Advances in Fuzzy Mathematics
ISSN 0973-533X Volume 5, Number 3 (2010), pp. 311–316

http://www.ripublication.com/afm.htm

Homomorphism on Intuitionistic L-Fuzzy BF/BG-Subalgebras

P. Muralikrishna[1] and M. Chandramouleeswaran[2]

[1]Assistant Professor in Mathematics, Latha Mathavan Engineering College,
Madurai -625 301, Tamilnadu, India
E-mail: pmkrishna@rocketmail.com
[2]Associate Professor in Mathematics, Saiva Bhanu Kshatriya College,
Aruppukkottai - 626 101. Tamilnadu, India
E-mail: moulee59@gmail.com

Abstract

This paper discusses the image and inverse image of homomorphic (anti-homomorpihic) Intuitionistic L-fuzzy subalgebras of a *BF*-algebra.

Keywords: *BF*-algebra, Sub algebra, Fuzzy subset, L-fuzzy Subset, Intuitionistic Fuzzy Subset, Intuitionistic L-fuzzy Subset, Intuitionistic L-fuzzy *BF*-subalgebra, Homomorphism and anti-homomorphism.

1. Introduction

In 1965, Lofti A. Zadeh[1] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. As a generalization of this, Intuitionistic Fuzzy Subset was defined by K.T.Atanassov[2] in 1986.

In 1966 Y.Imai and K.Iseki[4] introduced two classes of abstract algebras; BCK-algebras and BCI-algebra. It is known that the BCK-algebras is proper sub class of the class of BCI-algebras. Neggers and H.S.Kim[5] introduced the notion of B-algebras.

With these ideas, fuzzy *BF*_subalgebras of were developed by A. Borumand Saeid and M. A. Rezvani[6] in 2009. Motivated by this, we have introduced the Intuitionistic L-fuzzy *BF*-subalgebras[3] and by this paper we investigate the image and inverse image of intuitionistic L-fuzzy *BF*-subalgebra under homomorphism and anti-homomorphism.

2. Preliminaries

In this section the basic definitions of a *BF*-algebra, L-fuzzy subset and Intuitionistic L-fuzzy subset are recalled. We start with,

Definition 2.1. A *BF*-algebra is a non-empty set *X* with a consonant 0 and a binary operation * satisfying the following axioms:

(i) $x * x = 0$

(ii) $x * 0 = x$

(iii) $0 * (x * y) = y * x \qquad \forall\, x, y \in X$

Example. 2.2. Let X = $\{0,1,2,3,4\}$ be a set with the following table :

*	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then $(X, *, 0)$ is *BF*-algebra.

Definition 2.3. A non-empty subset S of a *BF*-algebra X is said to be a subalgebra if $x * y \in S \quad \forall x, y \in S$.

Definition 2.4. A fuzzy subset μ in a non-empty set X is a function $\mu : X \to [0,1]$.

Definition 2.5. Let $L = (L, \leq)$ be complete lattice with least element 0 and greatest element 1. Then a L-fuzzy subset A of a non-empty set X is defined as a function $A : X \to L$.

Definition 2.6. An Intuitionistic Fuzzy Subset (IFS) A in a non-empty set X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > / \ x \in X\}$ where $\mu_A : X \to [0,1]$ is the degree membership and $\nu_A : X \to [0,1]$ is the degree non-membership of the element $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.7. [3] Let $L = (L, \leq)$ be complete lattice with an involutive order reversing operation $N : L \to L$. Then an Intuitionistic L-fuzzy Subset (ILFS) A in a non-empty set X is defined as an object of the form

$A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ where $\mu_A : X \to L$ is the degree membership and $\nu_A : X \to L$ is the degree non-membership of the element $x \in X$ satisfying $\mu_A(x) \le N(\nu_A(x))$.

Definition 2.8. An Intuitionistic L-fuzzy Subset A in a *BF*-algebra X with the degree membership $\mu_A : X \to L$ and the degree non-membership $\nu_A : X \to L$ is said to have Sup-Inf property if for any subset $T \subseteq X$ there exists $x_0 \in T$ such that $\mu_A(x_0) = \underset{t \in T}{Sup}\, \mu_A(t)$ and $\nu_A(x_0) = \underset{t \in T}{Inf}\, \nu_A(t)$.

Definition 2.9. Let $f : X \to Y$ be a function and A and B be the intuitionistic L-fuzzy subsets of X and Y where $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > / \ x \in Y \}$.

Then the image of A under f is defined as $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / \ y \in Y \}$ such that

$$\mu_{f(A)}(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} \mu_A(z) & if \quad f^{-1}(y) = \{x : f(x) = y\} \ne \phi \\ 0 & otherwise \end{cases}$$ and

$$\nu_{f(A)}(y) = \begin{cases} \inf\limits_{z \in f^{-1}(y)} \nu_A(z) & if \quad f^{-1}(y) = \{x : f(x) = y\} \ne \phi \\ 0 & otherwise \end{cases}$$

Definition 2.10. Let $f : X \to Y$ be a function and A and B be the intuitionistic L-fuzzy subsets of X and Y where $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ and $B = \{ < x, \mu_B(x), \nu_B(x) > / \ x \in Y \}$. Then the inverse image of B under f is defined as $f^{-1}(B) = \{ < x, \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) > / \ x \in X \}$ such that $\mu_{f^{-1}(B)}(x) = \mu_{(B)}(f(x))$ *and* $\nu_{f^{-1}(B)}(x) = \nu_{(B)}(f(x)) \quad \forall\, x \in X$.

3. Homomorphism on Intuitionistic L-Fuzzy BF-Subalgebras

Here we introduce the notions of Intuitionistic L-fuzzy *BF*-subalgebra in a *BF*-algebra X. Here after unless otherwise specified X denotes a *BF*-algebra.

Definition 3.1. [3] An Intuitionistic L-fuzzy Subset A in a *BF*-algebra X is said to be an Intuitionistic L-fuzzy *BF*-subalgebra of X if

(i) $\mu_A(x * y) \ge \mu_A(x) \wedge \mu_A(y)$

(ii) $\nu_A(x * y) \le \nu_A(x) \vee \nu_A(y) \qquad \forall\, x, y \in X$

Example.3.2. Consider the *BF*-algebra $X = \{0, 1, 2, 3, 4\}$ in Example 2.2.and $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ is the Intuitionistic L-fuzzy Subset of X defined as

$$\mu_A(x) = \begin{cases} 0.6 & ; \quad x \neq 2 \\ 0.1 & ; \quad x = 2 \end{cases} \quad and \quad \nu_A(x) = \begin{cases} 0.2 & ; \quad x \neq 2 \\ 0.8 & ; \quad x = 2 \end{cases}.$$

Then $A = \{ < x, \mu_A(x), \nu_A(x) > / \ x \in X \}$ is intuitionistic L-fuzzy *BF*-subalgebra of X.

Definition.3.3. A function $f : X \to Y$ of *BF*-algebras is called homomorphism, if $f(x * y) = f(x) * f(y) \quad \forall\, x, y \in X$ and is called anti-homomorphism, if $f(x * y) = f(y) * f(x) \quad \forall\, x, y \in X$.

Remark 3.4. If $f : X \to Y$ is a homomorphism on *BF*-algebras then $f(0_X) = 0_Y$.

Theorem 3.5. Let f be a homomorphism from *BF*-algebras X onto Y and A be an intuitionistic L-fuzzy *BF*-subalgebra of X with Sup-Inf property. Then the image of A, $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / \ y \in Y \}$ is an intuitionistic L-fuzzy *BF*-subalgebra of Y.

Proof. Let $a, b \in Y$ with $x_0 \in f^{-1}(a)$ and $y_0 \in f^{-1}(b)$ such that

$$\mu_A(x_0) = \sup_{t \in f^{-1}(a)} \mu_A(t) \quad ; \quad \mu_A(y_0) = \sup_{t \in f^{-1}(b)} \mu_A(t) \qquad \text{and}$$

$$\nu_A(x_0) = \inf_{t \in f^{-1}(a)} \nu_A(t) \quad ; \quad \nu_A(y_0) = \inf_{t \in f^{-1}(b)} \nu_A(t).$$

Now by definition,

$$\begin{aligned} \mu_{f(A)}(a * b) &= \sup_{t \in f^{-1}(a*b)} \mu_A(t) \\ &\geq \mu_A(x_0 * y_0) \\ &\geq \mu_A(x_0) \ \wedge \ \mu_A(y_0) \\ &= \sup_{t \in f^{-1}(a)} \mu_A(t) \ \wedge \ \sup_{t \in f^{-1}(b)} \mu_A(t) \\ &= \mu_{f(A)}(a) \ \wedge \ \mu_{f(A)}(b) \end{aligned}$$

Also,

$$\begin{aligned} \nu_{f(A)}(a * b) &= \inf_{t \in f^{-1}(a*b)} \nu_A(t) \\ &\leq \nu_A(x_0 * y_0) \\ &\leq \nu_A(x_0) \ \vee \ \nu_A(y_0) \\ &= \inf_{t \in f^{-1}(a)} \nu_A(t) \ \vee \ \inf_{t \in f^{-1}(b)} \nu_A(t) \\ &= \nu_{f(A)}(a) \ \vee \ \nu_{f(A)}(b) \end{aligned}$$

Hence the image $f(A) = \{ < y, \mu_{f(A)}(y), \nu_{f(A)}(y) > / \ y \in Y \}$ is an intuitionistic L-fuzzy *BF*-subalgebra of Y.

Theorem 3.6. Let f be a homomorphism from *BF*-algebras X onto Y and B be an intuitionistic L-fuzzy *BF*-subalgebra of Y. Then the inverse image of B, $f^{-1}(B)=\{< x,\mu_{f^{-1}(B)}(x),\nu_{f^{-1}(B)}(x)> / \ x\in X\}$ is an intuitionistic L-fuzzy *BF*-subalgebra of X.

Proof. Let $x, y\in X$.

Then
$$\begin{aligned}\mu_{f^{-1}(B)}(x*y) &= \mu_B\big(f(x*y)\big)\\ &= \mu_B\big(f(x)*f(y)\big)\\ &\geq \mu_B\big(f(x)\big) \ \wedge \ \mu_B\big(f(y)\big)\\ &= \mu_{f^{-1}(B)}(x) \ \wedge \ \mu_{f^{-1}(B)}(y)\end{aligned}$$

Also
$$\begin{aligned}\nu_{f^{-1}(B)}(x*y) &= \nu_B\big(f(x*y)\big)\\ &= \nu_B\big(f(x)*f(y)\big)\\ &\leq \nu_B\big(f(x)\big) \ \vee \ \nu_B\big(f(y)\big)\\ &= \nu_{f^{-1}(B)}(x) \ \vee \ \nu_{f^{-1}(B)}(y).\end{aligned}$$

Then the inverse image of $f^{-1}(B)=\{< x,\mu_{f^{-1}(B)}(x),\nu_{f^{-1}(B)}(x)> / \ x\in X\}$ is an intuitionistic L-fuzzy *BF*-subalgebra of X.

In the similar way we can prove the following.

Theorem 3.7. Let f be an anti-homomorphism from X onto Y and A be an intuitionistic L-fuzzy *BF*-subalgebra of X with Sup-Inf property. Then the image of A, $f(A)=\{< y,\mu_{f(A)}(y),\nu_{f(A)}(y)> / \ y\in Y\}$ is an intuitionistic L-fuzzy *BF*-subalgebra of Y.

Theorem 3.8. Let f be an anti-homomorphism from X onto Y and B be an intuitionistic L-fuzzy *BF*-subalgebra of Y. Then the inverse image of B, $f^{-1}(B)=\{< x,\mu_{f^{-1}(B)}(x),\nu_{f^{-1}(B)}(x)> / \ x\in X\}$ is an intuitionistic L-fuzzy *BF*-subalgebra of X.

Conclusion

In this article we have extended Intuitionistic L-fuzzy *BF*-subalgebras under homomorphism and anti-homomorphism. In [7], we have discussed the Intuitionistic L-fuzzy subalgebras of BG-algebras. One can observe that all the results proved in this paper can verbatically be proved for Intuitionistic L-fuzzy subalgebras of BG-algebras. These concepts can further be generalized.

References

[1] L.A.Zadeh, Fuzzy sets,. Inform.Control. 8(1965), 338-353.
[2] K.T. Atanassov, Intuitionistic Fuzzy sets, Fuzzy Sets and Systems (1986).
[3] M.Chandramouleeswaran and P.Muralikrishna, The Intuitionistic L-Fuzzy *BF*_Subalgebras, Global Journal Pure and Applied Mathematics.(Vol 6, Number 1 (2010), pp. 1–6.).
[4] Y.Imai and K.Iseki, On Axiom Systems Of Propositional Calculi, 15, Proc. Japan Academy, 42(1966), 19-22.
[5] Neggers and H.S.Kim, On B_algebrs, Math. Vensik, 54(2002), 21-29.
[6] A. Borumand Saeid and M. A. Rezvani On Fuzzy *BF*-Algebras, International Mathematical Forum, 4, 2009, no. 1, 13 – 25.
[7] M.Chandramouleeswaran and P.Muralikrishna, On Intuitionistic L-Fuzzy Subalgebras of BG-algebras, International Mathematical Forum, 5, 2010, no. 20, 995 – 1000.