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An Application of Multi Objective Fuzzy Linear Programming for Sales Forecasting

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Abstract

Fuzzy Linear programming is developed for the evaluation of management strategies for various applications such as industries for their planning, production, inventory and other activities; this is also used in various government departments. This concept is very much useful in dealing with uncertain situations in day to day life. In this study, the solution procedure of Multi-objective Fuzzy Linear Programming Problem (MOFLPP) with mixed constraints and its application in sales forecasting is presented. The present paper demonstrates MOFLPP with fuzzy coefficient occurring in constraints and objective functions and fuzzy constraint goals has been considered. Fuzzy constraint goals and coefficients of objective and constraint functions are characterized by Triangular Fuzzy numbers (TFNs).Then the optimal solution of MOFLPP is solved using max-min operator. Here the study is dealt with a real life problem of sales forecasting for a particular product in a company to demonstrate the feasibility of the study. This Linear Programming Problem is much simplified than the conventional statistical methods which are sometimes ambiguous where there is always a problem of uncertainty in obtaining optimal solutions. MOFLPP effectively deals with flexible levels or goals.

Keywords and Phrases: Fuzzy multi-objective optimization, Triangular fuzzy number, Membership Function, Fuzzy Max-Min Operator,

Ams Subject Classification(2010): 90C05,90C08.

Introduction

In the present day situation we encounter a number of different kinds of situations which requires prediction due to the uncertainty involved in this fast world. These kinds of uncertainties can be dealt conveniently with probability, fuzzy set theory etc. Fuzzy linear programming problem associates fuzzy input data by fuzzy membership functions. Fuzzy Linear Programming assumes that the objectives and constraints are imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized.

Multi objective linear programming problem is an extension of linear programming. In a Multi objective linear programming problem (MOLPP) coefficients (of objective and constraint functions) as well as constraint goals are assumed to be fixed in value. But in practical situations these assumptions are not valid due to the uncertainties prevailing in the present day to day scenario. These coefficients as well as constraint goals may not be well defined due to lack of information of data. For the above reason the coefficients and constraint goals may be categorized by fuzzy numbers.

The idea of fuzzy set was first proposed by Zadeh **[2]**, as a mean of handling uncertainty that is due to imprecision rather than to randomness. Zimmerman [9] introduced fuzzy linear programming in fuzzy environment. Multi-objective linear Programming was introduced by Zeleny (1974) Lai-Hawng **[8]** considered MOLPP with all parameters having a triangular possibility distribution. They used an auxiliary model and it was solved by MOLPP. The concept of decision making in fuzzy environment involving several objectives was first proposed by Bellman and Zadeh [**2**] ` Zimmerman**[5]** applied their approach to vector maximum problem by transforming MOFLP problem to a single objective linear programming problem.

In this paper we have proposed MOFLPP with mixed constraint in which right hand side constraints are fuzzy numbers, the coefficient of constraint were taken as fuzzy numbers and it has been solved by max min operator. Fuzzy decision making process the MOFLPP is converted into an equivalent crisp LPP. Then it is solved by simplex method.

Preliminaries

Fuzzy Set: 2.1-[12]

If X is a collection of objects denoted generically by x, then a fuzzy set Ã in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership of x in Ã that maps X to the [0,1].

Alpha-cut 2.2-[12]

The alpha cut of a fuzzy set A is the crisp set that contains all the elements of the Universal set X whose membership grades in A is greater than or equal to the specified value of alpha. It is denoted as $\tilde{A}_\alpha$. i.e.

$$A_\alpha = \{x \in X / \mu_A(x) \geq \alpha\}$$

Height 2.3-[12]
The height of a fuzzy set A, is the largest (supremum) membership grade obtained by any element in that set. It is denoted as height (A) i.e.

$$hgt(A) = \sup_{x \in X} \mu_A(x)$$

Core 2.4:-[12]
The core of a fuzzy set is the crisp subset of A with membership grade equal to 1. It is denoted as Core (A) i.e.

$$Core(A) = \{ x \in X \mid \mu_A(x) = 1 \}$$

Normal: 2.5[12]
The fuzzy set Ã is normal if height (A) = 1. In other words there exist at least one x∈ X such that $\mu_{\tilde{A}}(x) = 1$.

Support: 2.6[12]
The support of a fuzzy set $\tilde{A}$,S (Ã), is the crisp set of all x∈ X such that $\mu_{\tilde{A}}(x) > 0$

Convex Fuzzy set: 2.7- [12]
A fuzzy set Ã is considered convex when its objective function increases and then decreases monotonically and there is no local minimum. A fuzzy set is convex if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \forall\ x_1, x_2 \in X, \lambda \in [0,1]$$

Fuzzy number: 2.8- [12]
A fuzzy set Ã of the real line R with membership function $\mu_{\tilde{A}}(x)$: R→ [0, 1] is called fuzzy number if

i. a must be normal and convex fuzzy set;
ii. the support of Ã, must be bounded
iii. $\propto_A$ must be a closed interval for every $\propto \in [0,1]$

Triangular Fuzzy number:2.9- [1]
Let F(R) be a set of all triangular fuzzy number in a real line R.A triangular fuzzy number Ã $(\epsilon F(R))$ is a normal and convex fuzzy set with the following membership function $\mu_{\tilde{A}}(x)$: R→[0,1] which satisfies both normality $\mu_{\tilde{A}}(x) = 1$ for at least one x ϵR and convexity i.e., $\mu_{\tilde{A}}(x') \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ where $\mu_{\tilde{A}}(x)\ \epsilon\ [0, 1]$ and $\forall$ x'∈[x_1,x_2]).

$$\mu_{\tilde{A}}^{(x)} = \frac{x-a_1}{a_2-a_1} \text{ for } a_1 \leq x \leq a_2$$

$$= \frac{a_3-x}{a_3-a_2} \text{ for } a_2 \leq x \leq a_3$$

$$= 0 \text{ elsewhere}$$

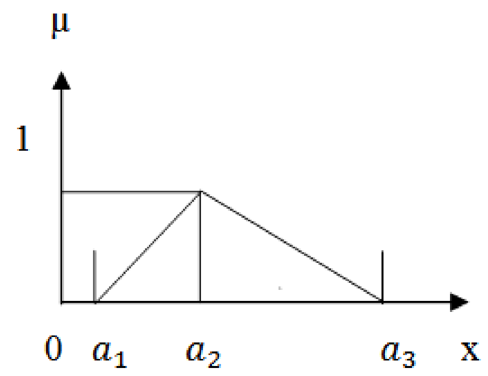

Figure 2.9.1: TFN $\widetilde{A} = (a_1, a_2, a_3)$.

Here we consider the left TFN$\tilde{A}$ = (a_1, a_2, a_2) provided that $a_2 > a_1$.It is represented by the following membership functions:

$$\mu_{\tilde{A}}^{(x)} = 0 \text{ for } x \le a_1,$$

$$= \frac{x - a_1}{a_2 - a_1} \text{ for } a_1 \le x \le a_2$$

$$= 1 \text{ for } x \ge a_2$$

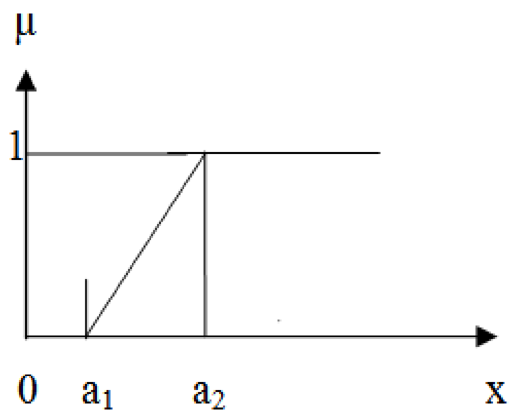

Figure 2.9.2: Left TFN $\widetilde{A} = (a_1, a_2, a_2)$.

Multi-objective Linear Programming problem (MOLPP) with Fuzzy resources

General Multi- objective linear programming problem with mixed constraints may be written as follows;

$$\text{Max (or Min) } Z = [Z^1, Z^2, Z^3 \ldots\ldots.. Z^k] \qquad (3.1.1)$$

Subject to,

$\sum_{j=1}^{n} a_{ij}\, x_j \geq b_i$ for i =1,2,3.....m_1

$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i$ for i=m_1 +1,m_1 + 2......m_2

$\sum_{j=1}^{n} a_{ij}\, x_j = b_i$ for i= m_2 +1,m_2 + 2......m

$x_j \geq 0$ for j=1,2,3.....n

where $Z^k = \sum_{j-1}^{n} c_j^k\, x_{j,}$ k= 1,2,....K

MOLPP with fuzzy resources

When constraint goals are triangular fuzzy numbers (3.1.1) becomes

$$\text{Max (or Min) } Z = [Z^1, Z^2, Z^3 \ldots\ldots\ldots Z^k] \qquad (3.1.2)$$

Subject to,

$\sum_{j=1}^{n} a_{ij}\, x_j \geq \breve{b}_i$ for i=1,2,3.....m_1

$\sum_{j=1}^{n} a_{ij}\, x_j \leq \breve{b}_i$ for i=m_1 +1,m_1 + 2......m_2

$\sum_{j=1}^{n} a_{ij}\, x_j = \breve{b}_i$ for i= m_2 +1,m_2 + 2......m

$x_j \geq 0$ for j=1,2,3.....n

where $Z^k = \sum_{j-1}^{n} c_j^k\, x_{j,}$ k= 1,2,....k

Fuzzy programming technique for the solution of MOLPP with fuzzy resources

Let L_k and U_k be lower and upper bound for k^{th} objective.

L_k = aspired level of the k^{th} objective function.

U_k = highest acceptable of achievement for the k^{th} objective function

When the aspiration level of each objective level has been specified we formed a fuzzy model

Algorithm

Step 1: The MOLPP is solved as a single objective LPP using each time only one objective and ignoring all others

Step 2: From the result of step 1 determine the corresponding value for every objective function at each solution.

Step 3: The upper and lower bounds for the k^{th} objective are found from the objective values derived in step 2.

Step 4: Using the max min operator (as Zimmermann [10]) crisp LPP of equation (3.1.2) is formulated and the solution is solved by any simplex method.

Case study

In this case study we consider a sales performance of PVC pipes and PVC fittings of a company for two quarters i.e., First quarter consisting of months April to June and Second quarter consisting of months July to September. The details of the sales performance is formulated as an LPP and the sales forecast is given for the Third and Fourth quarter which we consider as Second half of the year.

Table 4.2.1

Product	First Quarter (Numbers in units)	Second quarter(Numbers in units)
PVC Pipes	21	22
PVC Fittings	62	68
Target	100	100

In this problem we consider two objective functions wherein the cost coefficient of the first objective function is Rs.9 for PVC pipes and Rs.13 for PVC fittings. If the cost coefficient of the second objective functions is Rs 13 for PVC pipes and Rs.15 for PVC fittings.

The above problem can be formulated as

$$\text{Max}Z^1=c_1^1 x_1+c_2^1 x_2$$

$$\text{Max}Z^2=c_1^2 x_1+c_2^2 x_2$$

Subject to

$$a_{11}x_1+a_{12}x_2 \leq \tilde{b}_1,$$

$$a_{21}x_1 + a_{22}x_2 \leq \tilde{b}_2,$$

$$x_1, x_2 \geq 0$$

Notation

The following symbols are used in this paper;

Z^1 - objective function (Break even Point)

Z^2 - objective function (profit)

x_1 -number of units of PVC pipes sold (decision variable)

x_2 - number of units PVC fittings sold (decision variable)

c_1^1 - Cost coefficient of the of the first objective function

c_2^1 - Cost coefficient of the of the first objective function

c_1^2 - Cost coefficient of the of the second objective function

c_2^2 - Cost coefficient of the of the second objective function

a_{ij} - Constraint matrix

$\widetilde{b}_1$ - Triangular fuzzy numbers

$\widetilde{b}_2$ - Triangular fuzzy numbers

where $c_j^k = \begin{pmatrix} 9 & 13 \\ 13 & 15 \end{pmatrix}$, $a_{ij} = \begin{pmatrix} 21 & 62 \\ 22 & 68 \end{pmatrix}$, i,j,k= 1,2

and $\widetilde{b1} = \widetilde{100}$ (90,100,100) ; $\breve{b}_2 = \widetilde{100}$ (90,100,100) respectively are constraint goals

To solve this problem we first solve the following four Sub Problems (SPS)

$$\text{Max } Z^{11} = 9x_1 + 13x_2 \qquad (4.3.1)$$

subject to

$$21x_1 + 62x_2 \le 100$$
$$22x_1 + 68x_2 \le 100$$
$$x_1, x_2 \ge 0$$

$$\text{Max } Z^{12} = 9x_1 + 13x_2 \qquad (4.3.2)$$

subject to

$$21x_1 + 62x_2 \le 90$$
$$22x_1 + 68x_2 \le 90$$
$$x_1, x_2 \ge 0$$

$$\text{Max } Z^{21} = 13x_1 + 15x_2 \qquad (4.3.3)$$

subject to

$$21x_1 + 62x_2 \le 100$$
$$22x_1 + 68x_2 \le 100$$
$$x_1, x_2 \ge 0$$

$$\text{Max } Z^{22} = 13x_1 + 15x_2 \qquad (4.3.4)$$

subject to

$$21x_1 + 62x_2 \le 90$$
$$22x_1 + 68x_2 \le 90$$
$$x_1, x_2 \ge 0$$

So the optimal solutions of (4.3.1), (4.3.2), (4.3.3), and (4.3.4) are

$$x^{11} = (x_1^{11}, x_2^{11}) = (4.55, 0),\ Z^{11}(x^{11}) = 40.91$$

$x^{12} = (x_1^{12}, x_2^{12}) = (4.09,0)$, $Z^{12}(x^{12}) = 36.82$

$x^{21} = (x_1^{21}, x_2^{21}) = (4.55,0)$, $Z^{21}(x^{21}) = 59.09$

$x^{22} = (x_1^{22}, x_2^{22}) = (4.09,0)$, $Z^{22}(x^{22}) = 53.18$

So $L_1 = \min \{ Z^1(x^{11}), Z^1(x^{12}), Z^1(x^{21}), Z^1(x^{22}), \}$

$= \min (40.91, 36.82, 40.91, 36.82)$

$=36.82$

$U_1 = \max \{ Z^1(x^{11}), Z^1(x^{12}), Z^1(x^{21}), Z^1(x^{22}), \}$

$= \max (40.91, 36.82, 40.91, 36.82)$

$= 40.91$

$L_2 = \min \{ Z^2(x^{11}), Z^2(x^{12}), Z^2(x^{21}), Z^2(x^{22})\}$

$= \min (59.09, 53.18, 59.09, 53.18)$

$=53.18$

$U_2 = \max \{ Z^2(x^{11}), Z^2(x^{12}), Z^2(x^{21}), Z^2(x^{22})\}$

$= \max \{59.09, 53.18, 59.09, 53.18) = 59.09$

Find $\{x_j, j= 1, 2\}$ (4.3.5)

So as to satisfy

$9x_1 + 13x_2 \gtrsim 40.91,$

$13x_1 + 15x_2 \gtrsim 59.09,$

$21x_1 + 62x_2 \lesssim 100,$

$22x_1 + 68x_2 \lesssim 90,$

$x_1, x_2 \geq 0$

Here membership functions for Fuzzy constraints of equation (4.3.5) are defined as

$\mu_{\tilde{G}_1}(9x_1 + 13x_2) = 1$ for $9x_1 + 13x_2 \geq 40.9$

$= \frac{9x1 + 13x2 - 40.9}{4.09}$ for $36.82 \leq 9x_1 + 13x_2 \leq 40.9$

$= 0$ for $9x_1 + 13x2 \leq 36.82$

$\mu_{\tilde{G}_2}(13x_1 + 15x_2) = 1$ for $13x_1 + 15x_2 \geq 59.09$

$= \frac{13x1 + 15x2 - 59.}{5.91}$ for $53.18 \leq 13x_1 + 15x_2 \leq 59.09$

$= 0$ for $13x_1 + 15x2 \leq 53.18$

$\mu_{\tilde{c}_1}(100) = 1$ for $21x_1 + 62x_2 \geq 100$

$= \frac{100-(21x1+62x2)}{10}$ for $90 \le 21x_1+62x_2 \le 100$

$= 0$ for $21x_1+62x_2<90$

$\mu_{\tilde{c}_2}(90) = 1$ for $22x_1+68x_2 \ge 100$

$= \frac{100-(22x1+68x2)}{10}$ for $90 \le 22x_1+68x_2 \le 100$

$= 0$ for $22x_1+68x_2<90$

Using the max- min operator crisp LPP for equation (3.1.1) is formulated as follows:

Max λ

$9x_1 + 13x_2 + 4.09\lambda \le 40.91$

$13x_1 + 15x_2 + 5.91\lambda \le 59.09$

$21x_1+62x_2 - 90\lambda \ge 100$

$22x_1+68x_2 - 90\lambda \ge 100$

$0 \le \lambda \le 1, x_1, x_2 \ge 0$

So optimal solution of MOFLPP for equation (3.1.1) are $x_1 = 0$, $x_2 = 2.87$

$Z^1 = 37.31$, $Z^2 = 43.05$ with aspiration level $\lambda = 0.87$

Conclusion

In the above study we have taken a real life situation. Generally statistical methods are being used for projection of sales in the future with the corresponding data of the previous performances.

The study reveals the optimum sales that have to be done in the second half of the year so as to reach the desired levels of profit in the organization. The mathematical working has been carried out in such a way that the breakeven is calculated and is ascertained as 37.31 as well as the profitability of the organization also has been forecast as 43.05 with aspiration levels of 87percentage respectively. This particular industry has got a lot of uncertainties due to the interference of rains; climate and other natural conditions that inhibit the usage and consumption of the particular product. Hence careful planning has to be done prior to produce of the particular product without any further loss occurring due to the prevailing conditions in the economy. This will enable the company to formulate strategies for achieving the desired levels of profit at different levels of sales output as well profit at different levels of pricing. Moreover we have taken for two products only and the optimal combination results are also given for the desired aspiration levels. This study is such a versatile one that more than two products and also with different combinations can be worked and the results can be obtained and forecast for the given condition.

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