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Solving A Multi-Objective And Multi-Index Real Life Fuzzy Transportation Problem Using Modified Fuzzy Programming Technique

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Abstract

The paper presents an application of the method-Modified Fuzzy Programming Technique(MFPT) for the Fuzzy optimal solution to a real life multi-objective and multi-index Fuzzy Transportation Problem with Fuzzy parameters in terms of Triangular Fuzzy Numbers (TFN) without defuzzifying the problem. The problem is formulated where parameters of the different objective functions like transportation cost, deterioration rate and underused capacity are taken as triangular fuzzy numbers and supply and demand parameters are also taken as TFN. The proposed method is applied for the first time to solve a multi-objective and multi-index fuzzy transportation problem. The supply and demand parameters are first defuzzified.The three different objective functions are obtained for each objective function and then solved by the MFPT.The method is demonstrated by a numerical illustration and the results are compared with that of the existing ones.

Keywords: Modified Fuzzy Programming Technique, Multi-objective and Multi-index Fuzzy Transportation Problem, Triangular fuzzy numbers.

1. Introduction

The Transportation Problem (TP) is a special type of linear programming problem which is concerned with the transportation of commodities (raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized. There are effective classical methods for solving the transportation problems when all the decision parameters, i. e the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there can be many diverse circumstances. The decision makers generally do not have exact, accurate and complete information related to different parameters in such situations. The conventional and classical transportation methods are not sufficient to meet the requirements of the decision makers in such cases. Thus, the uncertainties are common in real world. These uncertainties are handled with some efficient methods to solve such optimization problems.

Bellman [1] and Zadeh [2] introduced the fuzzy concept. Zimmermann [3], [4] extended the fuzzy set theory and its applications in linear programming. Chanas et al.[5] applied the fuzzy set theory to the transportation problems and solved them by using parametric approach. Chen Shan Huo [6] introduced the concept of function principle for operations on fuzzy numbers. The Graded Mean Integration Representation Method, used to defuzzify the fuzzy transportation cost, was also introduced by Chen Shan Huo [5].Using Fuzzy valued data different works on fuzzy TP have been done so far. S. Chanas [6] performed parametric programming in fuzzy linear programming. M. O'he'igeartaigh [7] proposed an algorithm to solve fuzzy transportation problems withtriangular fuzzy numbers.S. Chanas, W. Kolodziejczyk, A. Machaj [8] solved fuzzy transportation problems by integer programming. S. Chanas, D. Kuchta [9] performed fuzzy goal programming. E. E. Ammar, E. A. Youness [10] performed multi-objective and multi-item solid fuzzy transportation problems. H. Basirzadeh [11] has done multi-objective fuzzy solid transportation problems. H. A. Barough [12] solved multi-objective fuzzy transportation problem using goal programming approach. A. Chakraborty [13] performed fuzzy transportation with cost – time minimization by fuzzy parametric programming. A. Kaur, A. Kumar [14] introduced a new method for solving fuzzy transportation problems using ranking functions. Dalbinder et al. [15] solved Multi-objective Multi-index real life transportation problem with crisp objective function and interval valued supply and destination parameters. Dalbinder et al. [16] used interval valued objective function as well as supply and destination parameters in Multi-objective Multi-index real life transportation problem. Dalbinder et al. [17] applied goal programming approach to multi-index real life transportation problem with crisp objective function and interval valued supply and destination parameters. Dalbinder et al. [18] applied fuzzy non-linear goal programming in transportation problems. Dalbinder et al. [19] shows application of various membership functions to fuzzy transportation problems. B. Ramesh Kumar, S. Murugesan [20] used Modified Revised Simplex Method for fuzzy transportation problems with Triangular Fuzzy data. P. Kundu, S. Kar, M. Maiti [21] solved multi-objective solid fuzzy transportation problems by two methods-fuzzy programming technique and global criterion method. A.Ojha, S. Kr. Mondal, M. Maiti [22] presented Transportation Policies for single and multi-objective transportation problem using fuzzy logic. Lush Li, K. K Lai [23] presented afuzzy approach to solve multi-objective transportation problems. K. Maity, M. Maity [24] focused on fuzzy optimal control theory. M. K. Maiti [25] applied fuzzy genetic algorithm. Generally, the existing fuzzy linear techniques provide crisp solutions or provide the solutions after converting to crisp numbers. Liu and Kao [26] described a method to solve a Fuzzy Transportation problem based on extension principle. Gani Nagoor and Razak Abdul [27] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Pandian and Natarajan[28] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. Most of the works in fuzzy transportation problems are done by defuzzifying the fuzzy parameters. A. N. Gani, E. Samuel and Anuradha [29] used Arshamkhan’s Algorithm to solve a Fuzzy Transportation problem without the classical defuzzification method and the problem is solved by a revised simplex method without defuzzification. M. Shanmugasundari, K Ganesan [30] also developed Fuzzy version of Vogels and MODI algorithms for finding Fuzzy basic feasible and fuzzy optimal solution of fuzzy transportation problems without converting them to classical transportation problems. Dalbinder et al. [31] solved a single objective fuzzy transportataion problem using triangular fuzzy number without defuzzification introducing a new technique called modified fuzzy programming technique(MFPT), which is a modified form of Fuzzy programming technique introduced by Das et al [32]

In the present paper, the MFPT is applied to a multi-objective and multi-index real life fuzzy transportation problem for the first time. The fuzzy optimal solution of the problem is obtained directly by the mentioned method without defuzzifying the TFN in the objective functions.The method is more effective, easy to apply, easy to calculate and yield even better results. The proposed method was introduced for multi-objective transportation problems (S. K. Das, A. Goswami, S. S. Alam [32]) but a new application of the method on triangular fuzzy numbers to find a fuzzy optimal solution directly without defuzzification is made in Dalbinder et al. [31] for single objective Fuzzy Transportation problem (SOFTP) with fuzzy cost objective function and fuzzy supply and demand parameters. The present paper focuses on solving a real life multi-objective multi-index Fuzzy Transportation problem with more than one fuzzy objective functions and fuzzy supply and demand parameters by the method introduced in Dalbinder et al. [31]. The supply and demand parameters are first defuzzified. Then using the triangular fuzzy numbers of the different objective functions each objective function is converted into three objective functions. Then the Modified Fuzzy Programming Technique is applied and the fuzzy optimal solution is obtained. The problem in this paper is solved by defining linear membership number. Similar solution can be obtained by taking other membership functions like exponential and hyperbolic `as done for the similar transportation problem with crisp objective function and interval valued supply and demand parameters in Dalbinder et al. [19].The results obtained in this paper has been compared with those obtained by Dalbinder et al. [15][16] using linear membership functions.

2. Preliminaries

A. Fuzzy set

A fuzzy set is characterized by a membership function mapping element sof a domain, space or universe of discourse X to the unit interval [0, 1]

i.e. A = {(x, $\mu_A(x)$; x ∈ X},

Here $\mu_A(x)$: X →[0, 1] is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x∈ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

B. Triangular fuzzy number

A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (δ, m, β) where $\delta, m,$ and β are real number and its membership function $\mu_A(x)$ is given below (H. Basirzadeh [16]),

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \le \delta \\ (x-\delta)/(m-\delta) & \text{for } \delta \le x \le m \\ 1 & \text{for } x = m \\ (\beta - x)/(\beta - m) & \text{for } m \le x \le \beta \\ 0 & \text{for } \mathbf{x} \ge \beta \end{cases}$$

According to the above-mentioned definition of a triangular fuzzy number, let $\tilde{A}_\omega = (\underline{A}(r), \overline{A}(r)), 0 \le r \le \omega$, and $0 \le \omega \le 1$ be a fuzzy number, where $\overline{A}(r)$ is a bounded left continuous non-decreasing function and $\underline{A}(r)$ is a bounded left continuous non-increasing function over $[0, \omega]$ and $A(r) \le A(r)$.

If $A^{\sim}$ be an arbitrary fuzzy number then the α-cut of $A^{\sim}$ is $\tilde{A}_\alpha = (\underline{A}(\alpha), \overline{A}(\alpha)), 0 \le \alpha \le \omega$. If ω = 1, then the above-defined number is called a normal fuzzy number.

A measure of a fuzzy number A is a function $M: F(X) \rightarrow R+$ where $F(X)$ denotes the set of all fuzzy numbers on X.

The measure of a fuzzy number is obtained by the average of two side areas, left side area and right side area, from membership function to α axis.

Measure of a fuzzy number $M(A^{\sim})$, is assigned to $\tilde{A}$ is calculated as follows:

$$M_o^{Tri}(\tilde{A}) = \frac{1}{2}\int_0^1 \{\underline{A}(r) + \overline{A}(r)\} dr$$

$$= \frac{1}{4}\left[2m + \delta + \beta\right] \qquad (1)$$

which is very convenient for calculation.

If $\tilde{A}_w = (\underline{A}(r) + \overline{A}(r)) = \left\{\delta + \frac{m-\delta}{w} r, \beta + \frac{m-\beta}{w} r\right\}$

be an arbitrary triangular fuzzy number at decision level higher than " α " and $\alpha, w \in [0,1]$, then the value $M_o^{Tri}(\tilde{A}_w)$, assigned to $\tilde{A}_w$ may be calculated as follows:

If $w > \alpha$, then

$$M_o^{Tri}(\tilde{A}_w) = \frac{1}{2}\int_0^1 \{\underline{A}(r) + \overline{A}(r)\} dr$$

$$= \frac{1}{2w}\left[2m(w^2 - \alpha^2) + (\delta + \beta)(w - \alpha)^2\right] \qquad (2)$$

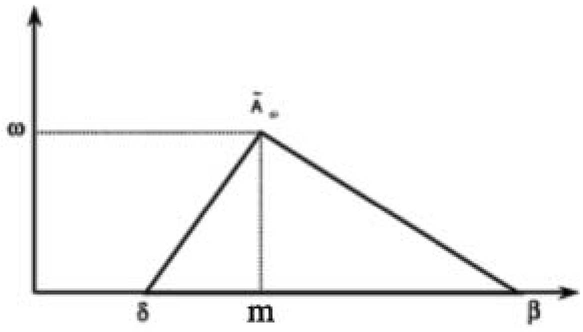

Figure 3: A triangular fuzzy number

C. Classical Transportation Problem

The conventional transportation problem in its basic form by Hitchcock [16] includes the minimization of transportation cost of some commodities from sources (supply) to Destination (demand) and is given by Model 1.

Model 1

Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$ (3)

Subject to the constraints

$$\sum_{j=1}^{n} X_{ij} \le a_i, i = 1,2,3,....,m \quad (4)$$

$$\sum_{i=1}^{m} X_{ij} \ge b_j, j = 1,2,3,....,n \quad (5)$$

$$\sum_{j=1}^{n} X_{ij} \ge 0 \quad (6)$$

D. Fuzzy Transportation Problem

The Mathematical Formulation of **Single objective Fuzzy Transportation Problem** is given by Model 2:

Model 2

Minimize $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{C}_{ij} X_{ij}$ *where* $\tilde{Z} = (Z_1, Z_2, Z_3)$, $\tilde{C}_{ij} = (C_{ij1}, C_{ij2}, C_{ij3})$ (7)

Subject to the constraints

$$\sum_{j=1}^{n} X_{ij} \le \tilde{a}_i, i = 1,2,3,....,m \text{ where } \tilde{a}_i = (a_{i1}, a_{i2}, a_{i3}) \quad (8)$$

$$\sum_{i=1}^{m} X_{ij} \geq \tilde{b}_j,\ j = 1,2,3,\ldots,n \ \textit{where} \quad \tilde{b}_j = (b_{j1}, b_{j2}, b_{j3}) \tag{9}$$

$$\sum_{j=1}^{n} X_{ij} \geq 0 \tag{10}$$

The Mathematical Formulation of **Multi-objective Fuzzy Transportation Problem** is given by Model 3:

Model 3

$$\text{Minimize } \tilde{Z}_k = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{C}_{kij} X_{ij} \tag{11}$$

$$\textit{where} \quad \tilde{Z}_k = (Z_{k1}, Z_{k2}, Z_{k3}),\ \tilde{C}_{kij} = (C_{kij1}, C_{kij2}, C_{kij3}),\ \text{k=1, 2, ...K}$$

Subject to the constraints

$$\sum_{j=1}^{n} X_{ij} \leq \tilde{a}_i,\ i = 1,2,3,\ldots,m \ \textit{where} \quad \tilde{a}_i = (a_{i1}, a_{i2}, a_{i3}) \tag{12}$$

$$\sum_{i=1}^{m} X_{ij} \geq \tilde{b}_j,\ j = 1,2,3,\ldots,n \ \textit{where} \quad \tilde{b}_j = (b_{j1}, b_{j2}, b_{j3}) \tag{13}$$

$$\sum_{j=1}^{n} X_{ij} \geq 0 \tag{14}$$

3. Proposed method

Step1. First the triangular fuzzy numbers of supply and demand parameters in Model 2 are defuzzified using equation (1) and values are obtained as a_i and b_j

Step2. Each objective function is converted into three objective functions. The Model 3 is modified as Model 4:

Model 4

$$\text{Minimize } Z_{k1} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{k1ij} X_{ij} \tag{15}$$

$$\text{Minimize } Z_{k2} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{k2ij} X_{ij} \tag{16}$$

$$\text{Minimize } Z_{k3} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{k3ij} X_{ij} \tag{17}$$

Subject to the constraints

$$\sum_{j=1}^{n} X_{ij} \leq a_i,\ i = 1,2,3,\ldots,m \tag{18}$$

a_i is the defuzzified value of $\tilde{a}_i$, where $a_i = \dfrac{2a_{i2} + a_{i1} + a_{i3}}{4}$

[From Equation (1)] (19)

$$\sum_{i=1}^{m} X_{ij} \geq b_j ,\ j = 1,2,3,....,n \tag{20}$$

b_j is the defuzzified value of $\tilde{b}_j$, where $b_j = \dfrac{2b_{j2} + b_{j1} + b_{j3}}{4}$

[From Equation(1)] (21)

$$\sum_{j=1}^{n} X_{ij} \geq 0 \tag{22}$$

Step3. Solve the multi-objective transportation problem as a single objective transportation problem using each time only one objective and ignoring others.

Step4. From the results of Step 1, determine the corresponding values for every objective at each solution derived.

Step5. Two Cases may arise:

Case 1: If the three values for each objective functions in Model2 are same, then the obtained solution is final fuzzy optimal solution and we proceed in the next step.

Case 2: If the three values for each objective functions in Model2 are different, then we can introduce any membership functions and proceed as:

From Step 4 we may find, for each objective, the best (Z_k^L) and worst (Z_k^U) values corresponding to the set of solutions. The initial fuzzy model can then be stated, in terms of the aspiration levels of each objective, as follows.

Find $x_{ij}, i = 1,2,......,m, j = 1,2,.....n,$ so as to satisfy $Z^k \tilde{\leq} Z_k^L$ where k=1, 2,, …., K, and the given constraints and non-negativity conditions. For the multi-objective transportation problem, a membership function can be defined.

Linear Membership Function

A Linear membership function $\mu_{k(x)}$ corresponding to k^{th} objective, is defined as

$$\mu_k(x) = \begin{cases} 1 & if \quad Z_k \leq Z_k^L , \\ 1 - \dfrac{Z_k - Z_k^L}{Z_k^U - Z_k^L} & if Z_k^L < Z_k < Z_k^U \\ 0 & if \quad Z_k \geq Z_k^U \end{cases} \tag{23}$$

Maximize λ

subject to $\lambda \leq \dfrac{Z_k - Z_k^L}{Z_k^U - Z_k^L}$ for all k (24) and the given constraints, the non-negativity conditions as well as $\lambda \geq 0$, where $\lambda = \min\{\mu_k(x)\}$.This linear programming problem can further be simplified as Model 4:

Maximize λ

Subject to $Z_k + \lambda(Z_k^U - Z_k^L) \leq Z_k^U$ (25)

with the given constraints and non-negativity restriction and $\lambda \geq 0$

(26) This problem can be solved by taking other membership functions like exponential and hyperbolic also.

Step6. The final fuzzy optimal solution can be defuzzified using equation(1) as:

$$Z_k = \frac{2Z_{k2} + Z_{k1} + Z_{k3}}{4} \quad (27)$$

4. Numerical Illustration

The proposed approach was introduced in Kaur Dalbinder et al. [19] for single objective transportation problem. To illustrate the application of the proposed approach for a multi-objective and multi-index a real life transportation problem, following numerical example from Dalbinder et al.[15] is considered, previously taken as approximate past records from DSP Plant, Durgapur, West Bengal, INDIA.

The problem deals with the solution of the multi-objective multi-index real life transportation problem focusing on the minimization of the transportation cost, deterioration rate and underused capacity of the transported raw materials like coal, iron ore, etc from different sources to different destination sites at Durgapur Steel Plant (DSP) by different transportation modes like train, trucks, etc. The problem is formulated by taking different parameters in the objective function and supply and demand as triangular fuzzy numbers.

Consider a problem based on the mathematical formulation given in equations (11)-(14) in which we have three raw materials (m=3) i.e. q=1(Coal), 2(Iron-ore), 3(Limestone).These raw materials are transported from different i^{th} sources to j^{th} destination sites by different transportation modes ‘h’ where h=1(train), 2(truck) as per Table 1.

Table 1 Number of raw materials, sources, destinations and mode of transportation

q=1	i=1, 2	j=1, 2	h=1
q=2	i=1, 2, 3	j=1, 2	h=1, 2
q=3	i=1, 2, 3, 4, 5	j=1	h=1

Then we are considering the triangular fuzzy number values of three objective functions i.e. Transportation Cost functions $C_{ijh}^{q}(C_{ij1}^{1},\ C_{ij1}^{2}\ \ C_{ij2}^{2}, C_{ij1}^{3})$, Deterioration rate functions R_{ijh}^{q} (R_{ij1}^{1}, R_{ij1}^{2} R_{ij2}^{2}, R_{ij1}^{3}) and Underused capacity functions U_{ijh}^{q} (U_{ij1}^{1}, U_{ij1}^{2} U_{ij2}^{2}, U_{ij1}^{3}) in matrix form which are to be minimized. The data are given below.

$$C_{ij1}^{1} = \begin{bmatrix} [26,29,32] & [38,40,43] \\ [58,62,63] & [16,17,20 \end{bmatrix}_{h=1} \quad C_{ij1}^{2} = \begin{bmatrix} [6,7,10] & [4,5,6] \\ [6,8,9] & [7,8,9] \\ [2,3,4] & [1,3,5] \end{bmatrix}_{h=1}$$

$$C_{ij2}^{2} = \begin{bmatrix} [10,12,15] & [6,7,8] \\ [5,7,10] & [5,8,11] \\ [4,7,9] & [2,3,4] \end{bmatrix}_{h=2} \quad C_{ij1}^{3} = \begin{bmatrix} [12,13,14] \\ [13,15,17] \\ [12,14,16] \\ [14,15,16] \\ [16,18,20] \end{bmatrix}_{h=1}$$

$$R_{ij1}^{1} = \begin{bmatrix} [31,34,37] & [30,32,34] \\ [28,29,30] & [30,32,34] \end{bmatrix}_{h=1} \quad R_{ij1}^{2} = \begin{bmatrix} [20,22,24] & [14,15,16] \\ [15,18,21] & [18,20,22] \\ [24,25,26] & [20,22,24] \end{bmatrix}_{h=1}$$

$$R_{ij2}^{2} = \begin{bmatrix} [15,20,25] & [10,12,14] \\ [12,15,17] & [15,16,17] \\ [20,22,24] & [16,17,18] \end{bmatrix}_{h=2} \quad R_{ij1}^{3} = \begin{bmatrix} [78,80,82] \\ [108,110,112] \\ [99,100,101] \\ [88,90,92] \\ [84,85,86] \end{bmatrix}_{h=1}$$

$$U_{ij1}^{1} = \begin{bmatrix} [38,40,42] & [30,32,34] \\ [28,30,32] & [34,35,36] \end{bmatrix}_{h=1} \quad U_{ij1}^{2} = \begin{bmatrix} [16,20,24] & [13,15,18] \\ [18,22,24] & [15,18,20] \\ [14,16,20] & [17,19,23] \end{bmatrix}_{h=1}$$

$$U_{ij2}^{2} = \begin{bmatrix} [28,30,32] & [24,25,26] \\ [20,22,24] & [18,20,24] \\ [22,23,24] & [20,22,24] \end{bmatrix}_{h=1} \quad U_{ij1}^{3} = \begin{bmatrix} [99,100,101] \\ [58,60,62] \\ [78,80,82] \\ [107,110,113] \\ [86,90,94] \end{bmatrix}_{h=1}$$

The data in the form of triangular fuzzy numbers for supply and demand are as follows:

Table 2 TFN Supply data

Q	**h**	**I**	$\tilde{S}_{iq}$
1	1	1	(1.6, 1.8, 2.1)
1	1	2	(0.8, 1.1, 1.3)
2	1	1	(0.55, 0.58, 0.65)
2	1	2	(0.3, 0.4, 0.5)
2	1	3	(0.25, 0.3, 0.37)
2	2	1	(0.7, 0.76, 0.85)
2	2	2	(0.85, 0.89, 0.95)
2	2	3	(0.45, 0.5, 0.6)
3	1	1	(0.07, 0.08, 0.085)
3	1	2	(0.04, 0.048, 0.055)
3	1	3	(0.10, 0.12, 0.15)
3	1	4	(0.12, 0.15, 0.17)
3	1	5	(0.09, 0.12, 0.15)

Table 3 TFN Demand data

Q	**H**	**j**	$\tilde{D}_{jq}$
1	1	1	(0.6, 0.9, 1.2)
1	1	2	(1.7, 1.9, 2.3)
2	1	1	(0.45, 0.5, 0.55)
2	1	2	(0.75, 0.82, 0.85)
2	2	1	(0.85, 0.87, 0.92)
2	2	2	(1.25, 1.29, 1.33)
3	1	1	(0.45, 0.49, 0.53)

The data for supply $S_{iq}, \ \forall \ \ i,q$ are given in the Table 2. The data for demand $D_{jq}, \ \forall \ \ j,q$ are given in the Table 3.

The given problem is first written in the form of the formulated fuzzy model, Model 3 as:

$$\text{Minimize } \tilde{Z}_1 = \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p} \tilde{C}_{ijh}^{q} X_{ijh}^{q} \tag{28}$$

$$\text{Minimize } \tilde{Z}_2 = \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p} \tilde{R}_{ijh}^{q} X_{ijh}^{q} \tag{29}$$

$$\text{Minimize } \tilde{Z}_3 = \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p} \tilde{U}_{ijh}^{q} X_{ijh}^{q} \tag{30}$$

$$\text{Subject to } \sum_{j}\sum_{h} X_{ijh}^{q} = \tilde{S}_{iq}, \ \forall \ \ i,q \tag{31}$$

$$\sum_{i}\sum_{h} X_{ijh}^{q} = \tilde{D}_{jq}, \quad \forall \quad j, q \tag{32}$$

$$X_{ijh}^{q} \geq 0. \tag{33}$$

Where
q = type of raw material; m= number of raw materials;
n = number of sources;
o = number of destination sites;
h = transportation modes; p = number of transportation modes.
X_{ijh}^{q}=Quantity to be transported of q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h';
C_{ijh}^{q} =Transportation cost (in billion rupees per metric tonne) of transportation of q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h';
R_{ijh}^{q} = Deterioration rate (in tonnes per million metric tonne) while transporting q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h';
U_{ijh}^{q}= Underused capacity (in tonnes per thousand metric tonne) while transporting q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h';
S_{iq} = Supplied quantity of q^{th} raw material from i^{th} source (Availability) (in million metric tonnes)
S_{Liq} = Left limit; S_{Riq} = Right limit;
D_{jq} = Demand of q^{th} raw material at j^{th} destination (Requirement) (in million metric tonnes)
D_{Ljq} =Left limit; D_{Rjq}=Right limit.
Z_1, Z_2, Z_3 are the minimal values of the Transportation Cost, Deterioration rate and Underused capacity respectively.

Solution:
The given problem is then solved following the method proposed in.Section III.
The results are obtained as follows:
Step1. The triangular fuzzy numbers of supply and demand parameters in Table 2 and Table 3 are defuzzified using equation (19) and (21). The values are obtained as in Table 4 and Table 5 respectively.

Table 4 Defuzified Supply data

Q	H	I	S_{iq}
1	1	1	1.825
1	1	2	1.075
2	1	1	0.59
2	1	2	0.4
2	1	3	0.305
2	2	1	0.7675
2	2	2	0.895
2	2	3	0.5125
3	1	1	0.07875
3	1	2	0.04775
3	1	3	0.1225
3	1	4	0.1475
3	1	5	0.12

Table 5 Defuzified Demand data

Q	H	J	D_{jq}
1	1	1	0.9
1	1	2	1.95
2	1	1	0.5
2	1	2	0.81
2	2	1	0.8775
2	2	2	1.29
3	1	1	0.49

Step 2. Each objective function is converted into three objective functions and modified as Model 4:

$$\text{Minimize } Z_{11} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{11ij} X_{ij} \tag{34}$$

$$\text{Minimize } Z_{12} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{12ij} X_{ij} \tag{35}$$

$$\text{Minimize } Z_{13} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{13ij} X_{ij} \tag{36}$$

$$\text{Minimize } Z_{21} = \sum_{i=1}^{m}\sum_{j=1}^{n} R_{21ij} X_{ij} \tag{37}$$

Minimize $Z_{22} = \sum_{i=1}^{m}\sum_{j=1}^{n} R_{22ij} X_{ij}$ (38)

Minimize $Z_{23} = \sum_{i=1}^{m}\sum_{j=1}^{n} R_{23ij} X_{ij}$ (39)

Minimize $Z_{31} = \sum_{i=1}^{m}\sum_{j=1}^{n} U_{31ij} X_{ij}$ (40)

Minimize $Z_{32} = \sum_{i=1}^{m}\sum_{j=1}^{n} U_{32ij} X_{ij}$ (41)

Minimize $Z_{33} = \sum_{i=1}^{m}\sum_{j=1}^{n} U_{33ij} X_{ij}$ (42)

Subject to the constraints

$\sum_{j=1}^{n} X_{ij} = S_{iq_i}, i = 1,2,3,....,m$ S_{iq} is the defuzzified value of $\tilde{S}_{iq}$ (43)

$\sum_{i=1}^{m} X_{ij} = D_{jq}, j = 1,2,3,....,n$ D_{jq} is the defuzzified value of $\tilde{D}_{jq}$ (44)

$\sum_{j=1}^{n} X_{ij} \geq 0$ (45)

Step 3. The multi-objective transportation problem is solved as a single objective transportation problem taking only one objective function at a time and ignoring others. The values are obtained as

1. $Z_{11} = 97.456$ 2. $Z_{12} = 108.948$ 3. $Z_{13} = 123.73$
4. $Z_{21} = 179.161$ 5. $Z_{22} = 191.875$ 6. $Z_{23} = 203.658$
7. $Z_{31} = 197.054$ 8. $Z_{32} = 210.341$ 9. $Z_{33} = 225.339$

Step4. From the results of Step1, the corresponding values for each objective for each solution are determined as:

$Z_{11} = 97.456$ $Z_{11} = 97.456$ (for Z_{12} solution set) ; $Z_{11} = 97.896$ (for Z_{13} solution set); $Z_{11} = 148.816$ (for Z_{21} solution set)

$Z_{11} = 148.8$ (for Z_{22} solution set); $Z_{11} = 148.801$ (for Z_{23} solution set); $Z_{11} = 150.762$ (for Z_{31} solution set)

$Z_{11} = 150.222$ (for Z_{32} solution set); $Z_{11} = 150.222$ (for Z_{33} solution set);

$Z_{12} = 108.948$ $Z_{12} = 108.948$ (for Z_{11} solution set); $Z_{12} = 108.948$ (for Z_{13} solution set); $Z_{12} = 162.223$ (for Z_{21} solution set)

$Z_{12} = 162.178$ (for Z_{22} solution set); $Z_{12} = 162.178$ (for Z_{23} solution set); $Z_{12} = 165.08$ (for Z_{31} solution set)
$Z_{12} = 164.72$ (for Z_{32} solution set); $Z_{12} = 164.712$ (for Z_{33} solution set)
$Z_{13} = 123.73$ $Z_{13} = 123.95$ (for Z_{11} solution set); $Z_{13} = 123.95$ (for Z_{12} solution set); $Z_{13} = 175.325$ (for Z_{21} solution set)
$Z_{13} = 175.265$ (for Z_{22} solution set); $Z_{13} = 175.265$ (for Z_{23} solution set); $Z_{13} = 178.835$ (for Z_{31} solution set)
$Z_{13} = 178.115$ (for Z_{32} solution set); $Z_{13} = 178.107$ (for Z_{33} solution set)
$Z_{21} = 179.161$ $Z_{21} = 182.146$ (for Z_{11} solution set); $Z_{21} = 182.146$ (for Z_{12} solution set); $Z_{21} = 183.686$ (for Z_{13} solution set)
$Z_{21} = 179.206$ (for Z_{22} solution set); $Z_{21} = 179.206$ (for Z_{23} solution set); $Z_{21} = 186.069$ (for Z_{31} solution set)
$Z_{21} = 184.449$ (for Z_{32} solution set); $Z_{21} = 184.426$ (for Z_{33} solution set)
$Z_{22} = 191.875$ $Z_{22} = 196.73$ (for Z_{11} solution set); $Z_{22} = 182.146$ (for Z_{12} solution set); $Z_{22} = 183.686$ (for Z_{13} solution set)
$Z_{22} = 191.875$ (for Z_{21} solution set); $Z_{22} = 191.875$ (for Z_{23} solution set); $Z_{22} = 197.784$ (for Z_{31} solution set)
$Z_{22} = 196.164$ (for Z_{32} solution set); $Z_{22} = 196.157$ (for Z_{33} solution set)
$Z_{23} = 203.658$ $Z_{23} = 210.428$ (for Z_{11} solution set); $Z_{23} = 210.428$ (for Z_{12} solution set); $Z_{23} = 211.088$ (for Z_{13} solution set)
$Z_{23} = 203.703$ (for Z_{21} solution set); $Z_{23} = 203.658$ (for Z_{22} solution set); $Z_{23} = 209.135$ (for Z_{31} solution set)
$Z_{23} = 207.515$ (for Z_{32} solution set) $Z_{23} = 207.515$ (for Z_{33} solution set)
$Z_{31} = 197.054$ $Z_{31} = 211.69$ (for Z_{11} solution set); $Z_{31} = 211.69$ (for Z_{12} solution set); $Z_{31} = 210.37$ (for Z_{13} solution set)
$Z_{31} = 198.45$ (for Z_{21} solution set); $Z_{31} = 198.330$ (for Z_{22} solution set); $Z_{31} = 198.33$ (for Z_{23} solution set)
$Z_{31} = 197.054$ (for Z_{32} solution set); $Z_{31} = 197.070$ (for Z_{33} solution set)
$Z_{32} = 210.341$ $Z_{32} = 224.76$ (for Z_{11} solution set); $Z_{32} = 224.76$ (for Z_{12} solution set); $Z_{32} = 210.37$ (for Z_{13} solution set)
$Z_{32} = 212.5$ (for Z_{21} solution set); $Z_{32} = 212.35$ (for Z_{22} solution set); $Z_{32} = 212.35$ (for Z_{23} solution set)

$Z_{32} = 210.521$ (for Z_{31} solution set); $Z_{32} = 210.357$ (for Z_{33} solution set)
$Z_{33} = 225.339$ $Z_{33} = 238.279$ (for Z_{11} solution set); $Z_{33} = 238.279$ (for Z_{12} solution set);
$Z_{33} = 236.739$ (for Z_{13} solution set)
$Z_{33} = 226.94$ (for Z_{21} solution set); $Z_{33} = 226.804$ (for Z_{22} solution set); $Z_{33} = 226.804$ (for Z_{23} solution set)
$Z_{33} = 225.7$ (for Z_{31} solution set); $Z_{33} = 225.339$ (for Z_{32} solution set)

Step 5.
As the different values are obtained for each objective functions we opt for Case 2 and proceeds to find the best and worst values for each objectives.

$Z_{11}^L = 97.456$ $Z_{11}^U = 150.762$ $Z_{11}^U - Z_{11}^L = 53.306$
$Z_{12}^L = 108.948$ $Z_{12}^U = 165.08$ $Z_{12}^U - Z_{12}^L = 56.132$
$Z_{13}^L = 123.75$ $Z_{13}^U = 178.835$ $Z_{13}^U - Z_{13}^L = 55.105$
$Z_{21}^L = 179.161$ $Z_{21}^U = 186.069$ $Z_{21}^U - Z_{21}^L = 6.908$
$Z_{22}^L = 191.875$ $Z_{22}^U = 197.83$ $Z_{22}^U - Z_{22}^L = 5.955$
$Z_{23}^L = 203.658$ $Z_{23}^U = 211.088$ $Z_{23}^U - Z_{23}^L = 7.43$
$Z_{31}^L = 197..054$ $Z_{31}^U = 211.69$ $Z_{31}^U - Z_{31}^L = 14.636$
$Z_{32}^L = 210.341$ $Z_{32}^U = 224.76$ $Z_{32}^U - Z_{32}^L = 14.419$
$Z_{33}^L = 225.339$ $Z_{33}^U = 238.279$ $Z_{33}^U - Z_{33}^L = 12.94$

Corresponding to the three objective functions, we can define a linear membership function and then the problem can be solved.
Substituting the values obtained above in the equation (25), the problem is transformed as follows:
Maximize λ
Subject to $Z_{11} + 53.306\lambda \le 150.762$
$Z_{12} + 56.132\lambda \le 165.08$
$Z_{13} + 55.105\lambda \le 178.835$
$Z_{21} + 6.908\lambda \le 186.069$
$Z_{22} + 5.955\lambda \le 197.83$
$Z_{23} + 7.43\lambda \le 211.088$
$Z_{31} + 14.636\lambda \le 211.69$
$Z_{32} + 14.419\lambda \le 224.76$
$Z_{33} + 12.94\lambda \le 238.279$
with the given constraints and non-negativity restriction as in Step 2 and $\lambda \ge 0$
The final solution is obtained as
$\lambda = 0.511$,

$Z_{11} = 123.533$, $Z_{12} = 135.665$, $Z_{13} = 149.601$

$Z_{21} = 181.453$, $Z_{22} = 194.788$, $Z_{23} = 207.245$ $Z_{31} = 203.976$, $Z_{32} = 217.395$,

$Z_{33} = 231.381$

Thus the fuzzy optimal solution is obtained as

$Z_1 = (123.533, 135.665, 149.601)$

$Z_2 = (181.453, 194.788, 207.245)$

$Z_3 = (203.976, 217.395, 231.381)$

Step 6 The defuzzified values can be calculated as in Equation (26)

$Z_1 = 136.116$ $Z_2 = 194.569$ $Z_3 = 217.537$

5. Result Discussion and Conclusions

The results obtained by our proposed method are discussed with the previous results obtained earlier in previous papers as in Table 6, 7and 8.

Table 6: Solution using crisp objective functions and interval valued supply and demand parameters and Linear membership function in paper Dalbinder [15]

λ	Z_1	Z_2	Z_3
0.88	106.18	172.83	192.21

Table 7 Solution using interval valued objective functions and interval valued supply and demand parameters and linear membership function in Dalbinder [16]

Z_c^1	Z_w^1	Z_c^2	Z_w^2	Z_c^3	Z_w^3	Z^1	Z^2	Z^3
103.88	172.38	196.59	17.35	32.94	42.63	[86.53, 121.23]	[139.44, 205.32]	[153.96, 239.22]

Table 8 Solution using TFN objective functions and TFN supply and demand parameters and linear membership function in Present paper

λ	Z_1	Z_2	Z_3
0.511	136.116	194.569	217.537

Following conclusions are obtained:

- A simple yet effective method-Modified Fuzzy Programming Technique(MFTP) is proposed for the Fuzzy optimal solution to the multi-objective and multi-index real life Transportation Problem.

- This method can be used for all kinds of fuzzy transportation problem, whether balanced or unbalanced transportation problems with fuzzy data.
- The paper provides a new application of the method to multi-objective and multi-index real life transporation problems.
- All the parameters are not defuzzified and the fuzzy solution is obtained without converting it to classical transportation problem.
- The data is considered in term of triangular fuzzy number and can also be applied for trapezoidal fuzy numbers.
- The method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem.
- The paper focusses on triangular fuzzy numbers, modified fuzzy programming techniques, their application in Multi-objective and multi-indexreal life fuzzy Transportation Problem.
- The proposed method optimal solutions were obtained which is evident from the above mentioned tables.

REFERENCES

[1] R. E. Bellman, L. A. Zadeh, (1970), " Decision-making in a fuzzy environment", Management Science 17 (4), B141–B164

[2] L. A. Zadeh (1965), "Fuzzy Sets" Information Control 8, 338-353.

[3] H. J. Zimmermann(1978), "Fuzzy Linear Programming with several objective functions, Fuzzy Sets and Systems, 1(1978), 46-55.

[4] H. J. Zimmermann, 2001. Fuzzy Set Theory and Its Applications. Kluwer Academic Publishers, USA.

[5] S. Chanas, (1983), "The use of parametric programming in fuzzy linear programming", Fuzzy Sets and Systems 11, 43-251.

[6] Shan-Huo Chen (1985), "Operations on fuzzy numbers with function principle", Tamkang Journal of Management Sciences, 6(1) (1985), 13-25.

[7] M. O'he'igeartaigh, (1982), "A fuzzy transportation algorithm", Fuzzy Sets and Systems 8, 235-243.

[8] S. Chanas, W. Kolodziejczyk, and A. Machaj, (1984), "A fuzzy approach to the transportation problem", Fuzzy Sets and Systems 13, 211-221.

[9] S. Chanas, D. Kuchta, (2002), " Fuzzy goal programming – One notion, many meanings", Control and Cybernetics 31 (4), 871–890.

[10] E. E. Ammar, E. A Youness (2005), " Study on multiobjective transportation problem with fuzzy numbers", *Applied Mathematics and Computation, Volume 166, Issue 2, 15 July 2005, Pages 241-253*

[11] Hadi Basirzadeh (2011), "An Approach for Solving Fuzzy Transportation Problem", Applied Mathematical Sciences, Vol. 5, 2011, no. 32, 1549-1566

[12] Hossein Abdollahnejad Barough (2011), "A Multi-objective Goal Programming Approach to a Fuzzy Transportation Problem: The Case of a General Contractor Company", The Journal of Mathematics and Computer Science, Vol.2 No. 1 (2011) 9-19

[13] Ananya Chakraborty, Chakraborty M. (2010), "Cost-time Minimization in a Transportation Problem with Fuzzy Parameters: A Case Study", *Journal of Transportation Systems Engineering and Information Technology, Volume 10, Issue 6, December 2010, Pages 53-63*

[14] Amarpreet Kaur, Amit Kumar (2011), "A new method for solving fuzzy transportation problems using ranking function" *Applied Mathematical Modelling, Volume 35, Issue 12, December 2011, Pages 5652-5661.*

[15] Dalbinder Kaur, Sathi Mukherjee, Kajla Basu (2011), "Multi-objective Multi-index real life transportation problem with crisp objective function and interval valued supply and destination parameters", Proceedings to ICMCM'2011, PSG College of Technology, Coimbatore *ISBN 978-81-8487-1647*

[16] Dalbinder Kaur, Sathi Mukherjee, Kajla Basu (2012), "Multi-objective multi-index real life transportation problem with interval valued parameters", Proceedings to RAMAES'2012, BCET, Durgapur. *ISBN 978-93-5067-395-9*

[17] Dalbinder Kaur, Sathi Mukherjee, Kajla Basu (2012), "Goal programming approach to multi-index real life transportation problem with crisp objective function and interval valued supply and destination parameters", Proceedings to ICOCBA'2012, ORSI. *ISBN 978-81-8424-8142.*

[18] Dalbinder Kaur, Sathi Mukherjee, Kajla Basu (2013), "Solution of a Multi-index Real Life Transportation Problem by Fuzzy Non-linear Goal Programming", Proceedings to RAMA'2013, ISM, Dhanbad. *ISBN 978-81-8424-821-0*

[19] Dalbinder Kaur, Sathi Mukherjee, Kajla Basu (2014), "Solution of a Multi-objective and Multi-index Real Life Transportation Problem using different Fuzzy membership functions", J. Optim Theory Appl (2015) 164: 666-678, DOI – 10.1007/s10957-014-0579-6, Received:10 December 2013, Accepted: 29 April 2014.(2014), published online: 31 May 2014 @ springer Science + Business Media New York 2014

[20] B. Ramesh Kumar, S. Murugesan (2012), "On Fuzzy Transportation Problem Using Triangular Fuzzy Numbers With Modified Revised Simplex Method", International Journal of Engineering Science and Technology (IJEST) ISSN: 0975-5462 Vol. 4 No.01 January 2012, 285-294.

[21] Pradip Kundu, Samarjit Kar, Manoranjan Maiti (2012), "Multi-objective Multi-item Solid Transportation Problem in Fuzzy Environment" Appl. Math Modelling, Accepted Date:21 April 2012"

[22] Anupam Ojha, Shyamal Kr. Mondal, Manoranjan Maiti (2011), "Transportation Policies for single and multi-objective transportation problem using fuzzy logic", *Mathematical and Computer Modelling, Volume 53, Issues 9-10, May 2011, Pages 1637-1646*

[23] Lushu Li, K. K. Lai (2000), "A fuzzy approach to the multiobjective transportation problem" *Computers & Operations Research, Volume 27, Issue 1, January 2000, Pages 43-57*

[24] K. Maity, M. Maity, (2006) "Possibility and necessity constraints and their defuzzifications – a multi-item production-inventory scenerio via optimal control theory", European Journal of Operational Research 177, 882–896.

[25] M. K. Maiti, (2011), " A fuzzy genetic algorithm with varying population size to solve an inventory model with credit-linked promotional demand in an imprecise planning horizon", European Journal of Operational Research 213 (1), 96–106.

[26] S. T. Liu, C. Kao (2004), "Solving Fuzzy transportation problem based on extension principle", European Journal of Operations Research, 153 (2004), 661-674.

[27] A. Nagoor Gani and K. Abdul Razak (2004), " Two stage fuzzy transportation problem", *European Journal of Operational Research*, **153** (2004), 661-674.

[28] P. Pandian and G. Natarajan (2010), "A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems", *Applied Mathematical Sciences*, **4**(2010), 79-90.

[29] A. Nagoor Gani, A. Edward Samuel, D. Anuradha (2011), "Simplex Type Algorithm for Solving Fuzzy Transportation Problem" Tamsui Oxford Journal of Information and Mathematical Sciences 27 (1) (2011) 89-98 Aletheia University Received March 5, 2010, Accepted April 7, 2010

[30] Mohanaselvi S., K. Ganesan (2012), "Fuzzy Optimal Solution to Fuzzy Transportation Problem: A New Approach", International Journal on Computer Science and Engineering (IJCSE) ISSN: 0975-3397 Vol. 4 No. 03 March 2012, 367-375

[31] Dalbinder Kaur, Sathi Mukherjee, Kajla Basu (2014), " A new fuzzy programming technique approach to solve fuzzy transportation problem ", 2nd International Conference on Business and Information Management (ICBIM) 978-1-4799-3264-1/14/$31.00 ©2014 IEEE (2014)144-150.

[32] S. K. Das, A. Goswami, S. S. Alam, "Multi-objective transportation problem with interval cost, source and destination parameters", *European Journal of Operational Research, Volume 117, Issue 1, 16 August 1999, Pages 100-112*

[33] F. L. Hitchcock, (1941), "The distribution of a product from several sources to numerous localities", Journal of Mathematics and Physics. 20, 224-230.