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Introduction to g-Fuzzy Product Topology

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Abstract

The objective of this paper is to introduce product topology on g-fuzzy topological spaces. Also we make an attempt to obtain some properties of g-fuzzy Product topological Spaces.

Key words: g-fuzzy topology, Base for fuzzy topological space, g-fuzzy Subspace Topology, g-fuzzy Compactness, g-fuzzy connectedness, g-fuzzy Hausdorff space and g-fuzzy product topology.

1. Introduction

Zadeh[12] introduced the concept of fuzzy set which motivated a lot of mathematical activity on the generalization of the notion of fuzzy set. The fuzzy topological space was introduced and developed by Chang and since then various notions in classical topology have been extended to fuzzy topological spaces. Mathews and Samuel [9] suggested an alternate and more general definition of fuzzy topological spaces called g-fuzzy topological spaces.

2. Basic Concepts

A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.1 The *sum* of two fuzzy sets A and B in a set X,denoted by $A\oplus B$,is a fuzzy set in X defined by $(A\oplus B)(x) = \min(1, A(x) + B(x))$ for all $x\in X$.

The *conjunction* of two fuzzy sets A and B,denoted by A&B, is a fuzzy set in X defined by $(A\&B)(x) = \max(0,A(x)+B(x)-1)$ for all $x \in X$

Definition 2.2 Let X be a non empty set. A family T of fuzzy sets in X is called a g-*fuzzy topology* on X if

1 $X \in T$ and $\Phi \in T$

2 $A \& B \in T$ whenever $A, B \in T$ and

3 $(\oplus_\alpha A_\alpha) \in T$ for any subfamily $\{A_\alpha\}_{\alpha \in J}$ in T

The set X together with a g-fuzzy topology T, denoted by (X, T) is called a g-*fuzzy topological space* or gfts. Members of T are called g-*open* fuzzy sets in X. The complement of a g-open fuzzy set is called g-*closed* fuzzy set.

Remark 2.3 If $A, B \in P(X)$, then $A \oplus B = A \cup B$, $A \& B = A \cap B$ and $A \theta B = A \setminus B$. Thus the ordinary topology and ordinary topological spaces become special cases of g-fuzzy topology and g-fuzzy topological spaces.

Definition 2.4 Let (X,T) and (Y,S) be two g-fuzzy topological spaces and $f : X \to Y$ be a function. Then f is said to be a g-fuzzy continuous function if $f^{-1}(V) \in T$ for each $V \in S$.

The g-fuzzy topology on X can be specified by a smaller collection of fuzzy subsets called base of the g-fuzzy topology.

Definition 2.5 Let (X, T) be a gfts. A sub family ß of T is called a *base* for T, if and only if, for each A in T, there exists $(A_i)_{i \in J} \subset$ ß such that $A = (\oplus_{i \in J} A_i$

Definition 2.6 Let A and $\{A_\alpha\}_{\alpha \in J}$ be fuzzy sets in X.Then $\{A_\alpha : \alpha \in J\}$ is called a *cover* of A iff $\oplus\{A_\alpha : \alpha \in J\} \supset A$. If there exists a subset J_1 of J such that $\oplus \{A_\alpha : \alpha \in J_1\} \supset A$, then $\{A_\alpha : \alpha \in J\}$ is called a sub cover.

Definition 2.7 A fuzzy set K of a g-fuzzy topological space X is g-*fuzzy compact* if every g-fuzzy open cover of K has a finite sub cover.

Definition 2.8 A fuzzy topological space X is g-*fuzzy compact* if each fuzzy open cover of X has a finite fuzzy sub cover.

Definition 2.9 Let A, B are fuzzy sets of a g-fuzzy topological space X. Then, A and B are *separated* if $A^- \& B = A \& B^- = \Phi$.

A fuzzy set F is *connected* if there does not exist separated $A, B \in I(X) \setminus \Phi$ such that $F = A \oplus B$. (X, T) is connected if the largest fuzzy subset X is connected.

Definition 2.10 A gfts (X,δ) is said to have the *Hausdorff property* or to be a *Hausdorff* if for each pair $x, y \in X$ with $x \neq y$, implies that there exist fuzzy open sets μ and ν with $\mu(x) = \underline{1} = \nu(y)$ and $\mu \& \nu = \underline{0}$.

Definitions 2.11 Let (X, T) be a g-fts and $Y \subset X$; then we call the family $T_Y = \{U \& Y : U \in T\}$ which is a g-fuzzy topology for Y, the relative g-fuzzy topology.

T_Y contains Φ and Y because $\Phi = Y \& \Phi$ and $Y = Y \& X$, Φ and X are elements of T.

It is closed under finite disjunctions and arbitrary sums of g-fuzzy open sets follows from the equations $\&_{i=1}^{n} (A_i \& Y) = (\&_{i=1}^{n} A_i) \& Y$ and $\oplus_{i\in J} (A_i \& Y) = (\oplus_{i\in J} A_i) \& Y$

Such a fuzzy topological space (Y, T_Y) is called a *g-fuzzy subspace* of (X, T).

Note 2.12 g-fuzzy open set in T with respect to T_Y need not be g-fuzzy open set with respect to T.

Example 2.13 Let X = {a, b, c, d}. Define A, B, C: X → [0, 1] by
A (x) = 1 if x = a, c, d and 0 elsewhere
B (x) = 1 if x = a, b, d and 0 elsewhere
C (x) = 1 if x = a, d and 0 elsewhere
Let T = {Φ, X, A, B,C }.Then, (X,T) is a g-fts.
Let Y = {a, b, c} and $T_Y = \{U\& Y: U \in T\}$
= {Φ, Y, A&Y, B&Y, C&Y} is a g-fuzzy topology on Y.
The theorem follows immediately from the results
$(A\&Y) \oplus (B\&Y) = Y$, $(A\&Y) \oplus (C\&Y) = A \&Y$
$(B\&Y) \oplus (C\&Y) = B \&Y$, $(A\&Y) \& (B\&Y) = C\& Y$ and so on.

Theorem 2.14 If ß is a base for the f-fuzzy topology of X, then the collection
$ß_Y = \{B\& Y / B \in ß\}$ is a base for the g-fuzzy subspace topology on Y.

Proof: Let U be a g-open fuzzy set in X and $y \in U \& Y$. We can choose an element B of ß such that $y\in B \subset U$. Then, $y \in B \& Y \subset U \& Y$. Hence $ß_Y$ is a base for the g-fuzzy subspace topology on Y.

Theorem 2.15 Let Y be a g-fuzzy subspace of X. If U is g-fuzzy open in Y and Y is g-open in X, and then U is g-open in X

Proof: Since U is g-fuzzy open in Y, U = Y & V for some fuzzy set V g-open in X. since both Y and V both g-fuzzy open in X, so is Y&V.

3. g-fuzzy product topology on $X \times Y$

Definition 3.1 Let X and Y be non-empty sets.
Let $A \in I(X)$ and $B \in I(Y)$. Then by $A \times B$ we denote the fuzzy set in $X \times Y$ for which $(A\times B) (x, y) = \min \{A(x), B(y)\}$ for every $(x, y)\in X \times Y$

Example 3.2 Let X = Y = I. Consider fuzzy sets A and B of I defined as
A(x) = 1/6, if x = 2/3 and 0 otherwise.
B(x) = 2/5, if x = 4/5 and 0 otherwise.

Then, $A \times B$ is given by $(A \times B)(x, y) = \min\{1/6, 2/5\} = 1/6$ if $(x, y) = (2/3, 4/5)$ and 0 otherwise

Definition 3.3 Let X and Y be g-fuzzy topological spaces.
The g-fuzzy product topology on $X \times Y$ is the topology having as basis the collection β of all fuzzy sets of the form $U \times V$ where U is g-fuzzy open set of X and V is a g-fuzzy open set of Y.
The collection β is a basis because $X \times Y$ is itself a basis element and the intersection of any two basis elements $U \times V$ and $U_1 \times V_1$ is another basis element: For
$(U \times V) \& (U_1 \times V_1) = (U \& U_1) \times (V \& V_1)$
is a basis element because $U \& U_1$ and $(V \& V_1)$ are g-fuzzy open in X and Y respectively.

Theorem 3.4 If A is a g-fuzzy subspace of X and B is a g-fuzzy subspace of Y, then the g-fuzzy product topology on $A \times B$ is the same as the g-fuzzy topology $A \times B$ inherits as a fuzzy subspace of $X \times Y$

Proof: The fuzzy set $U \times V$ is the general basis element for $X \times Y$ where U is g-fuzzy open set of X and V is a g-fuzzy open set of Y. Therefore $(U \times V) \& (A \times B)$ is the general basis element for the g-fuzzy subspace topology on $A \times B$. Now
$(U \times V) \& (A \times B) = (U \& A) \times (V \& B)$.
Since $U \& A$ and $V \& B$ are the general open sets for the subspace topologies on A and B respectively, the set $(U \& A) \times (V \& B)$ is the general basis element for the g-fuzzy product topology on $A \times B$.
Thus, the bases for the g-fuzzy subspace topology on $A \times B$ and for the g-fuzzy product topology on $A \times B$ are the same. Hence the topologies are the same.

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