Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 8, Number 1 (2013), pp. 41-53

http://www.ripublication.com/afm.htm

New Type of Intuitionistic fuzzy boundary

V. Thiripurasundari and S. Murugesan
Department of Mathematics,
Sri S. Ramasamy Naidu Memorial College,
Sattur-626203, Tamil Nadu, INDIA,
E-mail: thiripurasund@gmail.com, satturmuruges@gmail.com

Abstract

The aim of this paper is to introduce a new type of intuitionistic fuzzy boundary and investigate its properties. Here, this new type of intuitionistic fuzzy boundary is compared with the existing intuitionistic fuzzy boundary.

AMS subject classification: 54A05, 54A40, 03E72.
Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy topological spaces, intuitionistic fuzzy open set, intuitionistic fuzzy closed set, intuitionistic fuzzy boundary, intuitionistic fuzzy product related spaces.

1. Introduction

In the year 1965, Zadeh [10] introduced the concepts of fuzzy sets. After that the notion of intuitionistic fuzzy sets as a generalization of fuzzy set was introduced by Atanassov [1] in the year 1986. Using the concept of intuitionistic fuzzy sets, Coker [4] introduced intuitionistic fuzzy topological spaces.

In general topology, the boundary of a set is obtained as the set of elements which are both in the closure of the set and closure of its complement. In many real-life situations, boundary of a spatial object is not well defined due to the inherent of fuzziness. The concept of fuzzy boundary naturally came as a sequel.

Topologically, fuzzy boundary was defined by Warren [9] in 1977. Later, Pu and Liu [6], gave another definition of fuzzy boundary based on the intersection of closure of the set and closure of the complement of the set in 1980. To generalize the concept of fuzzy boundary Ahmed and Athar [2], proposed the concept of fuzzy semi-boundary generalizing the notion of Pu and Liu. Recently many fuzzy topological concepts have been generalized for intuitionistic fuzzy topological spaces. In [7, 8], the authors of this paper extend the concepts of fuzzy boundary and fuzzy semi-boundary due to Ahmed and Athar to intuitionistic fuzzy topological space.

In this paper, we introduce the new type of intuitionistic fuzzy boundary and investigate its properties. Further, new type of intuitionistic fuzzy boundary in product related space is analysed. Finally, this new type of intuitionistic fuzzy boundary is compared with the existing intuitionistic fuzzy boundary.

2. Preliminaries

In this section, basic definition and results which needed in sequel are summarized.

Definition 2.1. [1] An intuitionistic fuzzy set (IFS) A in X is an object of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leqslant \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B be IFS's of the forms $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle | x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) The complement of A is denoted by $\bar{A}$ and is defined by

$$\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle | x \in X\},$$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle | x \in X\}$,

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle | x \in X\}$.

For the sake of simplicity, we will use the notation $A = \langle x, \mu_A, \gamma_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$. The IFS's $0_\sim$ and $1_\sim$ are defined to be $0_\sim = \{\langle x, \underline{0}, \underline{1}\rangle | x \in X\}$ and $1_\sim = \{\langle x, \underline{1}, \underline{0}\rangle | x \in X\}$ respectively.

Definition 2.3. [4] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFS's in X satisfying the following axioms:

(1) $0_\sim, 1_\sim \in \tau$,

(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(3) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement $\bar{A}$ of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4. **[4]** Let (X, τ) be an IFTS and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by $int(A) = \bigcup\{G | G$ is an IFOS in X and $G \subseteq A\}$, $cl(A) = \bigcap\{K | K$ is an IFCS in X and $A \subseteq K\}$.

Lemma 2.5. **[7]** For any IFS's A and B in IFTS (X, τ),

(a) $\overline{int(A)} = cl(\bar{A})$;

(b) $\overline{cl(A)} = int(\bar{A})$;

(c) $A \leqslant B \Rightarrow int(A) \leq int(B)$ and $cl(A) \leq cl(B)$

(d) A is an intuitionistic fuzzy closed (IFC) $\Leftrightarrow cl(A) = A$. (resp. intuitionistic fuzzy open (IFO) $\Leftrightarrow int(A) = A$).

(e) $A \leq B \Leftrightarrow cl(A) \leq cl(B); (int(A) \leq int(B))$.

(f) $cl(cl(A)) = cl(A); (int(int(A)) = int(A))$.

(g) $cl(A) \vee cl(B) = cl(A \vee B)$.

(h) $cl(A) \wedge cl(B) \geq cl(A \wedge B)$.

(i) $int(A) \vee int(B) \leq int(A \vee B)$.

(j) $int(A) \wedge int(B) = int(A \wedge B)$.

Definition 2.6. **[7]** Let A be a intuitionistic fuzzy set in an IFTS X. Then the intuitionistic fuzzy boundary of A is defined as $IBd(A) = cl(A) \wedge cl(\overline{A})$. IBd(A) is a intuitionistic fuzzy closed set(IFCS).

3. New type of Intuitionistic fuzzy Boundary

In this section, we introduce a new type of intuitionistic fuzzy boundary and its properties are discussed analogously with the properties of intuitionistic fuzzy boundary [7].

Definition 3.1. Let A be a intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, τ). Then the new intuitionistic fuzzy boundary of A is denoted by $IBd_1(A)$ and defined as $IBd_1(A) = intcl(A) \wedge intcl(\overline{A})$.

Note 3.2. $IBd_1(0_\sim) = 0_\sim$ and $IBd_1(1_\sim) = 0_\sim$.

Proposition 3.3. For any IFS A in an IFTS (X, τ), then the following condition hold.

(1) $IBd_1(A) = IBd_1(\overline{A})$.

(2) If A is IFC, then $IBd_1(A) \leq A$.

(3) If A is IFO, then $IBd_1(A) \leq \overline{A}$.

(4) $\overline{IBd_1(A)} = clint(A) \vee clint(\overline{A})$.

Proof.

(1)

$$\begin{aligned} IBd_1(A) &= intcl(A) \wedge intcl(\overline{A}) \\ &= intcl(\overline{A}) \wedge intcl(A) = intcl(\overline{A}) \wedge intcl(\overline{\overline{A}}) = IBd_1(\overline{A}). \end{aligned}$$

(2) $IBd_1(A) = intcl(A) \wedge intcl(\overline{A}) \leq intclA \leq clA = A$. Since A is IFC. Hence, $IBd_1A \leq A$.

(3) A is IFO implies $\overline{A}$ is IFC.
By (1),

$$\begin{aligned} IBd_1(A) = IBd_1(\overline{A}) &= intcl(\overline{A}) \wedge intcl(\overline{\overline{A}}) \\ &\leq intcl(\overline{A}) \leq cl(\overline{A}) = \overline{A} \text{ by (2).} \end{aligned}$$

Hence $IBd_1A \leq \overline{A}$.

(4)

$$\begin{aligned} (\overline{IBd_1(A)}) &= \overline{intcl(A) \wedge intcl(\overline{A})} = \overline{intcl(A)} \vee \overline{intcl(\overline{A})} \\ &= cl\overline{cl(A)} \vee cl\overline{cl(\overline{A})} = clint(\overline{A}) \vee clint\overline{\overline{A}} = clint(\overline{A}) \vee clint(A) \end{aligned}$$

Hence, $\overline{IBd_1(A)} = clint(A) \vee clint(\overline{A})$.

The converse of (2) and (3) of Proposition 3.3 is not true as seen in the following example. ■

Example 3.4. Let $X = \{a, b\}$ be a set and $\tau = \{0_\sim, 1_\sim, A, B\}$ be a intuitionistic fuzzy topology on X.
where

$$A = \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.5}{a} + \frac{0.3}{b} \rangle$$

and

$$B = \langle x, \frac{0.6}{a} + \frac{0.7}{b}, \frac{0.3}{a} + \frac{0.2}{b} \rangle.$$

Choose

$$M = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.2}{b} \rangle$$

to be an IFS in IFTS (X,τ). Then,

$$\begin{aligned} IBd_1(M) &= intcl(M) \wedge intcl(\overline{M}), \\ cl(M) &= 1_\sim, intcl(M) = 1_\sim. \\ cl(\overline{M}) &= \langle x, \frac{0.5}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.5}{b} \rangle, \\ intcl(\overline{M}) &= 0_\sim. \end{aligned}$$

Hence, $IBd_1(M) = 0_\sim \leq$ M, but M is not intuitionistic fuzzy closed. Also, $IBd_1(M) = 0_\sim \leq \overline{M}$, but M is not intuitionistic fuzzy open.

Proposition 3.5. For any IFS A in an IFTS (X, τ), the following condition hold.

(1) $IBd_1(A) \leq cl(A)$.

(2) $A \vee IBd_1(A) \leq cl(A)$.

Proof.

(1) $IBd_1(A) = intcl(A) \wedge intcl(\overline{A}) \leq intcl(A) \leq cl(A)$.
Hence, $IBd_1(A) \leq cl(A)$.

(2) Since, $A \leq cl(A)$ and by (1) $IBd_1(A) \leq cl(A)$.
Hence $A \vee IBd_1(A) \leq cl(A)$. ■

The following example shows that the equality may not hold in Proposition 3.5 (1 and 2).

Example 3.6. Let $X = \{a, b\}$ be a set and $\tau = \{0_\sim, 1_\sim, A, B, A \cup B, A \cap B\}$ be a intuitionistic fuzzy topology on X, where

$$A = \langle x, \frac{0.5}{a} + \frac{0.3}{b}, \frac{0.1}{a} + \frac{0.4}{b} \rangle$$

and

$$B = \langle x, \frac{0.7}{a} + \frac{0.2}{b}, \frac{0.2}{a} + \frac{0.3}{b} \rangle.$$

Choose

$$M = \langle x, \frac{0.2}{a} + \frac{0.3}{b}, \frac{0.5}{a} + \frac{0.4}{b} \rangle$$

to be an IFS in IFTS (X, τ). Then,

$$\begin{aligned}
&(1)\ IBd_1(M) = intcl(M) \wedge intcl(\overline{M}),\\
&cl(M) = \langle x, \frac{0.2}{a} + \frac{0.4}{b}, \frac{0.5}{a} + \frac{0.2}{b}\rangle,\\
&cl(\overline{M}) = 1_{\sim}, intcl(M) = 0_{\sim}, intcl(\overline{M}) = 1_{\sim},\\
&IBd_1(M) = 0_{\sim} \wedge 1_{\sim} = 0_{\sim}.\\
&\text{Hence, } IBd_1(M) = 0_{\sim} \lneqq \langle x, \frac{0.2}{a} + \frac{0.4}{b}, \frac{0.5}{a} + \frac{0.2}{b}\rangle = cl(M).\\
&(2)\ M \vee IBd_1(M) = \langle x, \frac{0.2}{a} + \frac{0.3}{b}, \frac{0.5}{a} + \frac{0.4}{b}\rangle \lneqq \langle x, \frac{0.2}{a}\\
&\quad + \frac{0.4}{b}, \frac{0.5}{a} + \frac{0.2}{b}\rangle = cl(M).
\end{aligned}$$

Proposition 3.7. Let A be a intuitionistic fuzzy set in an IFTS (X, τ). Then,

(1) $IBd_1(int(A)) \leq IBd_1(A)$.

(2) $IBd_1(cl(A)) \leq IBd_1(A)$.

(3) $int(A) \leq clint(A) \leq A \wedge (\overline{IBd_1(A)})$.

Proof. (1)

$$\begin{aligned}
IBd_1(int(A)) &= intcl(int(A)) \wedge intcl(\overline{int(A)})\\
&\leq intcl(A) \wedge int(\overline{intint(A)})\\
&\leq intcl(A)) \wedge int(\overline{int(A)}) = intcl(A) \wedge intcl(\overline{A}).
\end{aligned}$$

Thus $IBd_1(int(A)) \leq IBd_1(A)$.
(2)

$$\begin{aligned}
IBd_1(cl(A)) &= intcl(cl(A)) \wedge intcl(\overline{cl(A)}) = intcl(A) \wedge intcl(int\overline{A})\\
&\leq intcl(A) \wedge intcl(\overline{A})
\end{aligned}$$

Thus $IBd_1(cl(A)) \leq IBd_1(A)$.
(3)

$$\begin{aligned}
A \wedge (\overline{IBd_1(A)}) &= A \wedge \overline{intcl(A) \wedge intcl(\overline{A})})\\
&= A \wedge (\overline{intcl(A)} \vee \overline{intcl(\overline{A})})\\
&= A \wedge (cl(\overline{cl(A)}) \vee cl(\overline{cl(\overline{A})})\\
&= [A \wedge clint(\overline{A})] \vee [A \wedge clint(A)]\\
&= [A \wedge clint(\overline{A})] \vee clint(A) \geq clint(A) \geq int(A).
\end{aligned}$$

Thus $int(A) \leq clint(A) \leq A \wedge (\overline{IBd_1(A)})$. The following example shows that the equality may not hold in Proposition 3.7 (1-3). ■

Example 3.8. Choose intuitionistic fuzzy sets

$$P = \langle x, \frac{0.3}{a} + \frac{0.2}{b}, \frac{0.6}{a} + \frac{0.7}{b} \rangle$$

and

$$Q = \langle x, \frac{0.2}{a} + \frac{0.3}{b}, \frac{0.5}{a} + \frac{0.4}{b} \rangle,$$

in the IFTS (X, τ) of example 3.6.

(1) Now, $cl(P) = 1_\sim, int(P) = 0_\sim, cl(\overline{P}) = 1_\sim$.

$$IBd_1(P) = intcl(P) \wedge intcl(\overline{P}) = 1_\sim.$$

$$\begin{aligned} IBd_1(int(P)) &= intcl(int(P)) \wedge intcl(\overline{int(P)}) = intcl(0_\sim) \wedge intcl(1_\sim) \\ &= 0_\sim \wedge 1_\sim = 0_\sim. \end{aligned}$$

Hence, $IBd_1(int(P)) = 0_\sim \lneqq 1_\sim = IBd_1(P)$.

(2)

$$\begin{aligned} IBd_1(cl(P)) &= intcl(cl(P)) \wedge intcl(\overline{cl(P)}) \\ &= intcl(1_\sim) \wedge intcl(\overline{1}_\sim) = 1_\sim \wedge 0_\sim = 0_\sim. \end{aligned}$$

Hence, $IBd_1(cl(P)) = 0_\sim \lneqq 1_\sim = IBd_1(P)$.

(3)

$$cl(Q) = \langle x, \frac{0.2}{a} + \frac{0.4}{b}, \frac{0.5}{a} + \frac{0.2}{b} \rangle,$$

$$cl(\overline{Q}) = 1_\sim, int(Q) = 0_\sim, clint(Q) = 0_\sim,$$

$$intcl(Q) = 0_\sim, intcl(\overline{Q}) = 1_\sim. IBd_1(Q) = 0_\sim \wedge 1_\sim = 0_\sim.$$

$$Q \wedge (\overline{IBd_1(Q)}) = Q \wedge \overline{0}_\sim = Q \wedge 1_\sim = Q.$$

Hence, $clint(Q) = 0_\sim \lneqq Q = Q \wedge (\overline{IBd_1(Q)})$.

Proposition 3.9. If A and B are IFS's in an IFTS (X, τ), then

$$IBd_1(A \vee B) \leq IBd_1(A) \vee IBd_1(B).$$

Proof.

$$\begin{aligned} IBd_1(A \vee B) &= intcl(A \vee B) \wedge intcl(\overline{A \vee B}) \\ &\leq (intcl(A) \vee intcl(B)) \wedge (intcl(\overline{A}) \wedge intcl(\overline{B})) \\ &= \{intcl(A) \wedge [intcl(\overline{A}) \wedge intcl(\overline{B})]\} \vee \{intcl(B) \wedge [intcl(\overline{A}) \wedge intcl(\overline{B})]\} \\ &= [IBd_1(A) \wedge intcl(\overline{B})] \vee [IBd_1(B) \wedge intcl(\overline{A})] \\ &\leq IBd_1(A) \vee IBd_1(B). \end{aligned}$$

Hence, $IBd_1(A \vee B) \leq IBd_1(A) \vee IBd_1(B)$. ■

Proposition 3.10. For any intuitionistic fuzzy sets A and B in an IFTS (X, τ),

$$IBd_1(A \wedge B) \leq IBd_1(A) \vee (IBd_1(B).$$

Proof.

$$\begin{aligned}
IBd_1(A \wedge B) &= intcl(A \wedge B) \wedge intcl(\overline{A \wedge B})\\
&\leq [intcl(A) \wedge intcl(B)] \wedge [intcl(\overline{A}) \vee intcl(\overline{B})]\\
&= \{[intcl(A) \wedge intcl(B)] \wedge intcl(\overline{A})\} \vee \{[intcl(A) \wedge intcl(B)] \wedge intcl(\overline{B})\}\\
&= [IBd_1(A) \wedge intcl(B)] \vee [IBd_1(B) \wedge intcl(A)]\\
&= IBd_1(A) \vee IBd_1(B).
\end{aligned}$$

Hence, $IBd_1(A \wedge B) \leq IBd_1(A) \vee IBd_1(B)$. In Propositions 3.9 and 3.10, the equality may not hold as seen in the following example. ■

Example 3.11. Let $X = \{a, b\}$ be a set and $\tau = \{0_\sim, 1_\sim, A, B\}$ be a intuitionistic fuzzy topology on (X, τ). where

$$A = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.5}{a} + \frac{0.3}{b}\rangle$$

and

$$B = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b}\rangle.$$

Choose

$$M = \langle x, \frac{0.3}{a} + \frac{0.6}{b}, \frac{0.7}{a} + \frac{0.3}{b}\rangle$$

and

$$N = \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.3}{a} + \frac{0.4}{b}\rangle$$

to be an IFS in IFTS (X, τ). Then,

$$\begin{aligned}
cl(M) = 1_\sim, cl(\overline{M}) = 1_\sim, intcl(M) &= 1_\sim, intcl(\overline{M}) = 1_\sim,\\
cl(N) &= \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.3}{a} + \frac{0.4}{b}\rangle = cl(\overline{N}),\\
intcl(N) &= \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b}\rangle = intcl(\overline{N}).\\
IBd_1 M = 1_\sim, IBd_1(N) &= \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b}\rangle
\end{aligned}$$

$$IBd_1(M) \vee IBd_1(N) = 1_\sim.$$

$$(1)\ M \vee N = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.3}{a} + \frac{0.3}{b} \rangle, cl(M \vee N) = 1_\sim,$$
$$cl(\overline{M \vee N}) = \langle x, \frac{0.5}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle,$$
$$intcl(M \vee N) = 1_\sim, intcl(\overline{M \vee N}) = 0_\sim$$
$$IBd_1(M \vee N) = 1_\sim \wedge 0_\sim = 0_\sim.$$

Hence, $IBd_1(M \vee N) \lneqq IBd_1(M) \vee IBd_1(N)$.

$$(2)\ M \wedge N = \langle x, \frac{0.3}{a} + \frac{0.5}{b}, \frac{0.7}{a} + \frac{0.4}{b} \rangle,$$
$$cl(M \wedge N) = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.3}{a} + \frac{0.4}{b} \rangle,$$
$$intcl(M \wedge N) = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle,$$
$$cl(\overline{M \wedge N}) = 1_\sim, intcl(\overline{M \wedge N}) = 1_\sim,$$
$$IBd_1(M \wedge N) = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle.$$

Hence, $IBd_1(M \wedge N) \lneqq IBd_1 M) \vee IBd_1(N)$.

Proposition 3.12. For any IFS A in an IFTS X,

(1) $IBd_1(IBd_1(A)) \leq IBd_1(A)$

(2) $IBd_1(IBd_1(IBd_1(A))) \leq IBd_1(IBd_1(A))$

Proof. (1)

$$\begin{aligned} IBd_1(IBd_1(A)) &= intcl(IBd_1(A)) \wedge intcl(\overline{IBd_1(A)}) \\ &\leq intcl(IBd_1(A)) = intcl[intcl(A) \wedge intcl(\overline{A})] \\ &\leq intcl(A) \wedge intcl(\overline{A}) = IBd_1(A) \end{aligned}$$

Hence $IBd_1(IBd_1(A)) \leq IBd_1(A)$.
(2) Follows from (1) and hence $IBd_1(IBd_1(IBd_1(A))) \leq IBd_1(IBd_1(A))$. ■

Remark 3.13. The equality in proposition 3.12(1)may not hold as seen in the following example.

Example 3.14. In example 3.11 $IBd_1 M = 1_\sim$,

$$\begin{aligned} IBd_1(IBd_1(M) &= intcl(IBd_1(M)) \wedge intcl(\overline{IBd_1(M)}) \\ &= intcl(1_\sim) \wedge intcl(0_\sim) = 1_\sim \wedge 0_\sim = 0_\sim. \end{aligned}$$

Hence, $IBd_1(IBd_1(M)) \lneqq IBd_1(M)$.

4. Intuitionistic fuzzy product related spaces

In this section a new type of intuitionistic fuzzy boundary is analysed in product related spaces. K. K. Azad [3] introduced the concept of fuzzy product related spaces. H. M. Hanafy [5] extend this concept to intuitionistic fuzzy topological spaces as follows:

Definition 4.1. [5] Let X,Y be non-empty sets and

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle\},$$
$$B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle\}$$

be IFS's of X and Y respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by,

$$(A \times B)(x, y) = \langle (x, y), \mu_{A\times B}(x, y), \gamma_{A\times B}(x, y)\rangle.$$

Where,

$$\mu_{A\times B}(x, y) = min\{\mu_A(x), \mu_B(y)\} \text{ for each } (x, y) \in X \times Y;$$
$$\gamma_{A\times B}(x, y) = max\{\gamma_A(x), \gamma_B(y)\} \text{ for each } (x, y) \in X \times Y$$

and

$$0 \leq \mu_{A\times B}(x, y) + \gamma_{A\times B}(x, y) \leq 1.$$

Definition 4.2. [5] Let (X, τ) and (Y, δ) be IFTS's and $A \in \tau$ and $B \in \delta$. We say that (X, τ) is product related to (Y, δ) if for any IFS's U of X and V of Y, Whenever ($\bar{A} \ngeq U$ and $\bar{B} \ngeq V$) imply ($\bar{A} \times 1_\sim \vee 1_\sim \times \bar{B} \geq U \times V$), there exist $A_1 \in \tau$ and $B_1 \in \delta$ such that $\bar{A}_1 \geq U$ or $\bar{B}_1 \geq V$ and

$$\bar{A}_1 \times 1_\sim \vee 1_\sim \times \bar{B}_1 = \bar{A} \times 1_\sim \vee 1_\sim \times \bar{B}.$$

Theorem 4.3. [5] Let (X, τ) and (Y, δ) be IFTS's such that X is product related to Y. Then for a IFS's A of X and B of Y,

(i) $cl(A \times B) = cl(A) \times cl(B)$;

(ii) $int(A \times B) = int(A) \times int(B)$.

Lemma 4.4. [7] Let A, B, C and D be a IFS in IFTS X, then $(A \wedge B) \times (C \wedge D) = (A \times D) \wedge (B \times C)$.

Proposition 4.5. Let X_i, $i = 1, 2, 3, \ldots, n$, be a family of product related intuitionistic fuzzy topological spaces. If each A_i is a IFS in X_i, then

$$IBd_1 \prod_{i=1}^{n} A_i = [IBd_1(A_1) \times intcl(A_2) \times \cdots \times intcl(A_n)]$$
$$\vee [intcl(A_1) \times IBd_1(A_2) \times intcl(A_3) \cdots \times intcl(A_n)]$$

$$\vee \cdots \vee [intcl(A_1) \times intcl(A_2) \times \cdots \times IBd_1(A_n)].$$

Proof. It suffices to prove this for $n = 2$.

$$\begin{aligned}
IBd_1(A_1 \times A_2) &= intcl(A_1 \times A_2) \wedge intcl(\overline{A_1 \times A_2}) \\
&= intcl(A_1 \times A_2) \wedge int(\overline{int(A_1 \times A_2)}) \\
&= intcl(A_1 \times A_2) \wedge \overline{clint(A_1 \times A_2)} \\
&= (intcl(A_1) \times intcl(A_2)) \wedge (\overline{clint(A_1) \times clint(A_2)}) \\
&\text{(by theorem 4.3)} \\
&= (intcl(A_1) \times intcl(A_2)) \\
&\wedge \overline{[(clint(A_1) \wedge clcl(A_1)) \times (clint(A_2) \wedge clcl(A_2))]} \\
&= (intcl(A_1) \times intcl(A_2)) \\
&\wedge [\overline{(clint(A_1) \wedge clcl(A_1))} \times 1 \vee 1 \times \overline{(clint(A_2) \wedge clcl(A_2))}] \\
&= (intcl(A_1) \times intcl(A_2)) \\
&\wedge [(intcl(\overline{A_1}) \vee intint(\overline{A_1})) \times 1 \vee 1 \times (intcl(\overline{A_2}) \vee intint(\overline{A_2}))] \\
&= (intcl(A_1) \times intcl(A_2)) \wedge [intcl(\overline{A_1}) \times 1 \vee 1 \times intcl(\overline{A_2})] \\
&= [(intcl(A_1) \times intcl(A_2)) \wedge (intcl(\overline{A_1}) \times 1)] \\
&\vee [(intcl(A_1) \times intcl(A_2)) \wedge (1 \times intcl(\overline{A_2}))] \\
&= [(intcl(A_1) \wedge intcl(\overline{A_1})) \times (intcl(A_2) \wedge 1)] \\
&\vee [(intcl A_1 \times 1) \times (intcl(A_2) \wedge intcl(\overline{A_2}))]
\end{aligned}$$

(by lemma 4.4). Hence, $IBd_1(A_1 \times A_2) = (IBd_1(A_1) \times intcl(A_2)) \vee (intcl(A_1) \times IBd_1(A_2))$. ■

5. Comparative Study

In this section, we compare a new type of intuitionistic fuzzy boundary (IBd_1) with intuitionistic fuzzy boundary (IBd) which already exist.

Proposition 5.1. For any IFS A in an IFTS X, $IBd_1(A) \leq IBd(A)$.

Proof. For any IFS A $intcl(A) \leq cl(A)$ and $intcl(\overline{A}) \leq cl(\overline{A})$.

$$IBd_1(A) = intcl(A) \wedge intcl(\overline{A}) \leq cl(A) \wedge cl(\overline{A}) = IBd(A).$$

Hence, $IBd_1(A) \leq IBd(A)$. ■

The following example shows that the equality may not hold in Proposition 5.1.

Example 5.2. In Example 3.11, choose IFS's

$$P = \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.3}{a} + \frac{0.4}{b} \rangle.$$

Then,

$$cl(P) = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.3}{a} + \frac{0.4}{b} \rangle = cl(\overline{P}),$$

$$intcl(P) = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle = intcl(\overline{P}).$$

$$IBd_1(P) = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle.$$

$$IBd(P) = cl(P) \wedge cl(\overline{P}) = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.3}{a} + \frac{0.4}{b} \rangle.$$

Hence, $IBd_1(P) \lneqq IBd(P)$.

Acknowledgement

The authors are highly grateful to referees for valuable comments and suggestions for improving the paper.

References

[1] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and system, 20(1986), no.1, 87–96.

[2] M. Athar and B. Ahmad, Fuzzy boundary and Fuzzy semi boundary, Advances in Fuzzy systems,Article ID.586893 (2008) 1–9 pages.

[3] K.K. Azad, On fuzzy continuity, Fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl., 82(1981), 14–32.

[4] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and system, 88 (1997), no.1, 81–89.

[5] I. M. Hanafy, Completely continuous functions in intuitionistic fuzzy topological spaces, Czechoslovak Math. J., 53(4) (2003), 793–803.

[6] P. M. Pu and Y. M. Liu, Fuzzy topology I-Neighborhood structure of a fuzzy point and Moore-smith convergence, J. Math. Anal. Appl., Vol. 76, No. 2(1980), 571–599.

[7] V. Thiripurasundari and S. Murugesan, Intuitionistic fuzzy boundary and intuitionistic fuzzy continuous functions, (accepted in 'The Journal of the Indian Academy of Mathematics').

[8] V. Thiripurasundari and S.Murugesan, Intuitionistic fuzzy semiboundary and intuitionistic fuzzy product related spaces, Bulletin of society for mathematical services & standards, Vol.1, No.2 (2012), 90–108.

[9] R.H. Warren, Boundary of a fuzzy set, Indiana Univ. Math. J. 26 (1977), 191–197.

[10] L.A. Zadeh, Fuzzy sets, Information and control, 8(1965), 338–353.