,\ldots,c_k)}}(x)=0$ for $x\le a_1, x\ge a_{2k+4}$.

If k=0, a k+1-trapezoidal Fuzzy Number is a trapezoidal Fuzzy Number.



(Figure3)

We define the general form of an exponential k+1- trapezoidal fuzzy number as follows :

$$f_{\overline{A}}(x) = \begin{cases} c_1 e^{[(x-a_2)/(a_2-a_1)]}, a_1 \le x \le a_2 \\ e^{[((a_3-x)\ln c_1+(x-a_2)\ln c_2)/(a_3-a_2)]}, a_2 \le x \le a_3 \\ . \\ . \\ e^{[((a_{k+1}-x)\ln c_{k-1}+(x-a_k)\ln c_k)/(a_{k+1}-a_k)]}, a_k < x < a_{k+1} \\ e^{[((a_{k+2}-x)\ln c_k)/(a_{k+2}-a_{k+1})]}, a_{k+1} < x < a_{k+2} \\ 1, a_{k+2} \le x \le a_{k+3} \\ e^{[((x-a_{k+3})\ln c_k)/(a_{k+4}-a_{k+3})]}, a_{k+3} < x < a_{k+4} \\ e^{[((x-a_{k+4})\ln c_{k-1}+(a_{k+5}-x)\ln c_k)/(a_{k+5}-a_{k+4})]}, a_{k+4} < x < a_{k+5} \\ . \\ . \\ e^{[((a_{2k+3}-x)\ln c_2+(x-a_{2k+2})\ln c_1)/(a_{2k+3}-a_{2k+2})]}, a_{2k+2} < x < a_{2k+3} \\ c_1 . e^{[(a_{2k+3}-x)/(a_{2k+4}-a_{2k+3})]}, a_{2k+3} \le x \le a_{2k+4} \end{cases}$$

We denote it by $\overline{A}_{(c_1,\dots,c_k)} = (a_1,\dots,a_{2k+4})_E$.

4. The Ranking Method

In this section we generalize the ranking method introduced in [16].

Definition 6: Cardinality of a fuzzy number A is the value of the integral

$$cardA = \int_a^b A(x)dx = \int_0^1 (b_\alpha - a_\alpha)d\alpha$$

Now, we apply definition 6 to exponential k+1- trapezoidal fuzzy numbers.

Theorem 7: Cardinality of an exponential k+1- trapezoidal fuzzy number A is the value of the integral

$$card\overline{A} = (a_{k+4}-a_{k+3}) + \sum_{i=1}^{k-1}\left[\frac{(a_{i+2}-a_{i+1})(c_{i+1}-c_i)}{\ln c_{i+1}-\ln c_i} + \frac{(a_{k+4+i}-a_{k+3+i})(c_{k+1-i}-c_{k-i})}{\ln c_{k+1-i}-\ln c_{k-i}}\right]$$
$$+\frac{(c_k-1)(a_{k+4}-a_{k+3}+a_{k+2}-a_{k+1})}{\ln c_k} + c_1(1-\frac{1}{e})(-a_{2k+4}+a_{2k+3}+a_2-a_1)$$

Proof :

$$card\overline{A}=\int_{a_1}^{a_2} c_1 e^{[(x-a_2)/(a_2-a_1)]}dx+\int_{a_2}^{a_3} e^{[((a_3-x)\ln c_1+(x-a_2)\ln c_2)/(a_3-a_2)]}dx+\ldots$$

$$+\int_{a_k}^{a_{k+1}} e^{[((a_{k+1}-x)\ln c_{k-1}+(x-a_k)\ln c_k)/(a_{k+1}-a_k)]}dx+\int_{a_{k+1}}^{a_{k+2}} e^{[((a_{k+2}-x)\ln c_k)/(a_{k+2}-a_{k+1})]}+\int_{a_{k+2}}^{a_{k+3}} 1dx$$

$$+\int_{a_{k+3}}^{a_{k+4}} e^{[((x-a_{k+3})\ln c_k)/(a_{k+4}-a_{k+3})]}dx+\int_{a_{k+4}}^{a_{k+5}} e^{[((x-a_{k+4})\ln c_{k-1}+(a_{k+5}-x)\ln c_k)/(a_{k+5}-c_{k+4})]}dx+\ldots$$

$$+\int_{a_{2k+2}}^{a_{2k+3}} e^{[((a_{2k+3}-x)\ln c_2+(x-a_{2k+2})\ln c_1)/(a_{2k+3}-a_{2k+2})]}+\int_{a_{2k+3}}^{a_{2k+4}} c_1 e^{[(a_{2k+3}-x)/(a_{2k+4}-a_{2k+3})]}dx$$

$$=(a_{k+4}-a_{k+3})+\sum_{i=1}^{k-1}\left[\frac{(a_{i+2}-a_{i+1})(c_{i+1}-c_i)}{\ln c_{i+1}-\ln c_i}+\frac{(a_{k+4+i}-a_{k+3+i})(c_{k+1-i}-c_{k-i})}{\ln c_{k+1-i}-\ln c_{k-i}}\right]$$

$$+\frac{(c_k-1)(a_{k+4}-a_{k+3}+a_{k+2}-a_{k+1})}{\ln c_k}+c_1(1-\frac{1}{e})(-a_{2k+4}+a_{2k+3}+a_2-a_1)$$

Finally, if $\overline{A}$ and $\overline{B}$ are two exponential k+1- trapezoidal fuzzy numbers, then

i) If $card\overline{A}>card\overline{B}$, then $\overline{A}\succ\overline{B}$.

ii) If $card\overline{A}<card\overline{B}$, then $\overline{A}\prec\overline{B}$.

iii) If $card\overline{A}\approx card\overline{B}$ then $\overline{A}\approx\overline{B}$

5. Numerical Examples

Example 5.1

Let $\overline{A}_{(0.35)}=(0.2,0.3,0.4,0.6,0.7,0.8)$ and $\overline{B}_{(0.7)}=(0.1,0.16,0.2,0.3,0.35,0.4)$.

Then

$$card\,\overline{A}=(a_5-a_4)+\frac{(c_1-1)(a_5-a_4+a_3-a_2)}{\ln c_1}+c_1(1-\frac{1}{e})(-a_6+a_5+a_2-a_1)$$

$$card\overline{A}=(0.7-0.6)+\frac{(0.35-1)(0.7-0.6+0.4-0.3)}{\ln 0.35}=0.35(1-\frac{1}{e})(-0.8+0.7+0.3-0.2)$$

= 0.2238

$$card\overline{B}=(0.35-0.3)+\frac{(0.5-1)(0.35-0.3+0.2-0.16)}{\ln 0.5}+0.5(1-\frac{1}{e})(-0.4+0.35+0.16-0.1)$$

= 0.1181

$$card\overline{A}>card\overline{B}\Rightarrow\overline{A}\succ\overline{B}$$

Example 5.2

Let $\overline{A}_{(0.5)} = (0.1,0.15,0.2,0.3,0.4,0.5)$ and $\overline{B}_{(0.5)} = (0.1,0.15,0.25,0.3,0.375,0.5)$.

Then

$$card\overline{A} = (0.4-0.3) + \frac{(0.5-1)(0.4-0.3+0.2-0.15)}{\ln 0.5} + 0.5(1-\frac{1}{e})(-0.5+0.4+0.15-0.1)$$

=0.1924

$$card\overline{B} = (0.375-0.3) + \frac{(0.5-1)(0.375-0.3+0.25-0.15)}{\ln 0.5} + 0.5(1-\frac{1}{e})(-0.5+0.375+0.15-0.1)$$

=0.1775

$$card\overline{A} > card\overline{B} \Rightarrow \overline{A} \succ \overline{B}$$

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