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Homomorphism in Intuitionistic Ql-Fuzzy Subhemiring of a Hemiring

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Abstract

In this paper, we made an attempt to study the algebraic nature of an intuitionistic QL-fuzzy subhemiring of a hemiring under homomorphism and anti-homomorphism.

Keywords: QL-fuzzy set, QL-fuzzy subhemiring, anti QL-fuzzy subhemiring, intuitionistic QL-fuzzy subset, intuitionistic QL-fuzzy subhemiring.

Introduction

There are many concepts of universal algebras generalizing an associative ring (*R;+;.)*. Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R; +,.) is said to be a semiring if (R; +) and (R;.) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c).a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introdution of fuzzy sets by L.A.Zadeh[5], several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic QL-fuzzy subset was introduced by K.T.Atanassov[1, 2], as a generalization of the notion of fuzzy set. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan&K.Arjunan [3]. Some properties of intuitionistic fuzzy subgroups was introduced by Palaniappan. N &K.Arjunan [4]. In this paper, we introduce the some Theorems in intuitionistic QL-fuzzy subhemiring of a hemiring under homomorphism and anti-homomorphism.

Section 1: Preliminaries: and basic definitions

Definition 1.1: Let X be a non-empty set and L = (L, $\leq$) be a lattice with least element 0 and greatest element 1. A QL-fuzzy subset A of X is a function A: X $\rightarrow$ L.

Definition 1.2: Let (R, +,.) be a hemiring. A QL-fuzzy subset A of R is said to be a QL-fuzzy subhemiring (LFSHR) of R if it satisfies the following conditions:

(i) μ_A (x+y, q) $\geq \mu_A$ (x, q) $\wedge\mu_A$ (y, q),

(ii) μ_A (xy, q) $\geq \mu_A$ (x, q) $\wedge\mu_A$ (y, q), for all x, q and y, q in R.

Definition 1.3: Let (R, +,.) be a hemiring. A QL-fuzzy subset A of R is said to be an anti QL-fuzzy subhemiring (ALFSHR) of R if it satisfies the following conditions:

(i) μ_A (x+y, q) $\leq \mu_A$ (x, q) $\vee\mu_A$ (y, q), μ_A (xy, q) $\leq$

(ii) μ_A (x, q) $\vee\mu_A$ (y, q), for all x, q and y, q in R.

Definition 1.4: Let (L, $\leq$) be a complete lattice with an involutive order reversing operation N : L $\rightarrow$ L. A intuitionistic QL-fuzzy subset (ILFS) A in X is defined as an object of the form A={< x, μ_A (x, q), ν_A (x, q) > / x, q in X }, where μ_A : X $\rightarrow$ L and ν_A : X $\rightarrow$ L define the degree of membership and the degree of non-membership of the element x, q$\in$X respectively and for every x, q$\in$X satisfying μA (x, q) $\leq$ N (νA (x, q)).

Definition 1.5: Let (R, +,.) be a hemiring. An intuitionistic QL-fuzzy subset A of R is said to be an intuitionistic QL-fuzzy subhemiring (ILFSHR) of R if it satisfies the following conditions:

(i) μ_A (x + y, q) $\geq \mu_A$ (x, q) $\wedge\mu_A$ (y, q);

(ii) μ_A (xy, q) $\geq \mu_A$ (x, q) $\wedge\mu_A$ (y, q);

(iii) ν_A (x + y, q) $\leq \nu_A$ (x, q) $\vee\nu_A$ (y, q); (iv) ν_A (xy, q) $\leq \nu_A$ (x, q) $\vee\nu_A$ (y, q), for all x, q and y, q in R.

Section 2:Properties on Intuitionistic L-fuzzy subsemiring

Theorem 2.1: Let (R, +,.) and (R', +,.) be any two hemirings. The homomorphic image of an intuitionistic QL-fuzzy subhemiring of R is an intuitionistic QL-fuzzy subhemiring of R'.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings. Let f: R $\rightarrow$ R'be a homomorphism.

Then, f (x+y, q) = f (x, q) + f (y, q) and f (xy, q) = f (x, q) f (y, q), for all x, q and y, q in R.

Let V = f (A), where A is an intuitionistic QL-fuzzy subhemiring of R.

We have to prove that V is an intuitionistic QL-fuzzy subhemiring of R'.

Now, for f (x, q), f (y, q) in R', μ_v (f (x, q) + f (y, q)) = μ_v (f (x+y, q)) $\geq \mu_A$ (x + y, q) $\geq \mu_A$ (x, q) $\wedge\mu_A$ (y, q), which implies that μ_v (f (x, q) + f (y, q)) $\geq \mu_v$ (f (x, q)) $\wedge$ μ_v (f (y, q)).

Again, μ_v (f (x, q) f (y, q)) = μ_v (f (xy, q)) ≥ μ_A (xy, q) ≥ μ_A (x, q) ∧ μ_A (y, q), which implies that μ_v (f (x, q) f (y, q)) ≥ μ_v (f (x, q)) ∧ μ_v (f (y, q)).

Now, for f (x, q), f (y, q) in R', ν_v (f (x, q) + f (y)) = ν_v (f (x+y, q)) ≤ ν_A (x+ y, q) ≤ ν_A (x, q) ∨ν_A (y, q), which implies that ν_v (f (x, q) + f (y, q)) ≤ ν_v (f (x, q)) ∨ν_v (f (y, q)).

Again, ν_v (f (x, q) f (y, q)) = ν_v (f (xy, q)) ≤ ν_A (xy, q) ≤ ν_A (x, q) ∨ν_A (y, q), which implies that ν_v (f (x, q) f (y, q)) ≤ ν_v (f (x, q)) ∨ν_v (f (y, q)).

Hence V is an intuitionistic QL-fuzzy subhemiring of R'.

Theorem 2.2: Let (R, +,.) and (R', +,.) be any two hemirings. The homomorphic preimage of an intuitionistic QL-fuzzy subhemiring of R'is a intuitionistic QL-fuzzy subhemiring of R.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings.

Let f: R → R'be a homomorphism.

Then, f (x+y, q) = f (x, q) + f (y, q) and f (xy, q) = f (x, q) f (y, q), for all x, q and y, q in R.

Let V = f (A), where V is an intuitionistic QL-fuzzy subhemiring of R'.

We have to prove that A is an intuitionistic QL-fuzzy subhemiring of R.

Let x and y in R. Then, μ_A (x + y, q) =μ_v (f (x + y, q)) = μ_v (f (x, q) + f (y, q)) ≥ μ_v (f (x, q)) ∧ μ_v (f (y, q)) = μ_A (x, q) ∧ μ_A (y, q), which implies that μ_A (x+y, q) ≥ μ_A (x, q) ∧ μ_A (y, q).

Again, μ_A (xy, q) = μ_v (f (xy, q)) = μ_v (f (x, q) f (y, q)) ≥ μ_v (f (x, q)) ∧ μ_v (f (y, q)) = μ_A (x, q) ∧ μ_A (y, q), which implies that μ_A (xy, q) ≥ μ_A (x, q) ∧μ_A (y, q).

Let x and y in R. Then, ν_A (x+ y, q) = ν_v (f (x + y, q)) = ν_v (f (x, q) +f (y, q)) ≤ ν_v (f (x, q)) ∨ ν_v (f (y, q)) = ν_A (x, q) ∨ ν_A (y, q), which implies that ν_A (x+ y, q) ≤ ν_A (x, q) ∨ ν_A (y, q).

Again, ν_A (xy, q) = ν_v (f (xy, q)) = ν_v (f (x, q) f (y, q)) ≤ ν_v (f (x, q)) ∨ν_v (f (y, q)) = ν_A (x, q) ∨ν_A (y, q), which implies that ν_A (xy, q) ≤ ν_A (x, q) ∨ν_A (y, q).

Hence A is an intuitionistic QL-fuzzy subhemiring of R.

Theorem 2.3: Let (R, +,.) and (R', +,.) be any two hemirings. The anti-homomorphic image of an intuitionistic QL-fuzzy subhemiring of R is an intuitionistic QL-fuzzy subhemiring of R'.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings.

Let f: R → R'be an anti-homomorphism.

Then, f (x+y, q) = f (y, q) + f (x, q) and f (xy, q) = f (y, q) f (x, q), for all x, q and y, q in R.

Let V = f (A), where A is an intuitionistic QL-fuzzy subhemiring of R.

We have to prove that V is an intuitionistic QL-fuzzy subhemiring of R'.

Now, for f (x), f (y) inR', μ_v (f (x, q) + f (y, q)) = μ_v (f (y + x, q)) ≥ μ_A (y + x, q) ≥ μ_A (y, q) ∧ μ_A (x, q) = μ_A (x, q) ∧μ_A (y, q), which implies that μ_v (f (x, q) + f (y, q)) ≥ μ_v (f (x, q)) ∧μ_v (f (y, q)).

Again, μ_v (f (x, q) f (y, q)) = μ_v (f (yx, q)) ≥ μ_A (yx, q) ≥ μ_A (y, q) ∧μ_A (x, q) = μ_A (x, q) ∧μ_A (y, q), which implies that μ_v (f (x, q) f (y, q)) ≥ μ_v (f (x, q)) ∧μ_v (f (y, q)).

Now, for f (x, q), f (y, q) in R', ν_v (f (x, q) + f (y, q)) = ν_v (f (y + x, q)) ≤ ν_A (y + x, q) ≤ν_A (y, q) ∨ν_A (x, q) = ν_A (x, q) ∨ν_A (y, q), which implies that ν_v (f (x, q) +f (y, q)) ≤ ν_v (f (x, q)) ∨ν_v (f (y, q)).

Again, ν_v (f (x, q) f (y, q)) = ν_v (f (yx, q)) ≤ ν_A (yx, q) ≤ ν_A (y, q) ∨ν_A (x, q) = ν_A (x, q) ∨ ν_A (y, q), which implies that ν_v (f (x, q) f (y, q)) ≤ ν_v (f (x, q)) ∨ ν_v (f (y, q)).

Hence V is an intuitionistic QL-fuzzy subhemiring of R'.

Theorem 2.4: Let (R, +,.) and (R|, +,.) be any two hemirings. The anti-homomorphic preimage of anintuitionistic QL-fuzzy subhemiring of R'is an intuitionistic QL-fuzzy subhemiring of R.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings. Let f: R → R'be an anti-homomorphism. Then, f (x+y, q) = f (y, q) + f (x, q) and f (xy, q) = f (y, q) f (x, q), for all x, q and y, q in R. Let V = f (A), where V is an intuitionistic QL-fuzzy subhemiring of R'.

We have to prove that A is an intuitionistic QL-fuzzy subhemiring of R.

Let x and y in R. Then, μ_A (x+y, q) =μ_v (f (x+y, q)) = μ_v (f (y, q) +f (x, q))
≥ μ_v (f (y, q)) ∧μ_v (f (x, q)) =μ_v (f (x, q)) ∧μ_v (f (y, q))
= μ_A (x, q) ∧μ_A (y, q) which implies that μ_A (x + y, q) ≥ μ_A (x, q) ∧ μ_A (y, q).

Again, μ_A (xy, q) =μ_v (f (xy, q)) =μ_v (f (y, q) f (x, q)) ≥μ_v (f (y, q)) ∧μ_v (f (x, q)) =μ_v (f (x, q)) ∧μ_v (f (y, q)) = μ_A (x, q) ∧μ_A (y, q), which implies that μ_A (xy, q) ≥μ_A (x, q) ∧μ_A (y, q).

Then, ν_A (x+y, q) =ν_v (f (x+y, q)) = ν_v (f (y, q) + f (x, q)) ≤ν_v (f (y, q)) ∨ν_v (f (x, q)) = ν_v (f (x, q)) ∨ν_v (f (y, q)) = ν_A (x, q) ∨ν_A (y, q), which implies that ν_A (x + y, q) ≤ ν_A (x, q) ∨ ν_A (y, q).again, ν_A (xy, q) = ν_v (f (xy, q)) = ν_v (f (y, q) f (x, q)) ≤ ν_v (f (y, q)) ∨ν_v (f (x, q)) = ν_v (f (x, q)) ∨ν_v (f (y, q)) =ν_A (x, q) ∨ν_A (y, q), which implies that ν_A (xy, q) ≤ ν_A (x, q) ∨ ν_A (y, q).

Hence A is an intuitionistic L-fuzzy subhemiring of R.

Theorem 2.5: Let A be an intuitionistic QL-fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then A◦f is an intuitionistic QL-fuzzy subhemiring of R.

Proof: Let x and y in R and A be an intuitionistic QL-fuzzy subhemiring of a hemiring H.

Then we have, (μ_A◦f) (x+y, q) = μ_A (f (x+y, q)) = μ_A (f (x, q) + f (y, q)) ≥μ_A (f (x, q)) ∧μ_A (f (y, q)) = (μ_A◦f) (x, q) ∧ (μ_A◦f) (y, q), which implies that (μ_A◦f) (x+y, q) ≥ (μ_A◦f) (x, q) ∧ (μ_A◦f) (y, q). and, (μ_A◦f) (xy, q) =μ_A (f (xy, q)) =μ_{Az} f (x, q) f (y, q)) ≥μ_A (f (x, q)) ∧μ_A (f (y, q)) = (μ_A◦f) (x, q) ∧ (μ_A◦f) (y, q), which implies that (μ_A◦f) (xy, q) ≥ (μ_A◦f) (x, q) ∧ (μ_A◦f) (y, q). Then we have, (ν_A◦f) (x+y, q) =ν_A (f (x+y, q)) =ν_A (f (x, q) +f (y, q)) ≤ν_A (f (x, q)) ∨ν_A (f (y, q)) = (ν_A◦f) (x, q) ∨ (ν_A◦f) (y, q), which implies that (ν_A◦f) (x+y, q) ≤ (ν_A◦f) (x, q) ∨ (ν_A◦f) (y, q).

And (ν_A◦f) (xy, q) =ν_A (f (xy, q)) =ν_A (f (x, q) f (y, q)) ≤ν_A (f (x, q)) ∨ν_A (f (y, q))

= $(\nu_A \circ f)$ (x, q) ∨ $(\nu_A \circ f)$ (y, q), which implies that $(\nu_A \circ f)$ (xy, q) ≤ $(\nu_A \circ f)$ (x, q) ∨ $(\nu_A \circ f)$ (y, q).

Therefore (A∘f) is an intuitionistic QL-fuzzy subhemiring of a hemiring R.

Theorem 2.6: Let A be an intuitionistic QL-fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then A∘f is an intuitionistic QL-fuzzy subhemiring of R.

Proof: Let x and y in R and A be an intuitionistic QL-fuzzy subhemiring of a hemiring H.

Then we have, $(\mu_A \circ f)$ (x+y, q) $=\mu_A$ (f (x+y, q)) $=\mu_A$ (f (y, q) +f (x, q)) $\geq\mu_A$ (f (x, q)) $\wedge\mu_A$ (f (y, q)) = $(\mu_A \circ f)$ (x, q) ∧ $(\mu_A \circ f)$ (y, q), which implies that $(\mu_A \circ f)$ (x+y, q) ≥ $(\mu_A \circ f)$ (x, q) ∧ $(\mu_A \circ f)$ (y, q). and, $(\mu_A \circ f)$ (xy, q) $=\mu_A$ (f (xy, q)) $=\mu_A$ (f (y, q) f (x, q)) $\geq\mu_A$ (f (x, q)) $\wedge\mu_A$ (f (y, q)) = $(\mu_A \circ f)$ (x, q) ∧ $(\mu_A \circ f)$ (y, q), which implies that $(\mu_A \circ f)$ (xy, q) ≥ $(\mu_A \circ f)$ (x, q) ∧ $(\mu_A \circ f)$ (y, q).

Then we have, $(\nu_A \circ f)$ (x+y, q) $=\nu_A$ (f (x+y, q)) $=\nu_A$ (f (y, q) +f (x, q)) $\leq\nu_A$ (f (x, q)) $\vee\nu_A$ (f (y, q)) = $(\nu_A \circ f)$ (x, q) ∨ $(\nu_A \circ f)$ (y, q), which implies that $(\nu_A \circ f)$ (x+y, q) ≤ $(\nu_A \circ f)$ (x, q) ∨ $(\nu_A \circ f)$ (y, q).

And, $(\nu_A \circ f)$ (xy, q) $=\nu_A$ (f (xy, q)) $=\nu_A$ (f (y, q) f (x, q)) $\leq\nu_A$ (f (x, q)) $\vee\nu_A$ (f (y, q)) = $(\nu_A \circ f)$ (x, q) ∨ $(\nu_A \circ f)$ (y, q), which implies that $(\nu_A \circ f)$ (xy, q) ≤ $(\nu_A \circ f)$ (x, q) ∨ $(\nu_A \circ f)$ (y, q).

Therefore A∘f is an intuitionistic QL-fuzzy subhemiring of the hemiring R.

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