Advances in Fuzzy Mathematics.
ISSN No. 973-533X Volume 6, Number 3 (2011), pp. 335-351

http://www.ripublication.com/afm.htm

Developing Fuzzy Analytical Hierarchy Process Approach to Select the Optimum Diameter of Water Transfer Tunnel

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Abstract

Currently, selecting the optimum diameter for tunnels has been a complicated problem for designers. Different methods have been suggested for optimum diameter selection. One of these methods is the Fuzzy Analytic Hierarchy Process (FAHP) that tries to obtain the best alternative via expert opinion. In this paper we have attempted to select the optimum diameter for the Kaka Reza water tunnel which is located in Khoramabad city of Lorestan Province in North West of Iran. To achieve the goal, we had three alternatives and five criteria. The alternatives were 2.7 m, 4.3 m and 6 meter, obtained on the basis of the last tunnel excavation experiences, regional position and different tunneling problems. The selected criteria include blasting, ventilation during excavation, tunnel support system, tunnel construction time and tunnel construction costs. At the beginning, some survey forms were provided, using 5 points scale to have the experts' opinions. They recorded each alternative score for different criteria in respective tables. Analyzing the obtained primary data via the FAHP method, we have concluded that diameter 2.7 m is the best option for the Kaka Reza tunnel excavation.

Keywords: Multi-criteria decision making, FAHP, Fuzzy set theory, TFN, Water transfer tunnel

Introduction

Selecting the optimum diameter for tunnels is a difficult task and needs a long time. To select the optimum diameter for tunnels numerous methods exist, each having advantages and disadvantages apiece. Decision making about tunnel diameter is a very sensitive activity and should be carried out very accurately to reduce the costs and time of the project. From the beginning of human's life, decision making has been often discussed as a major topic. At the beginning of history most decisions were simple and made personally, but today decisions are more complex. Complexity of the situations and changes in the operational environments of the projects has increased the sensitivity of decisions. Mathematical techniques of decision-making are one of the valuable results of researchers' activities as quantitative decision making methods are often discussed in scientific circles. One of the most useful and efficient tools for the decision making process is Fuzzy Analytical Hierarchy Process (fuzzy AHP). The fuzzy AHP is easier to understand and can effectively handle both qualitative and quantitative data in the multi-attribute decision making problems. The aim of this paper is to determine the best diameter for Kaka Reza water tunnel using the FAHP approach. To achieve this, we have three alternatives and five criteria.

AHP method was used at different tunneling or mining projects. Panou and Sofianos [39] applied the AHP as a suitable selection method out of two alternatives of tunnel way or surface way. Nakamura [37] used the MCDM method to determine the tunnel management system. Padma and Balasubramanie [38] used the AHP for the tunnel maintenance system selection. Naghadehi et al [36] used the fuzzy AHP to select the optimum underground mining method for Jajarm Bauxite Mine. Azadeh et al [3] represented a solution by the fuzzy AHP for the problem of mining method selection (MMS) in mining projects.

The remainder of this paper is organized as follows. In section 2, a brief review about the concept of the fuzzy sets and fuzzy numbers is provided. In section 3, the FAHP method is illustrated. This section is included in both the literature review and the methodology of FAHP. In Section 4, after the explanation of tunnel information and hierarchy structure of the problem, the FAHP method is applied to determine the weights of the criteria given by the experts. Then, subsequent calculations and analyses are done and finally the optimum excavation diameter is selected. As far as the author knows, the optimum tunnel diameter selection using the FAHP is a unique research.

Fuzzy set and Fuzzy number theory

Zadeh [51] in 1965 introduced fuzzy set theory to solve problems involving the absence of sharply defined criteria. If uncertainty (fuzziness) of human decision making is not taken into account, the results can be misleading. A commonality among terms of expression, such as ''very likely'', ''probably so'', ''not very clear'', ''rather dangerous'' that are often heard in daily life, is that they all contain some degree of uncertainty [45]. Fuzzy theory, thus, is used to solve such kinds of problems, and it has been applied for a variety of fields in the last four decades. The theory of fuzzy sets has evolved in various directions and two distinct directions are: treating fuzzy sets as precisely defined mathematical objects subject to the rules of classical logic, and the linguistic approach. The underlying logic of linguistic approach is that the truth-values are fuzzy sets and the rules of inference are approximate rather than exact [19]. A triangular fuzzy number (TFN), a special case of a trapezoidal fuzzy number, is very popular in fuzzy applications. As shown in Figure 1, the triangular fuzzy number $\tilde{M}$ is represented by (a, b, c), and the membership function is defined as:

$$\mu_{\tilde{M}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & Otherwise \end{cases} \tag{1}$$

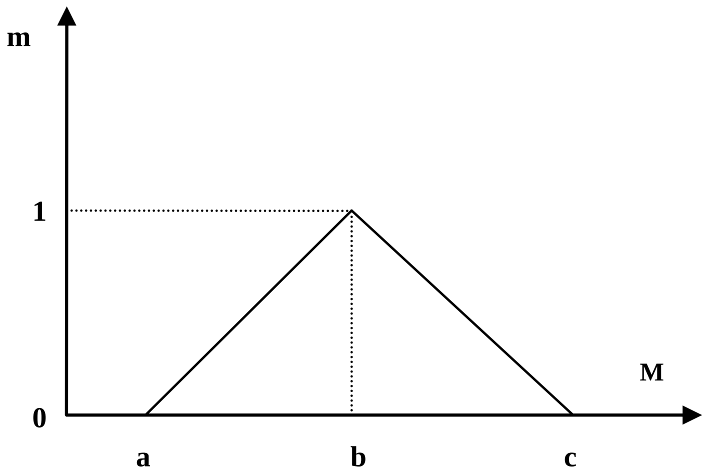

Figure 1: Membership function of a triangular fuzzy number $\tilde{M} = (a,b,c)$.

The strongest grade of membership is parameter b, that is $f_m(b) = 1$, while a and c are the lower and upper bounds. An important concept of fuzzy sets is the $\alpha - cut$. For a fuzzy number $\tilde{M}$ and any number $\alpha \in [0,1]$, the $\alpha - cut$, C_α is the crisp set [28]:

$$C_\alpha = \{x | C(x) \ge \alpha\} \tag{2}$$

The a-cut of a fuzzy number $\tilde{M}$ is the crisp set $\tilde{M}^\alpha$ that contains all the elements of the universal set U whose membership grades in $\tilde{M}$ are greater than or equal to the specified value of α, as shown in Fig. 2.

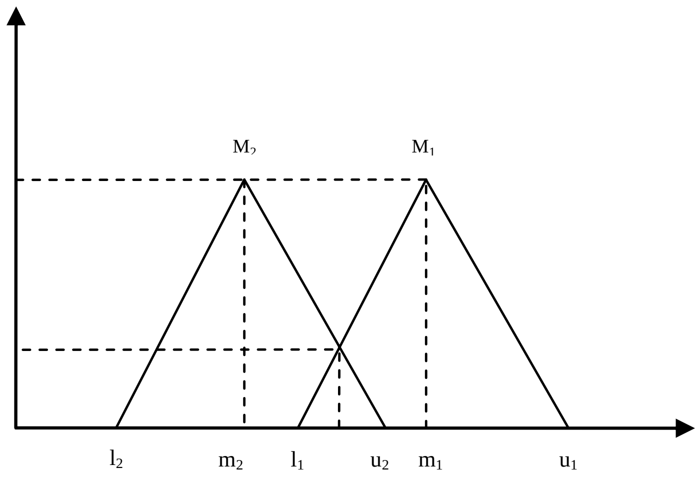

Figure 2: The intersection between M_1 and M_2

By defining the interval of confidence at level a, the triangular fuzzy number can be characterized as (Cheng, 1999; Cheng, 1996; Cheng and Mon, 1994) [11, 12, 13]:

$$\tilde{M}^{\alpha} = \left[a^{\alpha}, c^{\alpha}\right] = \left[(b-a)\alpha + a, -(c-b)\alpha + c\right], \forall \alpha \in \left[0,1\right] \tag{3}$$

The distance between two triangular fuzzy numbers can be defined by the vertex method. Letting $\tilde{M}_1 = (a_1, b_1, c_1)$ and $\tilde{M}_2 = (a_2, b_2, c_2)$ be two triangular fuzzy numbers, the distance between them is:

$$d(\tilde{M}_1, \tilde{M}_2) = \sqrt{\frac{1}{3}\left[(a_1 - a_2)^2 + (b_1 - b_2) + (c_1 - c_2)^2\right]} \tag{4}$$

Many different methods have been devised to rank the fuzzy numbers, and each method has its own advantages and disadvantages [28]. A popular method is the intuition ranking method, which ranks triangular fuzzy numbers by drawing their membership function curves. A higher mean value and lower spread fuzzy number is preferred by human intuition [30]. Another popular fuzzy number ranking method is the a-cut method [2]. The Centroid ranking method is also often used to rank fuzzy numbers [49]. A fuzzy mean and spread method was proposed by Lee and Li [30] by using a generalized mean and standard deviation based on the probability measures of fuzzy events.

A good decision-making model needs to tolerate vagueness or ambiguity because fuzziness and vagueness are common characteristics in many decision-making problems [50]. Since decision makers often provide uncertain answers rather than precise values, the transformation of qualitative preferences to point estimates may not be sensible. Conventional AHP that requires the selection of arbitrary values in pair-wise comparison may not be sufficient and uncertainty should be considered in some or all pair-wise comparison values [50]. Since the fuzzy linguistic approach can take the optimism/pessimism rating attitude of decision makers into account, linguistic values, whose membership functions are usually characterized by triangular fuzzy numbers, are recommended to assess the preference ratings instead of conventional numerical equivalence method [34]. As a result, the fuzzy AHP should be more appropriate and effective than conventional AHP in real practice where an uncertain pair-wise comparison environment exists.

With different daily decision making problems of diverse intensity, the results can be misleading if the fuzziness of human decision making is not taken into account [45]. Fuzzy sets theory provides a wider frame than the classic sets theory, which has been contributing to the capability of reflecting the real world [17]. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for common- sense reasoning in decision making in the absence of complete and precise information. Fuzzy set theory is a better means for modeling imprecision arising from mental phenomena which are neither random nor stochastic. Human beings are heavily involved in the process of decision analysis. This attitude towards the imprecision of human behavior led to the study of a new decision analysis: filed fuzzy decision making [29].

Fuzzy Analytic Hierarchy Process

Analytic hierarchy process (AHP) is one of the well-known Multi-criteria decision making techniques that was first proposed by Saaty [40]. Although the classical AHP includes the opinions of experts and makes a multiple criteria evaluation, it is not capable of reflecting human's vague thoughts. The classical AHP takes into consideration the definite judgments of decision makers [47]. Although the AHP aims to capture the expert's knowledge, the traditional AHP still cannot really reflect the human thinking style [26]. And AHP method is often criticized due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process [15]. To overcome all these shortcomings, FAHP was developed, being able to solve the hierarchical problems. Decision makers usually find it to be more confident to give interval judgments than to provide fixed value judgments. This is because he/she is usually unable to explicate his/her preference about the fuzzy nature of the comparison process [27].

Literature review

The first study on FAHP was carried out by Van Laarhoven and Pedrycz [46] and in this study fuzzy ratios which were defined by triangular membership functions were compared. Buckley [7] used the comparison ratios based on trapezoidal membership functions. Stam et al [41] revealed how to use artificial intelligence techniques in the determination or quasi- determination of preference ratings in the analytic hierarchy method. Chang [9] proposed the extent analysis method based on the utilization of triangular fuzzy numbers for pair-wise comparisons. Cheng [11] put forward a new algorithm for the assessment of tactical missile systems using fuzzy AHP. Kahraman et al., [25] proposed a fuzzy objective and subjective method based on the fuzzy AHP.

Deng [15] presented a multiple criteria analysis with fuzzy pair-wise comparisons to consider qualitative evaluations. Lee et al [31] revised the main ideas underlying AHP and proposed a methodology based on stochastic optimization to ensure global coherence and take into account the fuzzy character of the comparison process. Deng [15] presented a fuzzy approach for tackling qualitative multi-criteria analysis problems in a simple and straightforward manner.

Zhu et al., [53] proved the basic theory of the triangular fuzzy number and improved the formulation of comparing the triangular fuzzy number's size. On this basis, they introduced a practical example on petroleum prospecting. Leung and Cao [33] proposed a fuzzy consistency definition with consideration of a tolerance deviation. Essentially, the fuzzy ratios of relative importance, allowing certain tolerance deviation, were formulated as constraints on the membership values of the local priorities. Chou and Liang [14] proposed a fuzzy multi-criteria decision making model by combining fuzzy set theory, AHP and the concept of entropy, for shipping company performance evaluation. Bozdag et al., [6] proposed four different fuzzy multi-attribute group decision making methods to select the best computer integrated manufacturing system. One of these methods is FAHP and the others are Yager's weighted goals method, Blin's approach and fuzzy synthetic evaluation. Chang, Cheng, and Wang [9] developed a methodology for performance evaluation of airports. They used the gray statistics method in selecting the criteria, and FAHP method in determining the weights of criteria. And finally they adopted the fuzzy synthetic and TOPSIS approaches for the ranking of airport performance. Kahraman et al. [26] used the FAHP to select the best supplier firm providing the best satisfaction for the criteria determined. Kahraman et al., [27] used four different fuzzy multiattribute group decision making approaches for facility location selection. These approaches are: a fuzzy model proposed by Blin, fuzzy synthetic evaluation, Yager's weighted goals method and FAHP respectively. Hsieh et al., [22] presented a fuzzy multi-criteria analysis approach for the selection of planning and design alternatives in public office buildings. The FAHP method is used to determine the weightings for evaluation criteria among decision makers. Mikhailov and Tsvetinov [35] applied a new fuzzy modification of the AHP for evaluating services. The proposed fuzzy prioritization method uses fuzzy pair-wise comparison judgments rather than exact numerical values of the comparison ratios and transforms initial fuzzy prioritization problem into non-linear program. Enea and Piazza [16] focused on the constraints that have to be considered within FAHP. They used constrained FAHP in the project selection. Kahraman et al., [24] used the FAHP for comparing catering firms in Turkey. The means of the triangular fuzzy numbers produced by the customers and experts for each comparison were successfully used in the pair-wise comparison matrices. Tang and Beynon [42] used FAHP method for the application and development of a capital investment study. They tried to select the type of fleet car to be adopted by a car rental company. Basgil [4] provided an analytical tool to select the best software providing the most customer satisfaction. Tang et al., [42] proposed a multi-objective model for Taiwan notebook computer distribution problem. Their model involves a mixed integer programming and fuzzy analytic hierarchy process approach. Gu and Zhu [20] constructed the fuzzy symmetry matrix as an attribute evaluation space based on fuzzy decision matrix, and improved the FAHP method using the approximate fuzzy eigenvector of such a fuzzy symmetry matrix. Wang et al., [47] made a choice in optimum maintenance strategies using the fuzzy AHP. Different maintenance strategies were evaluated for different machineries in this study. Bozbura et al., [5] proposed a fuzzy AHP model to improve the quality of the prioritization of human capital measurement indicators under the fuzziness. Lee et al., [31] utilized the fuzzy AHP and Balanced Scorecard method for assessment of an IT department in the manufacturing sector in Taiwan. Ertugrul and Karakasoglu [17] used fuzzy AHP for performance evaluation of the Turkish cement firms and ranked the involved companies in terms of their performances by applying the TOPSIS method. Tang [44] provided a fuzzy multi-objective approach for Budget allocation in an aerospace company using FAHP and Artificial Neural Network (ANN). Abdi [1] proposed FAHP model for evaluating reconfigurable machines. This model is proposed to integrate the decisive factors for the equipment selection process under uncertainty. Torfi et al., [44] presented a fuzzy multi-criteria analysis approach. They used FAHP to determine the relative weights of evaluation criteria and fuzzy TOPSIS to rank the alternatives. Zhang et al., [52] developed a fuzzy analytic hierarchical process model for building energy conservation assessment. Hsu et al., [23] used FAHP in lubricant regenerative technology selection and applied the Fuzzy Analytic Hierarchy Process to find the importance degree of each criterion as the measurable indices of the regenerative technologies. Hadi-Vencheh and Mohamadghasemi [21] proposed an integrated fuzzy analytic hierarchy process-data envelopment analysis (FAHP-DEA) for multiple criteria ABC inventory classification. Chen et al. [10] described the design of a fuzzy decision support system in multi-criteria analysis approach for selecting the best plan alternatives or strategies in environment watershed.

FAHP methodology

In this study the extent FAHP is utilized, which was originally introduced by Chang [8]. Let $X = \{x_1, x_2, ..., x_n\}$ be an object set, and $G = \{g_1, g_2, ..., g_n\}$ be a goal set. According to Chang's extent analysis, each object is taken and extent analysis for each goal is performed respectively. Therefore, *m* extent analysis values for each object can be obtained, with the following signs:

$M_{g_i}{}^1, M_{g_i}{}^2, ..., M_{g_i}{}^m, i = 1, 2, ..., n$, where $M_{g_i}{}^j (j = 1, 2, ..., m)$ are all TFNs. The steps of Chang's extent analysis [8] can be given as follows:

Step 1. The value of fuzzy synthetic extent with respect to the *i*th object is defined as:

$$\mathrm{s_k} = \sum_{j=1}^{n} m_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} m_{g_i}^j \right]^{-1} \tag{5}$$

To obtain $\sum_{j=1}^{m} M_{gi}^{\ j}$, the fuzzy addition operation of *m* extent analysis values for a particular matrix is performed as:

$$\left[\sum_{j=1}^{m}, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right] \tag{6}$$

And to obtain $\left[\sum_{j=1}^{m}, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right]$, the fuzzy addition operation of $M_{g_i}^{\ j}(j=1,2,...,m)$ values is performed as:

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{\ j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \tag{7}$$

And then the inverse of the vector above is computed as:

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{\ j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \tag{8}$$

Step 2. As $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as: $V(M_2 \geq M_1) = \sup\left\lfloor \min(\mu_{m_1}(x), \mu_{m_2}(y)) \right\rfloor$, and can be expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) \tag{9}$$

$$= \begin{cases} 1 & m_2 \geq m_1 \\ 0 & l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & otherwise \end{cases}$$

Fig. 2 (Chang, 1996) illustrates Eq. (9) where *d* is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2} . To compare M1 and M2, we need both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 3. The degree possibility for a convex fuzzy number to be greater than k convex fuzzy $M_i(i=1,2,...,k)$ numbers can be defined by

$$V(M \geq M_1, M_2, ..., M_k) = V\left[(M \geq M_1) and (M \geq M_2) and ...(M \geq M_k)\right]$$
$$= \min V(M \geq M_i)$$

Assume that $d(A_i) = \min V(S_i \geq S_k)$ for $k = 1, 2, ..., n; k \neq i$. Then the weight vector is given by:

$$W' = (d'(A_1), d'(A_2, ..., d'(A_n)))^T \ ,$$

where $A_i = (i = 1, 2, ..., n)$ are n elements.

***Step* 4.** Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2, ..., d(A_n)))^T \ ,$$

Where W is a non-fuzzy number [17].

Application

The main goal of this study is to select the optimum diameter of Kaka Reza water transmission tunnel via Fuzzy AHP approach. Kaka Reza tunnel is being built to supply water in the west Iran at Lorestan province. It is located between the city of Khorramabad and Alashtar, With the Eastern longitude at 48.15̊~49.00̊ and Eastern latitude at 33.22̊~33.52̊. To perform this task, we suggested 3 options for this tunnel's diameter first based on the information we had from the tunnel and then the subsequent technical reasoning:

***Excavation diameter 2.7 m*:** Based on the existing standards the least diameter of the water tunnel is 2.1 meters. Considering the flow rate needed for this option with velocity of 5 m/s, the proposed diameter is able to meet the project objective. With the predicted minimum required thickness of 30 cm concrete lining excavation diameter, this option is considered 2.7 m.

***Excavation diameter 4.3 m*:** It is possible to perform the loading operations and tunnel excavation processes and transportations by mechanized or conventional equipments such as loaders and trucks when this diameter is chosen.

***Excavation diameter 6 m*:** The maximum diameter which is possible to be excavated full face is 6 meters based on executive considerations.

First of all ten experts from different areas evaluated the importance of these ratios with the help of questionnaires. FAHP is utilized to determine the weights of main criteria and sub-criteria. It is proposed to take the decision makers' subjective judgments into consideration and to reduce the uncertainty and vagueness in the decision making process.

Regarding the evaluation of the diameters, 10 experts were invited to survey four alternatives using the research framework shown in Fig. 3. By the investigation and experts' opinions, we finally adopted 5 criteria. These criteria are: blasting (c1), ventilation during excavation (c2), tunnel support system (c3), tunnel performance time (c4) and tunnel performance cost (c5). The hierarchical structure of the model in application is shown in table 3.

To gather the experts' opinions, the special survey forms were sent to them. For filling out the forms, 5 points scale was used. After preparing the forms and sending them to the experienced experts, we gathered the sent forms and the required matrixes for FAHP method was acquired from them. For example blasting (c_1) pair-wise comparison matrix is given in table 1.

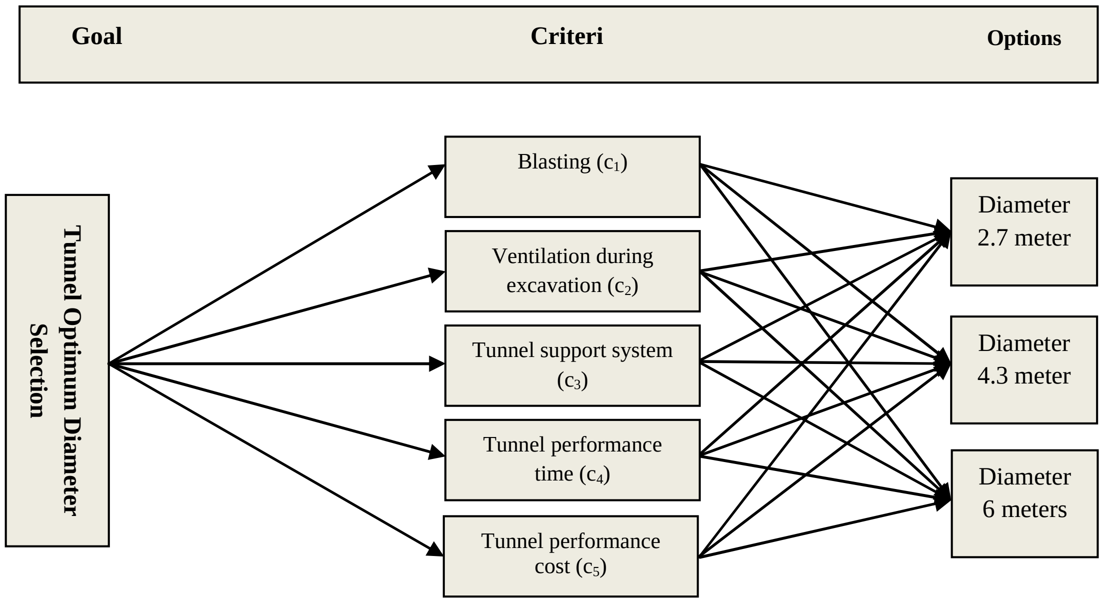

Figure 3: Hierarchical structure of model in application.

Data input and analysis

The comparison of the importance or preference of one criterion, attribute or alternative over another can be done with the help of the questionnaire. A questionnaire is designed with a conventional AHP questionnaire format (nine-point scale and pair-wise comparison) based on the hierarchy. Ten questionnaires are distributed to mining engineers in the tunneling industry.

The responses collected from the questionnaires act as input in the FAHP system, and the results are analyzed by the FAHP. Decision makers from different backgrounds may be defined different weight vectors. They usually cause not only the imprecise evaluation but also serious persecution during decision process. For this reason, we proposed a group decision based on FAHP to improve pair-wise comparison. Firstly, each decision maker carried out pair-wise comparison individually using Saaty's 1−9 scale given in table 1(Chen, 2004).

Table 1: Saaty's nine-point scale (1994).

Intensity of importance	Definition	Explanations
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over other	Experience and judgment slightly favor one activity over another
5	Essential or strong important	Experience and judgment strongly favor one activity over another
7	Demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Then, a comprehensive pair-wise comparison matrix is built in Table 1 by integrating three decision makers' grades through Eq. (12) (Chen, Lin, and Huang, 2006).

In this way, decision makers' pair-wise comparison values are transformed into triangular fuzzy numbers. Pair-wise comparison matrices of options with respect to all criteria are shown in tables 2-6 and pair-wise comparison matrix of all criteria together is shown in table 7.

Table 2: Fuzzy pair-wise comparison matrix with respect to criterion C_1.

Blasting(c_1)	2.7	4.3	6
2.7	(1,1,1)	(3,5.5,8)	(6,7,8)
4.3	(0.125,0.182,0.33)	(1,1,1)	(2,5,6)
6	(0.125,0.14,0.16)	(0.16,0.2,0.5)	(1,1,1)

Table 3: Fuzzy pair-wise comparison matrix with respect to criterion C_2

Ventilation while excavation(c_2)	2.7	4.3	6.1
2.7	(1,1,1)	(2,3.5,6)	(3,4.6,7)
4.3	(1/6,0.31,1/2)	(1,1,1)	(2,3.2,5)
6	(1/7,0.23,1/3)	(1/5,0.388,1/2)	(1,1,1)

Table 4: Fuzzy pair-wise comparison matrix with respect to criterion C_3

Tunnel support system(c_3)	2.7	4.3	6.1
2.7	(1,1,1)	(2,2.9,4)	(3,4.1,5)
4.3	(1/4,0.37,1/2)	(1,1,1)	(2,3.7,6)
6	(1/5,0.26,1/3)	(1/6,0.29,0.5)	(1,1,1)

Table 5: Fuzzy pair-wise comparison matrix with respect to criterion C_5.

Tunnel performance cost(c_5)	2.7	4.3	6.1
2.7	(1,1,1)	(1/4,0.39,1/2)	(1/5,0.28,1/2)
4.3	(2,2.7,4)	(1,1,1)	(1/6,0.29,1/2)
6	(2,3.8,5)	(2,3.7,6)	(1,1,1)

Table 6: Fuzzy pair-wise comparison matrix with respect to criterion C_4

Tunnel performance time(c_4)	2.7	4.3	6.1
2.7	(1,1,1)	(2,3.9,6)	(5,5.9,7)
4.3	(0.16,0.28,0.5)	(1,1,1)	(2,3.9,5)
6	(0.14,0.17,0.5)	(1/5,0.28,1/2)	(1,1,1)

Table 7: Fuzzy pair-wise comparison matrix with respect to all criteria.

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(2,5.1,7)	(1/6,0.2,1/4)	(1/8,0.16,1/4)	(1/9,0.12,1/7)
C_2	(1/7,0.24,1/2)	(1,1,1)	(1/8,0.15,1/6)	(1/9,0.13,1/6)	(1/9, 0.12,1/8)
C_3	(4,4.9,6)	(6,7.2,8)	(1,1,1)	(1/6,0.28,1/3)	(1/9,0.15,1/5)
C_4	(4,6,8)	(6,7.6,9)	(3,3.8,6)	(1,1,1)	(1/9,0.19,1/4)
C_5	(7,8.3,9)	(8,8.8,9)	(5,6.8,9)	(4,5.5,9)	(1,1,1)

$$\left(\tilde{x}_{ij}\right)=\left(a_{ij},b_{ij},c_{ij}\right),l_{ij}=\min\left\{a_{jk}\right\},m_{ij}=\frac{1}{k}\sum_{k=1}^{k}b_{ijk},u_{ij}=\max\left\{d_{ijk}\right\} \qquad (12)$$

After forming the fuzzy pair-wise comparison matrix, weights of all criteria and sub-criteria are determined by the help of FAHP. According to the FAHP method, synthesis values must be calculated firstly. From Table 1, synthesis values respecting the main goal are calculated like in Eq. (5)

$$S_{c_1} = (10,13.5,17) \otimes (.028,.048,.07) = (.28,.648,1.19)$$
$$S_{c_2} = (3.125,6.182,7.33) \otimes (.028,.048,.07) = (.087,.297,.513)$$
$$S_{c_3} = (1.285,1.34,1.66) \otimes (.028,.048,.07) = (.036,.064,.116)$$

These fuzzy values are compared using Eq. (8) and these values are obtained:

$$V(S_{c_1} \geq S_{c_2}) = 1, V(S_{c_1} \geq S_{c_3}) = 1, V(S_{c_2} \geq S_{c_1}) = 0.399, V(S_{c_2} \geq S_{c_3})$$
$$= 1, V(S_{c_3} \geq S_{c_1}) = 0, V(S_{c_3} \geq S_{c_2}) = 0.11$$

Then, the priority weights are calculated using Eq. (9):

$$V(S_1 \geq S_2, S_3) = Min(1,1) = 1$$
$$V(S_2 \geq S_1, S_3) = Min(0.399,1) = 0.399$$
$$V(S_3 \geq S_1, S_2) = Min(0, 0.11) = 0$$

Then again, the priority weights are calculated using Eq. (10):

$$W' = (1, 0.399, 0)^T$$

Priority weights form $W' = (1, 0.399, 0)^T$ vector. After the normalization of these values, the priority weights respecting the main goal are calculated as:

$$W_{c_1} = (0.57, 0.309, 0.634), W_{c_2} = (0.744, 0.223, 0.027),$$
$$W_{c_3} = (0.556, 0.337, 0.097)$$
$$W_{c_4} = (0.625, 0.225, 0.15), W_{c_5} = (0.57, 0.309, 0.634)$$

Then, weights of sub-criteria are calculated similarly. According to the mentioned relations and the calculated relative coefficients to each criterion, table 8 can be formed. According to table 8 and the performed calculations, tunnel with 2.7m diameter was considered the optimum diameter.

Table 8: Final calculating for optimum option.

Critteria / options	C_1	C_2	C_3	C_4	C_5	Coefficients of the options relative importance
	0.05	0.025	0.22	0.346	0.36	
2.7	0.656	0.754	0.566	0.625	0.57	0.410*
4.3	0.298	0.223	0.337	0.225	0.309	0.283
6	0.046	0.027	0.096	0.15	0.634	0.303

Conclusion

The purpose of this study is to investigate and apply the fuzzy analytic hierarchy process (FAHP) method of multi-criteria decision-making for the problem of selecting the optimum diameter for tunnels. Due to uncertainty and the fuzzy nature of the complicated problem for the decision makers, the FAHP is used to allow for the imprecision in the judgments. The issue of imprecision is reformulated in this paper which further allows a sensitivity analysis on the preferences' weights on changes in the levels of imprecision. In order to develop a new classification system for selecting the Kaka Reza water transfer tunnel excavation, the fuzzy sets theory and subsequently multifactorial fuzzy approach have been utilized. These parameters were considered for developing a new classification system for evaluating optimum diameter in the water tunnel. Utilizing the multifactorial fuzzy approach, 3 diameters have been suggested. By identifying the five criteria and determining the relative weights using FAHP, the proposed framework was applied to the given project. Selecting the optimum diameter was difficult based on the results, so we used FAHP for the appropriate selection method. Considering this method, diameter 2.7 meters is more optimal. Results show that blasting (c_1) and tunnel support system (c_3) are more important and weakest criteria respectively too. This method can be used for projects with similar conditions. This method can be employed to select the suitable diameter and significantly reduce the costs of the excavation. The existing criteria can be used in similar conditions, but to work in different conditions criteria should be changed, trying to reach the best options via appropriate analysis.

Nevertheless, difficulties in obtaining data and criteria selection may be posed as challenges for future research in related areas. In ongoing research, furthermore, it is also worth mentioning that the weights of criteria determined in this study were based on the FAHP technique; however, for a more independent result, the analytic network process (ANP) method is suggested to be adopted due to its feedback and interdependency properties. When the criteria weights and performance ratings are vague and inaccurate, the Fuzzy AHP is one of the best techniques. In addition, there exist other noteworthy MCDM investigating methods for decision making problems. This becomes one of the future research opportunities in this classical, yet still important, research area.

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