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More on pairwise fuzzy baire spaces

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Abstract

In this paper several characterizations of pairwise fuzzy Baire spaces are studied in terms of pairwise fuzzy semi-closed sets and pairwise fuzzy semi-open sets.

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1. Introduction

The theory of fuzzy sets was initiated by L.A. Zadeh in his classical paper [16] in 1965 as an attempt to develop a mathematically precise framework to treat systems or phenomena which cannot themselves be characterized precisely. In Mathematics, topoloy provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by C.L.Chang [6] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1989, A.Kandil [7] introduced the concept of fuzzy topological space as a generalization of fuzzy topological spaces. The class of Baire bitopological spaces have been studied extensively in classical topology in [1], [2] and [4]. S. Sampath Kumar

[8] defined a (τ_i, τ_j)-fuzzy semi-open set and a (τ_i, τ_j)-fuzzy pre-open set in fuzzy bitopological space. The concept of pairwise fuzzy Baireness in fuzzy bitopological space was introduced and studied by the authors in [12]. The purpose of this paper is to study several characterizations of pairwise fuzzy Baire spaces. The conditions for the pairwise fuzzy first category set to be pairwise fuzzy semi-closed in a fuzzy bitopological space are also established in this paper. The characterization of pairwise fuzzy first category space in terms of pairwise fuzzy semi-closed set is also obtained in this paper.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are two fuzzy topologies on a non-empty set X. Throughout this paper, the indices i and j take values in $\{1, 2\}$ and $i \neq j$.

Definition 2.1. Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define:

(i) $\lambda \vee \mu : \mathrm{X} \to [0,1]$ as follows: $(\lambda \vee \mu)(x) = \max \{\lambda(\mathrm{x}), \mu(\mathrm{x})\}$;

(ii) $\lambda \wedge \mu : \mathrm{X} \to [0, 1]$ as follows: $(\lambda \wedge \mu)(x) = \min \{\lambda(\mathrm{x}), \mu(\mathrm{x})\}$;

(iii) $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2. [3] Let (X, T) be a fuzzy topological space. For a fuzzy set λ of X, the interior and the closure of λ are defined respectively as $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Definition 2.3. [9] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a *pairwise fuzzy dense set* if $cl_{T_1}(cl_{T_2}(\lambda)) = cl_{T_2}(cl_{T_1}(\lambda)) = 1$.

Definition 2.4. [12] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy nowhere dense set* if $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$.

Definition 2.5. [15] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a *pairwise fuzzy open set* if $\lambda \in T_1$ and $\lambda \in T_2$.

Definition 2.6. [15] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a *pairwise fuzzy closed set* if $1 - \lambda$ is a pairwise fuzzy open set.

Definition 2.7. [8] Let μ be a fuzzy set on a fuzzy bitopological space X. Then we call μ;

(1) a (τ_i, τ_j)-fuzzy semiopen $[(\tau_i, \tau_j)$-fso$]$ set on X if $\mu \leq \tau_j - Cl(\tau_i - Int\mu)$,

(2) a (τ_i, τ_j)-fuzzy semiclosed $[(\tau_i, \tau_j)$-fsc$]$ set on X if $\tau_j - Int(\tau_i - Cl\mu) \leq \mu$.

Definition 2.8. [12] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called *pairwise fuzzy first category set* if $\lambda = \vee_{k=1}^{\infty} \lambda_k$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). A fuzzy set which is not a pairwise fuzzy first category set, is called a *pairwise fuzzy second category set* in (X, T_1, T_2).

Definition 2.9. [12] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a *pairwise fuzzy residual set* if its complement is a pairwise fuzzy first category set.

3. Pairwise fuzzy Baire spaces

Definition 3.1. [12] A fuzzy bitopological space (X, T_1, T_2) is called a *pairwise fuzzy Baire* if $int_{T_i}\big(\vee_{k=1}^{\infty} (\lambda_k)\big) = 0$, $(i = 1, 2)$ where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Theorem 3.2. [12] Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:

(i). (X, T_1, T_2) is a pairwise fuzzy Baire space.

(ii). $int_{T_i}(\lambda) = 0$, (i=1,2) for every pairwise fuzzy first category set λ in (X, T_1, T_2).

(iii). $cl_{T_i}(\mu) = 1$, (i=1,2) for every pairwise fuzzy residual set μ in (X, T_1, T_2).

Theorem 3.3. [13] If λ is a pairwise fuzzy residual set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set μ in (X, T_1, T_2), then μ is a pairwise fuzzy residual set in (X, T_1, T_2).

Proposition 3.4. Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:

(i). (X, T_1, T_2) is a pairwise fuzzy Baire space.

(ii). Each non-zero pairwise fuzzy open set is a pairwise fuzzy second category set in (X, T_1, T_2).

(iii). No non-zero pairwise fuzzy closed set is a pairwise fuzzy residual set in (X, T_1, T_2).

Proof. $(i) \implies (ii)$ Let (X, T_1, T_2) be a pairwise fuzzy Baire space. Suppose that λ is a non-zero pairwise fuzzy open set in (X, T_1, T_2) such that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 3.2, $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$. Since λ is a non-zero pairwise fuzzy open set in (X, T_1, T_2), we have, $int_{T_i}(\lambda) = \lambda$ and hence we have $int_{T_i}(\lambda) \neq 0$ $(i = 1, 2)$, a contradiction to $int_{T_i}(\lambda) = 0$ $(i = 1, 2)$. Therefore, no non-zero pairwise fuzzy open set is a pairwise fuzzy first category set in (X, T_1, T_2). Hence each non-zero pairwise fuzzy open set is a pairwise fuzzy second category set in (X, T_1, T_2).

$(ii) \implies (iii)$ Suppose that μ is a pairwise fuzzy closed set such that μ is a pairwise fuzzy residual set in (X, T_1, T_2). Then $1 - \mu$ is a pairwise fuzzy open set in (X, T_1, T_2) and $1 - \mu$ is a pairwise fuzzy first category set in (X, T_1, T_2). Thus, we have the pairwise fuzzy open set $1 - \mu$ is a pairwise fuzzy first category set in (X, T_1, T_2), a contradiction to the hypothesis that each non-zero pairwise fuzzy open set is a pairwise fuzzy category set in (X, T_1, T_2). Hence no non-zero pairwise fuzzy closed set is a pairwise fuzzy residual set in (X, T_1, T_2).

$(iii) \implies (i)$ Suppose that (X, T_1, T_2) is not a pairwise fuzzy Baire space. Then, by theorem 3.2, we have $cl_{T_i}(\mu) \neq 1$ $(i = 1, 2)$ for a pairwise fuzzy residual set μ in (X, T_1, T_2). Since $cl_{T_i}(\mu) \neq 1$, there exists a non-zero pairwise fuzzy closed set λ in (X, T_1, T_2) such that $\mu \leq \lambda$. Since μ is a pairwise fuzzy residual set and $\mu \leq \lambda$, by theorem 3.3, λ is a pairwise fuzzy residual set in (X, T_1, T_2), a contradiction to the hypothesis that no non-zero pairwise fuzzy closed set is a pairwise fuzzy residual set in (X, T_1, T_2). Hence we must have $cl_{T_i}(\mu) = 1$ $(i = 1, 2)$ for each pairwise fuzzy residual set μ in (X, T_1, T_2). Then, by theorem 3.2, (X, T_1, T_2) is a pairwise fuzzy Baire space. ■

Proposition 3.5. Let (X, T_1, T_2) be a fuzzy bitopological space. If λ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2), then λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Then $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$ and therefore $int_{T_1}(cl_{T_2}(\lambda)) \leq \lambda$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq \lambda$. Hence λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2). ■

Proposition 3.6. If λ is a pairwise fuzzy open and T_j (j=1,2) fuzzy dense set in (X, T_1, T_2), then λ is a pairwise fuzzy semi-open set in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy open and T_j $(j = 1, 2)$ fuzzy dense set in (X, T_1, T_2). Then $cl_{T_1}(\lambda) = cl_{T_2}(\lambda) = 1$. Now $int_{T_1}(cl_{T_2}(1 - \lambda)) = 1 - cl_{T_1}(int_{T_2}(\lambda)) = 1 - cl_{T_1}(\lambda) = 1 - 1 = 0$. Also $int_{T_2}(cl_{T_1}(1 - \lambda)) = 1 - cl_{T_2}(int_{T_1}(\lambda)) = 1 - cl_{T_2}(\lambda) = 1 - 1 = 0$. Hence, we have $int_{T_1}(cl_{T_2}(1 - \lambda)) = int_{T_2}(cl_{T_1}(1 - \lambda)) = 0$. This implies that $1 - \lambda$ is a pairwise fuzzy nowhere dense set. Therefore by proposition 3.5, $1 - \lambda$ is a pairwise fuzzy semi-closed set in (X, T_1, T_2) and hence λ is a pairwise fuzzy semi-open set in (X, T_1, T_2). ■

Proposition 3.7. If λ is a pairwise fuzzy open and T_j $(j = 1, 2)$ fuzzy dense set in (X, T_1, T_2) and $\mu \leq 1 - \lambda$ then μ is a pairwise fuzzy semi-closed set in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy open and T_j $(j = 1, 2)$ fuzzy dense set in (X, T_1, T_2). Then as in the proof of proposition 3.6, $1 - \lambda$ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2) and hence we have $int_{T_1}(cl_{T_2}(1-\lambda)) = int_{T_2}(cl_{T_1}(1-\lambda)) = 0$. Now $\mu \leq 1-\lambda$ implies that $int_{T_1}(cl_{T_2}(\mu)) \leq int_{T_1}(cl_{T_2}(1-\lambda)$ and $int_{T_2}(cl_{T_1}(\mu)) \leq int_{T_2}(cl_{T_1}(1-\lambda)$. Then we have $int_{T_1}(cl_{T_2}(\mu)) = int_{T_2}(cl_{T_1}(\mu)) = 0$ and this implies that μ is a pairwise fuzzy nowhere dense set in (X, T_1, T_2). Therefore by proposition 3.5, μ is a pairwise fuzzy semi-closed set in (X, T_1, T_2). ■

The following proposition gives a pairwise fuzzy first category set in terms of pairwise fuzzy semi-closed sets in a fuzzy bitopological space.

Proposition 3.8. If λ is a pairwise fuzzy first category set in (X, T_1, T_2), then $\lambda = \vee_{k=1}^{\infty}\lambda_k$, where (λ_k)'s are pairwise fuzzy semi-closed sets in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then $\lambda = \vee_{k=1}^{\infty}\lambda_k$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). By proposition 3.5, the pairwise fuzzy nowhere dense sets (λ_k)'s are pairwise fuzzy semi-closed sets in (X, T_1, T_2) and hence $\lambda = \vee_{k=1}^{\infty}\lambda_k$, where (λ_k)'s are pairwise fuzzy semi-closed sets in (X, T_1, T_2). ■

The following proposition gives a pairwise fuzzy residual set in terms of pairwise fuzzy semi-open sets in a fuzzy bitopological space.

Proposition 3.9. If λ is a pairwise fuzzy residual set in (X, T_1, T_2) then, $\lambda = \wedge_{k=1}^{\infty}\mu_k$, where (μ_k)'s are pairwise fuzzy semi-open sets in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy residual set in (X, T_1, T_2). Then $1 - \lambda$ is a pairwise fuzzy first category set and $1 - \lambda = \vee_{k=1}^{\infty}(\lambda_k)$ where λ_k's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Then $\lambda = 1 - (\vee_{k=1}^{\infty}(\lambda_k)) = \wedge_{k=1}^{\infty}(1 - \lambda_k)$. By proposition 3.5, the pairwise fuzzy nowhere dense sets λ_k's are pairwise fuzzy semi-closed sets and hence $(1 - \lambda_k)$'s are pairwise fuzzy semi-open sets in (X, T_1, T_2). Let $\mu_k = 1 - \lambda_k$. Then we have $\lambda = \wedge_{k=1}^{\infty}\mu_k$, where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2). ■

Proposition 3.10. If μ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$, for a fuzzy set λ in (X, T_1, T_2), then λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2).

Proof. Let μ be a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2). Then, we have $int_{T_1}(cl_{T_2}(\mu)) = int_{T_2}(cl_{T_1}(\mu)) = 0$. Now $\lambda \leq \mu$, implies that $int_{T_1}(cl_{T_2}(\lambda)) \leq int_{T_1}(cl_{T_2}(\mu))$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq int_{T_2}(cl_{T_1}(\mu))$, hence

$int_{T_1}(cl_{T_2}(\lambda)) \leq 0$ and $int_{T_2}(cl_{T_1}(\lambda)) \leq 0$. That is, $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Therefore λ is a pairwise fuzzy nowhere dense set and by proposition 3.5, λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2). ■

Proposition 3.11. If μ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set λ in (X, T_1, T_2), then $\lambda = \vee_{k=1}^{\infty} \nu_k$, where ν_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2).

Proof. Let μ be a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2). Then, we have $\mu = \vee_{k=1}^{\infty} \mu_k$, where μ_k's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Now $\lambda \wedge \mu = \lambda \wedge (\vee_{k=1}^{\infty} \mu_k) = \vee_{k=1}^{\infty} (\lambda \wedge \mu_k)$. Also $\lambda \leq \mu$, implies that $\lambda \wedge \mu = \lambda$. Therefore $\lambda = \vee_{k=1}^{\infty} (\lambda \wedge \mu_k)$. Let $\nu_k = \lambda \wedge \mu_k$. Since $\nu_k = \lambda \wedge \mu_k \leq \mu_k$ and μ_k's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2), by Proposition 3.10, ν_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2). Hence $\lambda = \vee_{k=1}^{\infty} (\nu_k)$, where (ν_k)'s are pairwise fuzzy semi-closed sets in (X, T_1, T_2). ■

Proposition 3.12. If λ is a pairwise fuzzy residual set in a fuzzy bitopological space (X, T_1, T_2) and if $\lambda \leq \mu$ for a fuzzy set μ in (X, T_1, T_2), then $\mu = \wedge_{k=1}^{\infty} \mu_k$, where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy residual set in a fuzzy bitopological space (X, T_1, T_2). Then, $1 - \lambda$ is a pairwise fuzzy first category set in (X, T_1, T_2). Now $\lambda \leq \mu$, for a fuzzy set μ in (X, T_1, T_2), implies that $1 - \lambda \geq 1 - \mu$. Then, by proposition 3.8, $1 - \mu = \vee_{k=1}^{\infty} \nu_k$, where ν_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2). Now $\mu = 1 - (\vee_{k=1}^{\infty} \nu_k) = \wedge_{k=1}^{\infty} (1 - \nu_k)$. Let $\mu_k = (1 - \nu_k)$. Since ν_k's are pairwise fuzzy semi-closed sets, $\mu_k = (1 - \nu_k)$'s are pairwise fuzzy semi-open sets in (X, T_1, T_2). Hence $\mu = \wedge_{k=1}^{\infty} \mu_k$, where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2). ■

Proposition 3.13. If the pairwise fuzzy first category set λ, is a pairwise fuzzy closed set, in a pairwise fuzzy Baire space (X, T_1, T_2), then λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2).

Proof. Let λ be a pairwise fuzzy first category set in a pairwise fuzzy Baire space (X, T_1, T_2) such that $cl_{T_i}(\lambda) = \lambda$ ...(1) (i = 1,2) By theorem 3.2, $int_{T_i}(\lambda) = 0$ ...(2) (i = 1, 2), for the pairwise fuzzy first category set λ in (X, T_1, T_2). Then, from (1) and (2), we have $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Hence λ is a pairwise fuzzy nowhere dense set and by proposition 3.5, λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2). ■

Theorem 3.14. [13] If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then each pairwise fuzzy residual set is a pairwise fuzzy dense set in (X, T_1, T_2).

The following propositions give the conditions for pairwise fuzzy first category sets to be pairwise fuzzy semi-closed sets in (X, T_1, T_2).

Proposition 3.15. If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy submaximal and pairwise fuzzy Baire space, then each pairwise fuzzy first category set in (X, T_1, T_2) is a pairwise fuzzy semi-closed set in (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be a pairwise fuzzy submaximal and pairwise fuzzy Baire space and λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then, $1-\lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 3.14, we have $1-\lambda$ is a pairwise fuzzy dense set. Also since (X, T_1, T_2) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense set $1-\lambda$ is a pairwise fuzzy open set in (X, T_1, T_2) and hence we have $1-\lambda \in T_i$ $(i = 1, 2)$. Hence the pairwise fuzzy first category set λ is a pairwise fuzzy closed set in (X, T_1, T_2). Then, by proposition 3.13, λ is a pairwise fuzzy semi-closed set in (X, T_1, T_2). ■

Proposition 3.16. If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable and pairwise fuzzy Baire space, then each pairwise fuzzy first category set in (X, T_1, T_2) is a pairwise fuzzy semi-closed set in (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable and pairwise fuzzy Baire space and λ be a pairwise fuzzy first category set in (X, T_1, T_2). Then, $1-\lambda$ is a pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by Theorem 3.14, we have $1-\lambda$ is a pairwise fuzzy dense set . Also since (X, T_1, T_2) is a pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set $1-\lambda$, we have $cl_{T_1}(int_{T_2}(1-\lambda)) = cl_{T_2}(int_{T_1}(1-\lambda)) = 1$. Then $int_{T_1}(cl_{T_2}(\lambda)) = int_{T_2}(cl_{T_1}(\lambda)) = 0$. Hence the pairwise fuzzy first category set λ is a pairwise fuzzy nowhere dense set and by proposition 3.5, pairwise fuzzy semi-closed set in (X, T_1, T_2). ■

Theorem 3.17. [14] Let (X, T_1, T_2) be a pairwise fuzzy strongly irresolvable space. Then λ is a pairwise fuzzy dense set in (X, T_1, T_2) if and only if $1-\lambda$ is a pairwise fuzzy nowhere dense set.

Proposition 3.18. If λ is a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then λ is a pairwise fuzzy semi-open set.

Proof. Let λ be a pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable space (X, T_1, T_2). Then by Theorem 3.17, $1-\lambda$ is a pairwise fuzzy nowhere dense set. Hence by proposition 3.5, $1-\lambda$ is a pairwise fuzzy semi-closed set and so λ is a pairwise fuzzy semi-open set. ■

Proposition 3.19. If (X, T_1, T_2) is a pairwise fuzzy Baire space, then $int_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, (j=1,2) where λ_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be a pairwise fuzzy Baire space. Then by theorem 3.2, we have $int_{T_j}(\lambda) = 0$, $(j = 1, 2)$ for a pairwise fuzzy first category set λ in (X, T_1, T_2) and by proposition 3.8, $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where λ_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2). Therefore, we have $int_{T_j}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(j = 1, 2)$ where λ_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2). ■

Proposition 3.20. If (X, T_1, T_2) is a pairwise fuzzy Baire space, then $cl_{T_j}(\wedge_{k=1}^{\infty}\mu_k) = 1$, $(j = 1, 2)$ where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be a pairwise fuzzy Baire space. Then by theorem 3.2, we have $cl_{T_j}(\mu) = 1$, $(j = 1, 2)$ for a pairwise fuzzy residual set μ in (X, T_1, T_2) and by proposition 3.9, $\mu = \wedge_{k=1}^{\infty}\mu_k$, where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2). Therefore $cl_{T_j}(\wedge_{k=1}^{\infty}\mu_k) = 1$, $(j = 1, 2)$ where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2). ∎

Proposition 3.21. If (X, T_1, T_2) is a pairwise fuzzy Baire space, then $cl_{T_j}(\wedge_{k=1}^{\infty}\mu_k) = 1$, $(j = 1, 2)$ where μ_k's are pairwise fuzzy dense sets in (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be a pairwise fuzzy Baire space. Then by proposition 3.20, we have $cl_{T_j}(\wedge_{k=1}^{\infty}\mu_k) = 1$ (j=1,2), where μ_k's are pairwise fuzzy semi-open sets in (X, T_1, T_2). Now $cl_{T_j}(\wedge_{k=1}^{\infty}\mu_k) \leq \wedge_{k=1}^{\infty}(cl_{T_j}(\mu_k))$, implies that $1 \leq \wedge_{k=1}^{\infty}(cl_{T_j}(\mu_k))$. That is $\wedge_{k=1}^{\infty}(cl_{T_j}(\mu_k)) = 1$, hence $cl_{T_j}(\mu_k) = 1$ and so $cl_{T_1}(cl_{T_2}(\mu_k)) = cl_{T_2}(cl_{T_1}(\mu_k)) = 1$. Therefore we have $cl_{T_j}(\wedge_{k=1}^{\infty}\mu_k) = 1$, $(j = 1, 2)$ where μ_k's are pairwise fuzzy dense sets in (X, T_1, T_2). ∎

Proposition 3.22. If (X, T_1, T_2) is a pairwise fuzzy first category space then, $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where λ_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be a pairwise fuzzy first category space. Then $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where λ_k's are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). By proposition 3.5, the pairwise fuzzy nowhere dense sets λ_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2) and hence $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where λ_k's are pairwise fuzzy semi-closed sets in (X, T_1, T_2). ∎

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