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Fuzzy Neutrosophic Groups

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Abstract

In this paper we introduce the concept of fuzzy neutrosophic groups and investigate some of their properties.

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1. Introduction

Smarandache [11] initiated the concept of neutrosophic set which overcomes the inherent difficulties that existed in fuzzy sets and intuitionistic fuzzy sets. Following this, the neutrosophic sets are explored to different heights in all fields of science and engineering. Many researchers [3, 4, 5, 6, 8, 9, 13] applied the concept of fuzzy sets and intuitionistic fuzzy sets to algebra. In this paper we initiate the concept of fuzzy neutrosophic groups and some of its properties are discussed.

2. Preliminary Notes

Definition 2.1. **[1]** A Fuzzy Neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ where $T, I, F : X \longrightarrow [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2. **[1]** Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ are fuzzy neutrosophic sets. Then A is a subset of B if $\forall x \in X$

$$T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)), F_A(x) \geq F_B(x))$$

Definition 2.3. [1] Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ are fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$
$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4. [2] Let $D = \langle x, T_D(x), I_D(x), F_D(x)\rangle$. Let f be a mapping from a set X to a set Y. If $B = \{\langle y, T_B(y), I_B(y), F_B(y)\rangle\, / y \in Y\}$ is a fuzzy neutrosophic set in Y, then the preimage of B under f denoted by $f^{-1}(B)$ is the fuzzy neutrosophic set in X defined by

$$f^{-1} = \{< x, f^{-1}(T_B)(x), f^{-1}(I_B)(x), f^{-1}(F_B)(x) > /x \in X\}$$

where

$$f^{-1}(T_B)(x) = T_B(f(x)),$$
$$f^{-1}(I_B)(x) = I_B(f(x))$$

and

$$f^{-1}(F_B)(x) = F_B(f(x)) \text{ for all } x \in X.$$

Definition 2.5. [1] A Fuzzy neutrosophic set A over the non-empty set X is said to be empty fuzzy neutrosophic set if $T_A(x) = 0$, $I_A(x) = 0$,$F_A(x) = 1, \forall x \in X$. It is denoted by 0_N. A Fuzzy neutrosophic set A over the non-empty set X is said to be universe fuzzy neutrosophic set if $T_A(x) = 1$, $I_A(x) = 1$, $F_A(x) = 1$, $\forall x \in X$. It is denoted by 1_N.

Definition 2.6. [1] The complement of Fuzzy neutrosophic set A denoted by A^c and is defined as

$$A^c(x) = \langle x, T_{A^c}(x) = F_A(x), I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x)\rangle$$

Let $p, q, r \in [0, 1]$ and $p + q + r \leq 3$. An fuzzy neutrosophic point $x_{(p,q,r)}$ of X is the fuzzy neutrosophic set in X defined by

$$x_{(p,q,r)}(y) = \begin{cases} (p, q, r), & \text{if } x = y \\ (0, 0, 1), & \text{if } y \neq x \end{cases}, \text{ for each } y \in X.$$

A fuzzy neutrosphic point $x_{(p,q,r)}$ is said to belong to an fuzzy neutrosophic set $A =< T_A, I_A, F_A >$ in X denoted by $x_{(P,q,r)} \in A$ if $p \leq T_A(x)$, $q \leq I_A(x)$ and $r \geq F_A(x)$. We denote the set of all fuzzy neutrosophic points in X as FNP(X).

Definition 2.7. Let X be a set and let $p, q, r \in [0, 1]$ with $0 \leq p + q + r \leq 3$. Then the fuzzy neutrosophic set $C_{(p,q,r)} \in X$ is defined by for each $x \in x$,$C_{(p,q,r)}(x) = (p, q, r)$ ie., $T_{C_{(p,q,r)}}(x) = p$, $I_{C_{(p,q,r)}}(x) = q$ and $F_{C_{(p,q,r)}}(x) = r$.

3. Fuzzy Neutrosophic groups

Definition 3.1. Let $(X, .)$ be a group and let A and B be fuzzy neutrosophic sets in X. Then the fuzzy neutrosophic product of A and B, $A \circ B$ is defined as follows, for any $x \in X$

$$T_{A\circ B}(x) = \begin{cases} \bigvee_{yz=x} [T_A(y) \wedge T_A(z)], & \text{for each } (y,z) \in X \times X with\ yz = x \\ 0, & \text{otherwise} \end{cases}$$

$$I_{A\circ B}(x) = \begin{cases} \bigvee_{yz=x} [I_A(y) \wedge I_A(z)], & \text{for each } (y,z) \in X \times X with\ yz = x \\ 0, & \text{otherwise} \end{cases}$$

$$F_{A\circ B}(x) = \begin{cases} \bigwedge_{yz=x} [F_A(y) \vee F_A(z)], & \text{for each } (y,z) \in X \times X with\ yz = x \\ 1, & \text{otherwise} \end{cases}$$

Definition 3.2. Let $(X, .)$ be a group and let A be fuzzy neutrosophic sets in X. Then A is called a fuzzy neutrosophic group(in short, FNG) in X if it satisfies the following conditions:

(i) $T_A(xy) \geq T_A(x) \wedge T_A(y)$, $I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$

(ii) $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$.

Example 3.3. Let $G = \{e, x_1, x_2, x_3, x_4, x_5\}$ be the group given by

$*$	e	x_1	x_2	x_3	x_4	x_5
e	e	x_1	x_2	x_3	x_4	x_5
x_1	x_1	x_2	x_3	x_4	x_5	e
x_2	x_2	x_3	x_4	x_5	e	x_1
x_3	x_3	x_4	x_5	e	x_1	x_2
x_4	x_4	x_5	e	x_1	x_2	x_3
x_5	x_5	e	x_1	x_2	x_3	x_4

Then the fuzzy neutrosophic set

$$A =< x, \frac{e}{(0.6, 0.5, 0.3)}, \frac{x_1}{(0.3, 0.4, 0.6)}, \frac{x_2}{(0.5, 0.4, 0.5)}, \frac{x_3}{(0.3, 0.4, 0.6)}, \frac{x_4}{(0.5, 0.4, 0.5)}, \frac{x_5}{(0.3, 0.4, 0.6)} >$$

is a fuzzy neutrosophic group on G.

Proposition 3.4. Let A be a fuzzy neutrosophic group in a group X, then

(i) $T_A(x^{-1}) = T_A(x)$, $I_A(x^{-1}) = I_A(x)$ and $F_A(x^{-1}) = F_A(x)$

(ii) $T_A(e) \geq T_A(x)$, $I_A(e) \geq I_A(x)$ and $F_A(e) \leq F_A(x)$ for each $x \in X$, where e is the identity element of X.

Proof. Let $x \in X$. Then

(i) $T_A(x) = T_A[(x^{-1})^{-1}] \geq T_A(x^{-1}) \geq T_A(x)$ implies $T_A(x^{-1}) = T_A(x)$, $I_A(x) = I_A[(x^{-1})^{-1}] \geq I_A(x^{-1}) \geq I_A(x)$ implies $I_A(x^{-1}) = I_A(x)$ and $F_A(x) = F_A[(x^{-1})^{-1}] \leq F_A(x^{-1}) \leq F_A(x)$ implies $F_A(x^{-1}) = F_A(x)$ for each $x \in X$

(ii) $T_A(e) = T_A(xx^{-1}) \geq T_A(x) \wedge T_A(x^{-1}) = T_A(x)$ implies $T_A(e) \geq T_A(x)$, $I_A(e) = I_A(xx^{-1}) \geq I_A(x) \wedge I_A(x^{-1}) = I_A(x)$ implies $I_A(e) \geq I_A(x)$ and $F_A(e) = F_A(xx^{-1}) \leq F_A(x) \vee F_A(x^{-1}) = F_A(x)$ implies $F_A(e) \leq F_A(x)$for each $x \in X$. ■

Proposition 3.5. Let A is a fuzzy neutrosophic group in a group X if and only if $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y)$, $I_A(xy^{-1}) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy^{-1}) \leq F_A(x) \vee F_A(y)$ for each $x, y \in X$.

Proof. Let A is a fuzzy neutrosophic group on X then we have $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y^{-1}) = T_A(x) \wedge T_A(y)$. Similarly we get $I_A(xy^{-1}) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy^{-1}) \leq F_A(x) \vee F_A(y)$ for each $x, y \in X$.

Conversely if $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y)$ and let $y = x$ to obtain $T_A(e) \geq T_A(x)$ for all $x \in X$. Hence $T_A(y^{-1}) = T_A(ey^{-1}) \geq T_A(e) \wedge T_A(y) = T_A(y)$ and it follow that $T_A(xy) = T_A(x(y^{-1})^{-1})) \geq T_A(x) \wedge T_A(y^{-1}) = T_A(x) \wedge T_A(y)$ Similarly $I_A(y^{-1}) \geq I_A(y)$ and $I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(y^{-1}) \leq F_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$. ■

Proposition 3.6. If A is a fuzzy neutrosophic group in a group X then $A_e = \{x \in X : T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$ then A_e is a subgroup of X.

Proof. Let $x, y \in A_e$. Then $T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$ Now $T_A(xy^{-1}) \geq T_A(e) \wedge T_A(y^{-1}) \geq T_A(x) \wedge T_A(y) = T_A(e)$. Similarly we have $I_A(xy^{-1}) \geq I_A(e)$ and $F_A(xy^{-1}) \leq F_A(e)$. Now by proposition 3.8 we have $T_A(xy^{-1}) \leq T_A(e)$, $I_A(xy^{-1}) \leq I_A(e)$ and $F_A(xy^{-1}) \geq F_A(e)$. So $T_A(xy^{-1}) = T_A(e)$, $I_A(xy^{-1}) = I_A(e)$ and $F_A(xy^{-1}) = F_A(e)$. Hence $xy^{-1} \in A_e$. Therefore A_e is a subgroup of X. ■

Proposition 3.7. Let A be a fuzzy neutrosophic group in a group X. If $T_A(xy^{-1}) = T_A(e)$, $I_A(xy^{-1}) = I_A(e)$ and $F_A(xy^{-1}) = F_A(e)$ for any $x, y \in A$ then $T_A(x) = T_A(y)$,$I_A(x) = I_A(y)$ and $F_A(x) = F_A(y)$.

Proof. $T_A(x) = T_A[(xy^{-1})y] \geq T_A(xy^{-1}) \wedge T_A(y) = T_A(e) \wedge T_A(y) = T_A(y) = T_A[(yx^{-1})x] \geq T_A(e) \wedge T_A(x) = T_A(x)$. So $T_A(x) = T_A(y)$. Similarly we can prove $I_A(x) = I_A(y)$ and $F_A(x) = F_A(y)$. ■

Proposition 3.8. Let A be a fuzzy neutrosophic group in a group X and let $x \in X$. Then $T_A(xy) = T_A(y)$, $I_A(xy) = I_A(y)$ and $F_A(xy) = F_A(y)$ for each $y \in X$ if and only if $T_A(x) = T_A(e)$, $I_A(x) = I_A(e)$ and $F_A(x) = F_A(e)$, where e is the identity of X.

Proof. Let $T_A(xy) = T_A(y)$ for each $y \in X$. Then we have $T_A(x) = T_A(xe) = T_A(e)$. Similarly $I_A(x) = I_A(e)$ and $F_A(x) = F_A(e)$. Conversely let $T_A(x) = T_A(e)$ by proposition 3.8 $T_A(y) \leq T_A(x)$ for each $y \in X$. Since A is fuzzy neutrosophic group in a group X we have $T_A(xy) \geq T_A(x) \wedge T_A(y) = T_A(y)$ ie., $T_A(xy) \geq T_A(y)$, similarly we get $I_A(xy) \geq I_A(y)$ $F_A(xy) \leq F_A(y)$ for each $y \in X$. Using the proposition 3.8 we have $T_A(y) = T_A(x^{-1}xy) \geq T_A(x) \wedge T_A(xy) = T_A(xy)$ in the same way we obtain $I_A(y) \geq I_A(xy)$ and $F_A(y) \leq F_A(xy)$ for each $y \in X$. Hence $T_A(xy) = T_A(y)$, $I_A(xy) = I_A(y)$ and $F_A(xy) = F_A(y)$ for each $y \in X$. ■

Proposition 3.9. Let $f : X \to Y$ be a group homomorphism and let V be a fuzzy neutrosophic set in Y and if V is a fuzzy neutrosophic group in Y then $f^{-1}(V)$ is a fuzzy neutrosophic group in X.

Proof. Let $x, y \in G$. Then $T_{f^{-1}(B)}(xy) = f^{-1}(T_B)(xy) = (T_B)f(xy) = T_B[f(x)f(y)]$ $\geq T_B(f(x)) \wedge T_B(f(y)) = f^{-1}(T_B)(x) \wedge f^{-1}(T_B)(y)$. Similarly $I_{f^{-1}(B)}(xy) = f^{-1}(I_B)(x) \wedge f^{-1}(I_B)(y)$ and $F_{f^{-1}(B)}(xy) = f^{-1}(F_B)(x) \vee f^{-1}(F_B)(y)$.
Let $x \in X$. Then $T_{f^{-1}(B)}(x^{-1}) = f^{-1}(T_B)(x^{-1}) = (T_B)f(x^{-1}) = T_B[(f(x))^{-1}]$ $\geq T_B f(x) = T_{f^{-1}(B)}(x)$,in the same way we obtain $I_{f^{-1}(B)}(x^{-1}) \geq I_{f^{-1}(B)}(x)$ and $F_{f^{-1}(B)}(x^{-1}) \leq F_{f^{-1}(B)}(x)$. Hence $f^{-1}(V)$ is a fuzzy neutrosophic group in X. ■

Definition 3.10. Let $f : X \to Y$ be a group homomorphism and let A be a fuzzy neutrosophic group in a group X. A is said to be fuzzy neutrosophic- invariant if for any $x, y \in X$, $T_A(x) = T_A(y)$, $I_A(x) = I_A(y)$ and $F_A(x) = F_A(y)$.
It is clear that if A is fuzzy neutrosophic invariant then $f(A) \in FNG(Y)$. For each $A \in FNG(X)$, let $X_A = \{x \in X : T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)\}$. Then it is clear that X_A is a subgroup of X. For each $a \in X$, let $\lambda_a : X \to X$ and $\mu_a : X \to X$ be the right and left translations of X into itself, defined by $\lambda_a(x) = xa$ and $\mu_a(x) = ax$ respectively for each $x \in X$.

Proposition 3.11. Let A be a fuzzy neutrosophic group in a group X. Then for all $a \in A_e$, $\lambda_a[A] = \mu_a[A] = A$.

Proof. Let $a \in A_e$. Then $T_{\lambda_a[A]}(x) = T_A(xa^{-1}) \geq T_A(x) \wedge T_A(a^{-1}) = T_A(x) \wedge T_A(e) = T_A(x) = T_A(xa^{-1}a) \geq T_A(xa^{-1}) \wedge T_A(e) = T_A(xa^{-1}) = T_{\lambda_a[A]}(x)$, $I_{\lambda_a[A]}(x) = I_A(xa^{-1}) \geq I_A(x) \wedge I_A(a^{-1}) = I_A(x) \wedge I_A(e) = I_A(x) = I_A(xa^{-1}a) \geq I_A(xa^{-1}) \wedge I_A(e) = I_A(xa^{-1}) = I_{\lambda_a[A]}(x)$ and $F_{\lambda_a[A]}(x) = F_A(xa^{-1}) \leq F_A(x) \vee F_A(a^{-1}) = F_A(x) \vee F_A(e) = F_A(x) = F_A(xa^{-1}a) \leq F_A(xa^{-1}) \vee F_A(e) = F_A(xa^{-1}) = F_{\lambda_a[A]}(x)$ for all $x \in X$. Similarly we can prove for μ_a. Hence the proof. ■

Proposition 3.12. Let $f : X \to Y$ be a group homomorphism and let A be a fuzzy neutrosophic group in a group X. If A is fuzzy neutrosophic invariant, then $f(K \cap A) = f(K) \cap f(A)$ for each K a fuzzy neutrosophic group in a group X.

Proof. Let K be a fuzzy neutrosophic group in a group X and let $H = K \cap A$. Then $T_{f(H)}(y) = f_{T_H}(y) = \bigvee_{x \in f^{-1}(y)} T_H(x) = \bigvee_{x \in f^{-1}(y)} T_{K \cap A}(x) = \bigvee_{x \in f^{-1}(y)} [T_K(x) \wedge T_A(x)] = \left(\bigvee_{x \in f^{-1}(y)} T_K(x) \right) \wedge \left(\bigvee_{x \in f^{-1}(y)} T_A(x) \right) = T_{f(K)}(y) \wedge T_{f(A)}(y) = T_{f(K) \cap f(A)}(y)$. Similarly we have $I_{f(H)}(y) = I_{f(K) \cap f(A)}(y)$ and $F_{f(H)}(y) = F_{f(K) \cap f(A)}(y)$.Hence the proof. ■

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