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Neighborhood Critical Vertex of an M-Strong Fuzzy Graph

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Abstract

In this paper, the neighborhood critical vertex of an M-strong fuzzy graph is defined. Theorems related to these critical vertices are stated and proved.

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1. Introduction

The notion of fuzzy graph and several fuzzy analogs of graph theoretical concepts such as path, cycle and connectedness are introduced by Rosenfeld in the year 1975 [5]. Mordeson and Peng introduced the concept of fuzzy line graph and developed its basic properties in the year 1993 [4]. The neighborhood numbers (n_0) of various known fuzzy graphs are introduced by S. Ismail Mohideen and A. Mohamed Ismayil in the year 2010 [3]. Neighborhood critical vertex in crisp graph is introduced by E. Sambathkumar and Prabha S. Neeralagi in the year 1992 [6]. In this paper, Neighborhood critical vertices of an M-strong fuzzy graph are discussed. Theorems related to these critcal vertices are stated and proved.

2. Preliminaries

Definition 2.1. Let V be a finite non empty set and E be the collection of two element subsets of V. A *fuzzy graph* $G = (\sigma, \mu)$ is a set with two functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$for all $u, v \in V$.

Definition 2.2. Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $S \subset V$. Then the *scalar cardinality* of S is defined by $\sum_{u \in S} \sigma(u)$. The *order* (p) and *size* (q) of a fuzzy graph $G = (\sigma, \mu)$ are the scalar cardinality of V and E respectively.

Definition 2.3. A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *fuzzy sub graph* induced by V_1 if $\sigma_1(u) \leq \sigma(u)$ for all $u \in V_1$ and $\mu_1(u, v) \leq \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u, v)$ for all $u, v \in V_1$ and is denoted by $\langle V_1 \rangle$. A fuzzy graph $G_1 = (\sigma_1, \mu_1)$ is called the *full fuzzy sub graph* induced by V_1 if $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and $\mu_1(u, v) = \sigma_1(u) \wedge \sigma_1(v) \wedge \mu(u, v)$ for all $u, v \in V_1$ and is denoted by $\langle\langle V_1 \rangle\rangle$.

Definition 2.4. A vertex u of a fuzzy graph $G = (\sigma, \mu)$ is said to be *isolated* vertex if $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V \backslash u$. An edge $e = (u, v)$ of a fuzzy graph is called an *effective edge* if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. Here the vertex u is adjacent to v and the edge e is incident to u and v. A fuzzy graph $G = (\sigma, \mu)$ is said to be *M-strong fuzzy graph* [1] if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $(u, v) \in E$.

Definition 2.5. Let $u, v \in V$ and $e = (u, v)$ then $N(u) = \{v \in V : \mu(u, v) = \sigma(u) \wedge \sigma(v)\}$ is called *open neighborhood* of u and $N[u] = N(u) \cup \{u\}$ is called *closed neighborhood* of u.

Definition 2.6. Let $G = (\sigma, \mu)$ be a fuzzy graph on V and let $u, v \in V$. If $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ then u *dominates* v (or v is dominated by u) in G. A subset D of V is called a *dominating set* in G if for every $v \in V - D$ then there exist $u \in D$ such that u dominates v.The minimum fuzzy cardinality of a dominating set of G is called the *domination number* of G and is denoted by $\gamma(G)$ or γ.

Definition 2.7. Let $G = (\sigma, \mu)$ be an M-strong fuzzy graph. A set $S \in V$ is a *neighborhood set* of G if $G = \cup_{u \in S} \langle\langle N[u] \rangle\rangle$ and is denoted by $n - set$. The *neighborhood number* of G is the minimium scalar cardinality taken over all n-set and is denoted by n_0. n_0-set is a neighborhood set of G with minimium scalar cardinality.

In a fuzzy graph G, the beighborhood number may increase or decrcease or remain unaltered if a vertex is removed from G.

3. Neighborhood critical vertices

Definition 3.1. The vertex v of G is

(i) $\lambda - critical$ if $\lambda(G - v) \neq \lambda(G)$

(ii) $\lambda^{+} - critical$ if $\lambda(G - v) > \lambda(G)$

(iii) $\lambda^{-} - critical$ if $\lambda(G - v) < \lambda(G)$

(iv) $\lambda - fixed$ if v belongs to every λ-set

(v) $\lambda - free$ if v belongs to some λ-set but not all

(vi) $\lambda - totallyfree$ if v belongs to no λ-set.

Here the parameter λ is used as a common symbol for neighborhood number n_0 and domination number γ.

Definition 3.2. The set of all $\lambda - critical$ ($\lambda^{+} - critical$, $\lambda^{-} - critical$, $\lambda - fixed$, $\lambda - free$, $\lambda - totallyfree$) vertices are called $\lambda_c - set(\lambda^{+}, \lambda_c^{-}, \lambda_{fx}, \lambda_{fr}, \lambda_{tf} - set)$.

Example 3.3.

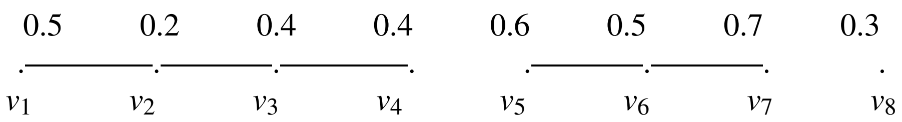

Figure: 3.1

Consider the fuzzy graph given in figure 3.1. Let us take the $\lambda - set$ be $\{v_2, v_4, v_6, v_8\}$.

(i) $\{v_1, v_2, v_4, v_6, v_8\}$ is λ_c-set

(ii) $\{v_2, v_6\}$ is λ_c^{+}-set

(iii) $\{v_1, v_4, v_8\}$ is λ_c^{-}-set

(iv) $\{v_2, v_6, v_8\}$ is λ_{fx}-set

(v) $\{v_3, v_4\}$ is λ_{fr}-set

(vi) $\{v_1, v_5, v_7\}$ is λ_{tf}-set.

Observation 3.4.

1. If the vertex v is isolated then $\lambda(G - v) < \lambda(G)$, that is $v \in \lambda_c^{-}$-set and $v \in \lambda_{fx}$-set.
2. λ_c^{+}-set and λ_c^{-}-set are always subsets of λ_c-set.
3. The union of λ_{fx}-set,λ_{fr}-set and λ_{tf}-set is V, that is $|\lambda_{fx} - set| + |\lambda_{fr} - set| + |\lambda_{tf} - set| = p$.
4. Every vertex of λ-set is $\lambda - critical$. Coverse is not true, for example, the figure given in example 3.3, v_1 is $\lambda - critical$ but not in λ-set.

Theorem 3.5. $\lambda_c^+ - set \subseteq \lambda_{fx} - set$.

Proof. Let $v \in \lambda_c^+ - set$. Hence v is

$$\lambda^+ - critical \Rightarrow \lambda(G - v) > \lambda(G) \tag{1}$$

If $v \notin \lambda_{fx} - set$, that is v is not a λ-fixed vertex. Therefore, v is not an element of atleast one λ-set. Without loss generality , let S is dominateing set or neighorhood set of $G - v$. Hence $\lambda(G - v) \leq \lambda(G)$, which is a contradiction to (1). Therefore $v \in \lambda_{fx} - set$. ■

Remark 3.6. Converse of the Theorem 3.5 need not be true. For example, the figure given in Example 3.3 v_8 is $\lambda - fixed$ but not $\lambda^+ - critical$.

Theorem 3.7. Let G be a fuzzy graph, if λ-set is unique in G, then every vertex of V is either in λ_{fx}-set or λ_{tf}-set. In this case the union of λ_{fx}-set and λ_{tf}-set is V. That is $|\lambda_{fx} - set| + |\lambda_{fr} - set| = p$.

Proof. **case (i)** let $v \in V$ be the vertex not in λ_{fx}-set. We claim that v is in λ_{tf}-set. Suppose v is not in λ_{tf}-set. Then v must be in λ_{fr}-set(Observation 3.4. 3). Since λ-set is unique, hence v must be in λ_{fx}-set, which is a contradiction. **case(ii)** similar we can prove this case. ■

Remark 3.8. If λ-set is unique, then

1. λ_{fr}-set is empty.

2. Intersection of λ_{fx}-set andλ_{tf}-set is empty.

Theorem 3.9. *Every vertex of V not in $\lambda - critical$ is either in λ_{fr}-set and λ_{tf}-set.*

Proof. Let $v \in V$ not in $\lambda - critical$
case (i) If v is not in λ_{fr}-set. We claim that v is in λ_{tf}-set. Suppose v is not in λ_{tf}-set, then v must be in λ_{fx}-set. Hence v is $\lambda - critical$,which is a contradiction.
case (ii) similar we can prove this case. ■

Theorem 3.10.

1. A vertex v is $\gamma^- - critical$ if and only if $N(u) \subset \cup_{u \in D - \{v\}} N(u)$ for some γ-set D containing v.

2. v is $n_0 - critical$ if and only if $\langle\langle N(v)\rangle\rangle$ is a full induced fuzzy subgraph of $\cup_{u \in D - \{v\}} \langle\langle N(u)\rangle\rangle$ for n_0-set D containing v.

Proof.

1. let v be $\gamma^- - critical$ and S be a γ-set of $G - v$. Then $D = S \cup \{v\}$is a γ-set of G. If S contains a vertex of $N(v)$, then S will be a dominating set of G, which is a

contradiction. Thus no vervex of $N(v)$ belong to S. Hence $N(v) \subset \cup_{u \in D-\{v\}} N(u)$. Conversely, if $N(v) \subset \cup_{u \in D-\{v\}} N(u)$ for some γ-set D containing v, then $D - \{v\}$ is a dominaing set of $G - v$. Hence v is $\gamma^- - critical$.

2. similarly we can prove this part. ■

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