Advances in Fuzzy Mathematics.
ISSN No. 973-533X Volume 6, Number 3 (2011), pp. 353-358

http://www.ripublication.com/afm.htm

Generating Function for Measures of Fuzzy Entropy

P. Jha and M. Verma

Department of Mathematics
Govt. Chhattisgarh P.G. College, Raipur, Chhattisgarh-492001, India
E-mail: mvmanjuverma28@gmail.com

Abstract

In this paper we obtain generating functions for measures of fuzzy entropies from which new measures of fuzzy entropies may be generated.

Keywords: Fuzzy Set / Fuzzy Entropy.

Introduction

Zadeh [9] introduced the concept of fuzzy sets. A fuzzy set A is represented as $A = \{x_i / \mu_A(x_i) : i = 1, 2, ..., n\}$ where $\mu_A(x_i)$ is a membership function.

A measure of fuzzy entropy of a fuzzy set should have at least the following properties.

1. It should be defined for all $\mu_A(x_i)$ in the range of $0 \leq \mu_A(x_i) \leq 1,\ i = 1,2,..........n$.
2. It should be continuous in this region.
3. It should be zero when $\mu_A(x_i) = 0 \ or \ 1$.
4. It should not change when any $\mu_A(x_i)$ is changed into $1 - \mu_A(x_i)$.
5. It should be maximum when $\mu_A(x_i) = \frac{1}{2},\ i = 1,2,....,n$.
6. It should be increasing function of $\mu_A(x_i)$ when $0 \leq \mu_A(x_i) \leq \frac{1}{2}$ and other variables are kept fixed and it should be decreasing function of $\mu_A(x_i)$,when $\frac{1}{2} \leq \mu_A(x_i) \leq 1$ and other variables are kept fixed.
7. The known measures of fuzzy entropies are:

Measure of fuzzy entropy corresponding to Shannon's [8] measure of entropy is:

$$F_1(A) = -\sum_{i=1}^{n} (\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))).$$

Measure of fuzzy entropy corresponding to Havrda-Charvat's [2] measure of entropy is:

$$F_2(A) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n} (\mu_A{}^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha}) - 1\right], \alpha > 0, \alpha \neq 1.$$

Measure of fuzzy entropy corresponding to Renyi's [5] measure of entropy is:

$$F_3(A) = \frac{1}{1-\alpha}\left[\sum_{i-1}^{n} \ln\,(\mu_A{}^{\alpha}(x_i) + (1-\mu_A(x_i))\right], \alpha > 0, \alpha \neq 1.$$

Measure of fuzzy entropy corresponding to Sharma and Taneja's [7] measure of entropy is:

$$F_4(A) = \frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left(\mu_A{}^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha} - \mu_A{}^{\beta}(x_i) - (1-\mu_A(x_i))^{\beta}\right)$$

$$\alpha \geq 1, \beta \leq 1 \; or \; \alpha \leq 1, \beta \geq 1 and \; \alpha \neq \beta.$$

Measure of fuzzy entropy corresponding to Kapur's [3] measure of entropy is:

$$F_5(A) = \frac{1}{\beta-\alpha}\ln\left[\frac{\sum_{i=1}^{n}\left(\mu_A{}^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha}\right)}{\sum_{i=1}^{n}\left(\mu_A{}^{\beta}(x_i) + (1-\mu_A(x_i))^{\beta}\right)}\right], \alpha \geq 1, \beta \leq 1 \; or \; \alpha \leq 1, \beta \geq 1 and \; \alpha \neq \beta.$$

Measure of fuzzy entropy corresponding to Kapur's [4] measure of entropy is:

$$\begin{aligned} F_6(A) = & -\sum_{i=1}^{n}\left(\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right) + \\ & \frac{1}{a}\sum_{i=1}^{n}\left((1+a\mu_A(x_i))\ln(1+a\mu_A(x_i)) + (1+a(1-\mu_A(x_i)))\ln(1+a(1-\mu_A(x_i)))\right) \\ & -\frac{1}{a}(1+a)\ln(1+a) \end{aligned}$$

Measure of fuzzy entropy corresponding to Behara-Chawala's [1] measure of entropy is

$$F_7(A) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)\right) + (1-\mu_A(x_i))^{\alpha}\right)^{\frac{1}{\alpha}-1} \; \alpha \neq 1, \alpha > 0\,.$$

Measure of fuzzy entropy corresponding to Sharma-Mittal's [6] measure of entropy is

$$F_8(A)=\frac{1}{(1-\alpha)\beta}\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)^{\beta}-1\right),\ \alpha\neq 1,\ \alpha>0,\ \beta\neq 0.$$

Generating Functions for Measures of Fuzzy Entropy

(i) Let

$$F(t)=-\left(\sum_{i=1}^{n}\left(\mu_A^{t}(x_i)+(1-\mu_A(x_i))^{t}\right)\right)$$

with the property that

$$F'(1)=-\sum_{i=1}^{n}\left(\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right)$$

which is fuzzy entropy $F_1(A)$

Thus $F(t)$ gives generating function for important fuzzy measure of information viz. $F_1(A)$.

(ii) Now we define another generating function.

$$F_\alpha(t)=\frac{1}{1-\alpha}\sum_{i=1}^{n}\left(\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)^{t}-1\right),\alpha\neq 1.\ \alpha>0.$$

Then

$$F_\alpha(1)=\frac{1}{1-\alpha}\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}-1\right),\alpha\neq 1,\alpha>0.$$

which is fuzzy entropy $F_2(A)$.

also

$$F_\alpha'(0)=\frac{1}{1-\alpha}\sum_{i=1}^{n}\ln\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right),\alpha\neq 1,\alpha>0.$$

which is fuzzy entropy $F_3(A)$.

Now

$$\lim_{\alpha\to 1}F_\alpha(1)=F_1(1)=-\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right]$$

and

$$\lim_{\alpha\to 1}F_\alpha'(1)=F_1'(1)=-\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right]$$

So that both $F_1(1)$ and $F_1'(1)$ gives $F_1(A)$. Thus $F_\alpha(t)$ is the generating function for fuzzy entropies $F_1(A), F_2(A)$ and $F_3(A)$.

(iii)Again we define another generating function.

$$F_{\alpha,\beta}(t)=\frac{1}{\beta-\alpha}\left[\left(\frac{\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)}{\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right)}\right)^{t}-1\right], \alpha\neq\beta, \alpha\geq 1, \beta\leq 1 \text{ or}$$

$\alpha\leq 1, \beta\geq 1$.

so that

$$F_{\alpha,\beta}(1)=\frac{1}{\beta-\alpha}\left[\frac{\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}-\mu_A^{\beta}(x_i)-(1-\mu_A(x_i))^{\beta}\right)}{\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right)}\right], \alpha\neq\beta, \alpha\geq 1, \beta\leq 1$$

or

$\alpha\leq 1, \beta\geq 1$

which is $\left(\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right)\right)^{-1}$ times of fuzzy entropy $F_4(A)$

Now

$$F'_{\alpha,\beta}(0)=\frac{1}{\beta-\alpha}\ln\left(\frac{\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)}{\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right)}\right), \alpha\neq\beta, \alpha\geq 1, \beta\leq 1 \text{ or}$$

$\alpha\leq 1, \beta\geq 1$

which is fuzzy entropy $F_5(A)$. Thus $F_{\alpha,\beta}(t)$ is the generating function for fuzzy entropies $F_4(A)$ and $F_5(A)$.

(iv)Again we define

$$\overline{F}_{\alpha,\beta}(t)=\frac{1}{\beta-\alpha}\left[\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)-(1-\mu_A(x_i))^{\alpha}\right)\right)^{t}-\left(\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)-(1-\mu_A(x_i))^{\beta}\right)\right)^{t}\right],$$

$\alpha\leq 1, \beta\geq 1$ or $\alpha\geq 1, \beta\leq 1, \alpha\neq\beta$.

so that

$$\overline{F}_{\alpha,\beta}(1)=\frac{1}{\beta-\alpha}\left[\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)-(1-\mu_A(x_i))^{\alpha}\right)\right)-\left(\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)-(1-\mu_A(x_i))^{\beta}\right)\right)\right]$$

$\alpha \le 1, \beta \ge 1$ or $\alpha \ge 1, \beta \le 1, \alpha \ne \beta.$

which is fuzzy entropy $F_4(A)$.

And

$$\overline{F}'_{\alpha,\beta}(0) = \frac{1}{\beta-\alpha} \ln\left[\frac{\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)}{\sum_{i=1}^{n}\left(\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\right)}\right] \alpha \ne \beta, \alpha \le 1, \beta \ge 1 \text{ or } \alpha \ge 1, \beta \le 1$$

which is fuzzy entropy $F_5(A)$

So that $\overline{F}_{\alpha,\beta}(t)$ can also be regarded as generating function for fuzzy entropies $F_4(A)$ and $F_5(A)$.

(v) Again we define

$$F_a(t) = -\sum_{i=1}^{n}\left((\mu_A(x_i))^t + (1-\mu_A(x_i))^t\right) + \frac{1}{a}\sum_{i=1}^{n}\left((1+a\mu_A(x_i))^t + (1+a(1-\mu_A(x_i)))^t\right) - \frac{1}{a}(1+a)^t, a \ge -1$$

.

Now

$$F_a'(1) = -\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right]$$

$$+\frac{1}{a}\sum_{i=1}^{n}\left[(1+a\mu_A(x_i))\ln(1+a\mu_A(x_i)) + (1+a(1-\mu_A(x_i)))\ln(1+a(1-\mu_A(x_i)))\right] + \frac{1}{a}(1+a)\ln(1+a)$$

which is fuzzy entropy $F_6(A)$. Thus $F_a(t)$ is the generating function for fuzzy entropy $F_6(A)$.

(vi) Again we define

$$\overline{F}_{\alpha}(t) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right]^{\left(\frac{1}{\alpha}-1\right)t}, \alpha \ne 1, \alpha > 0.$$

so that

$$\overline{F}_{\alpha}(1) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right]^{\left(\frac{1}{\alpha}-1\right)}, \alpha \ne 1, \alpha > 0.$$

which is fuzzy entropy $F_7(A)$ and

$$\overline{F}'_{\alpha}(0) = \left(\frac{1}{\alpha}-1\right)\left(\frac{1}{1-\alpha}\right)\ln\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right], \alpha \ne 1, \alpha > 0.$$

which is ($\frac{1}{\alpha}-1$) times of fuzzy entropy $F_3(A)$. Thus $\overline{F}_a(t)$is the generating function for fuzzy entropies $F_3(A)$and $F_7(A)$.

(vii) Again we define

$$\overline{\overline{F}}_{\alpha,\beta}(t)=\frac{1}{(1-\alpha)\beta}\left[\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right)^{\beta t}-1\right],\alpha\neq 1,\alpha>0,\beta\neq 0.$$

so that

$$\overline{\overline{F}}_{\alpha,\beta}(1)=\frac{1}{(1-\alpha)\beta}\left[\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right)^{\beta}-1\right],\alpha\neq 1,\alpha>0,\beta\neq 0.$$

which is fuzzy entropy $F_8(A)$.
and

$$\overline{\overline{F}}'_{\alpha,\beta}(0)=\frac{1}{(1-\alpha)}\ln\left(\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha}\right)\right),\alpha\neq 1,$$

which is fuzzy entropy $F_3(A)$. Thus $\overline{\overline{F}}_{\alpha,\beta}(t)$is the generating function for fuzzy entropies $F_3(A)$and $F_8(A)$.

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