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Some Remarkson Fuzzy Volterra Spaces

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Abstract

In this paper, the conditions under which fuzzy topological spaces become fuzzy Volterra spaces, are studied.

Key Words: Fuzzy dense set, fuzzy G_δ -set, fuzzy σ-nowhere dense set, fuzzy P-space, fuzzy Baire space, fuzzy D-Baire space, fuzzy sub maximal space, some what fuzzy continuous function, somewhat fuzzy open function.
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Introduction

The concept of fuzzysets and fuzzy set operations were first introduced by L. A. ZADEH [17] in 1965. This concept provides a natural foundation for treating mathematically the fuzzy phenomena, which exist pervasively in our real world. The first notion of fuzzy topological spaces had been defined by C.L.CHANG[3] in 1968 and this paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

The concepts of Volterraspaces have been studied extensively in classical topology in[4], [5],[6] , [7] and [8]. The concepts of Volterraspaces andweaklyVolterra spaces in fuzzy setting wereintroduced and studied by the authors in [15]. In this paper we study the conditions underwhich a fuzzy topological space becomes a fuzzy Volterra space and fuzzy Bairespaces, fuzzy D- Baire spaces, fuzzy submaximal spaces and fuzzy P-spaces are considered for this work.

Preliminaries

Now we introduce some basic notions and results used in this sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang [4]. A fuzzy set λ on a set X is a function from X to [0,1], that is,λ : X→ [0,1].

Definition 21 :Letλ andμbe fuzzy sets in X. Then, for all $x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$
(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$
(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$
(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$
(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$

For a family $\{\lambda_i / i \in I\}$of fuzzysets in(X, T), the unionψ $= \vee_i (\lambda_i)$ and theintersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(vi) $\psi(x) = sup_i \{\lambda_i(x) / x \in X\}$
(vii) $\delta(x) = inf_i\{\lambda_i(x) / x \in X\}$.

Definition 2.2 : Let (X,T) be a fuzzy topological space. For a fuzzy set λ of X, the fuzzy interiorint (λ) and the fuzzy closure cl(λ), are defined respectively as

(i) int (λ) = $\vee \{ \mu / \mu \leq \lambda, \mu \in T \}$,
(ii) cl (λ) = $\wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

Lemma 2.1 [1] : For a fuzzy set λ of a fuzzy topological space X,

(i). 1 − Int (λ) = Cl (1− λ),
(ii). 1 − Cl (λ) = Int (1 – λ).

Definition 2.3 [13] : A fuzzy set λin a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, cl(λ) = 1.

Definition 2.4 [13] : A fuzzy set λ in a fuzzy topological space (X,T) is calledfuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that μ < cl (λ). That is,int cl (λ) = 0.

Definition 2.5 [2]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(1 - \lambda_i) \in T$ for i∈I.

Definition 2.6 [2]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy G_δ -set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$ where $\lambda_i \in T$ for i∈I.

Definition 2.7 [2] : A fuzzy topological space (X,T) is called a fuzzy submaximalspace if cl(λ) = 1 for any non-zero fuzzy set λ in (X,T), then $\lambda \in T$.

Definition 2.8 [13]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where ($\lambda_i$)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T), is said to be of second categoryin (X,T).

Lemma 2.2 [1] : For a family of $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X,T), $\vee$ cl $(\lambda_\alpha) \leq$ cl $(\vee(\lambda_\alpha))$. In case $\mathcal{A}$ is a finite set, $\vee$ cl (λ_α) = cl $(\vee(\lambda_\alpha))$. Also $\vee$int$(\lambda_\alpha) \leq$int $(\vee(\lambda_\alpha))$.

Definition 2.9 [14] : Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy σ -nowhere dense set, if λ is a fuzzy F_σ- set in (X,T) such that int (λ) = 0.

Definition 2.10 [14]: A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy σ - first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Any other fuzzy set in (X,T) is said to be of fuzzy σ -second category .

Definition2.11[9] : Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Bairespace if int $(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where($\lambda_i$)'s are fuzzy nowhere dense sets in (X,T).

Definition 2.12 [16] : A fuzzy topological space (X,T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is , every non-zero fuzzy G_δ -set in (X,T), is fuzzy open in (X,T).

Fuzzy Volterra Spaces

Definition 3.1 [15]:A fuzzy topological space (X,T) is called a fuzzy Volterraspace if $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$,where (λ_i)'s arefuzzy dense and fuzzy G_δ -sets in (X,T).

Definition 3.2 [15]: A fuzzy topological space (X,T) is called a fuzzy weakly Volterra space if $cl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$, where (λ_i)'s arefuzzy denseand fuzzy G_δ -sets in (X,T).

Theorem 3.1 [9]:: Let (X,T) be a fuzzy topological space. Then the following are equivalent :

1) (X, T) is a fuzzy Baire space.
2) Int(λ) = 0 for every fuzzy first category set λ in (X,T).
3) cl (μ) = 1 for every fuzzy residual set μ in (X,T).

Theorem 3.2[10]: If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X,T), then 1 – λ is a fuzzy first category set in (X,T).

The conditions under which fuzzy Baire spaces become fuzzyVolterraspaces, are given in the following propositions :

Proposition 3.1 : If each fuzzy firstcategory set is afuzzy closed set inafuzzy Baire space (X,T), then (X, T) is a fuzzy Volterra space.

Proof: Let (λ_i)'s ($i = 1$ to$N$) be fuzzy dense and fuzzy G_δ-sets in (X,T). Then, by theorem 3.2, $(1 - \lambda_i)$' s are fuzzy first category sets in (X,T). By hypothesis $(1 - \lambda_i)'s$ are fuzzy closed sets in (X,T) and hence (λ_i)' s are fuzzy open sets in (X,T). Thus(λ_i)'s ($i = 1$ to$N$) are fuzzy dense and fuzzy open sets in (X,T). Letμ_α($\alpha = 1$ to ∞) be fuzzy dense and fuzzy open sets in (X,T) inwhich the first N fuzzy open and fuzzydense sets be(λ_i)($i = 1$ to$N$).Since (μ_α)'s are fuzzy open sets, $(1 - \mu_\alpha)$'s are fuzzy closed sets and hence we have cl $(1 - \mu_\alpha) = 1 - \mu_\alpha$. Also cl$(\mu_\alpha) = 1$, implies that int $(1 - \mu_\alpha) = 0$ andint cl $(1 - \mu_\alpha) =$ int $(1 - \mu_\alpha) = 0$. Thus $(1 - \mu_\alpha)$'s are fuzzy nowhere dense sets in (X,T). Then $\bigvee_{\alpha=1}^{\infty}(1 - \mu_\alpha)$ is a fuzzyfirst category set in (X,T). Since(X,T) is a fuzzy Baire space, by theorem3.1, int $[\bigvee_{\alpha=1}^{\infty}(1 - \mu_\alpha)] = 0$.This implies thatcl $(\bigwedge_{\alpha=1}^{\infty}(\mu_\alpha)) = 1$. But cl $[\bigwedge_{\alpha=1}^{\infty}(\mu_\alpha)] \leq$ cl $[\bigwedge_{\alpha=1}^{N}(\mu_\alpha)]$ .Hencewe have, $1 \leq$ cl $[\bigwedge_{\alpha=1}^{N}(\mu_\alpha)]$. That is,cl$[\bigwedge_{\alpha=1}^{N}(\mu_\alpha)] = 1$. This implies that,cl $[\bigwedge_{i=1}^{N}(\lambda_i)] = 1$. Hence cl$(\bigwedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ -sets in (X,T), implies that (X,T) is a fuzzy Volterra space.

Proposition 3.2: If each fuzzy residual set is a fuzzy open set in a fuzzy Baire space (X,T), then (X, T) is a fuzzy Volterra space.

Proof: Let fuzzy residual sets (δ_i)'s be fuzzy open setsina fuzzy Baire space(X,T). Then ($1-\delta_i$)'sare fuzzy first category sets in (X,T) and$(1-\delta_i)'s$are fuzzy closed sets in (X,T). That is,the fuzzy first category sets$(1 - \delta_i)'s$, are fuzzy closed sets in (X,T).Hence, byproposition 3.1, (X,T) is a fuzzy Volterraspace.

Theorem 3.3 [9]: If λ is a fuzzy dense and fuzzy open set in (X,T), then $1 - \lambda$ is a fuzzy nowhere dense set in (X,T).

Definition 3.3 [12]:A fuzzy topological space (X,T) is called a fuzzy D- Bairespace if every fuzzy first category set in (X,T), is a fuzzy nowhere dense set in (X,T).

Proposition 3.3: If the fuzzy topological space (X,T) is a fuzzy D-Baire space, then (X,T) is a fuzzy Volterra space.

Proof: Let (λ_i)' s ($i = 1$ to$N$) be fuzzy dense and fuzzy G_δ-sets in (X,T). Then, by theorem 3.2, $(1 - \lambda_i)$' s are fuzzy first category sets in (X,T).Since (X,T) is a fuzzy D-Baire space, the fuzzy first category sets $(1 - \lambda_i)$' s are fuzzy nowhere dense sets in (X,T). Let μ_α($\alpha = 1$ to ∞) be fuzzy nowhere dense sets in (X,T) in which the first N fuzzy nowhere dense setsbe $(1 - \lambda_i)$($i = 1$ to$N$). Then$\bigvee_{\alpha=1}^{\infty}(\mu_\alpha)$ is a fuzzy first category set in (X,T). Again since (X,T) is a fuzzy D-Baire space, the fuzzy first category set $\bigvee_{\alpha=1}^{\infty}(\mu_\alpha)$ is a fuzzy nowhere dense set in (X,T). Then,int cl $[\bigvee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$.But int $[\bigvee_{\alpha=1}^{\infty}(\mu_\alpha)] \leq$int cl $[\bigvee_{\alpha=1}^{\infty}(\mu_\alpha)]$, implies that, int [

$\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$. Then, $1-\text{int}[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 1$ and hence$\text{cl}[\wedge_{\alpha=1}^{\infty}(1-(\mu_\alpha))] = 1$.But,$\text{cl}[\wedge_{\alpha=1}^{\infty}(1-(\mu_\alpha))] \leq \text{cl}[\wedge_{\alpha=1}^{N}(1-(\mu_\alpha))]$. Hence we have, $1 \leq \text{cl}[\wedge_{\alpha=1}^{N}(1-(\mu_\alpha))]$. That is,$\text{cl}[\wedge_{\alpha=1}^{N}(1-(\mu_\alpha))] = 1$. This implies that,$\text{cl}[\wedge_{i=1}^{N}(1-((1-\lambda_i))] = 1$. That is, $\text{cl}(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)' s are fuzzy dense and fuzzy G_δ -sets in (X,T).Hence (X,T) is a fuzzy Volterra space.

Proposition 3.4 : If the fuzzy topological space (X,T) is a fuzzyBaire and fuzzy P-space , then (X,T) is a fuzzy Volterra space.

Proof: Let (λ_i)'s (i = 1 to N) be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X, T) is a fuzzy P-space, the fuzzy G_δ-sets (λ_i)' s in (X,T), are fuzzy open sets in (X,T).Hence (λ_i)' s (i = 1 toN) are fuzzy dense and fuzzyopen sets in (X,T). Then,by theorem 3.3, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Let $\mu_\alpha(\alpha = 1 \text{ to } \infty)$ be fuzzy nowhere dense sets in (X,T) in which the first N fuzzy nowhere dense sets be $(1-\lambda_i)$'s(i = 1 toN). Since (X,T) is a fuzzy Bairespace, $\text{int}[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$.Now $\text{int}[\vee_{i=1}^{N}(1-\lambda_i)] \leq \text{int}[\vee_{\alpha=1}^{\infty}(\mu_\alpha)]$, implies thatint $[\vee_{i=1}^{N}(1-\lambda_i)] \leq 0$. That is, $\text{int}[\vee_{i=1}^{N}(1-\lambda_i)] = 0$. Then, $\text{int}[1-(\wedge_{i=1}^{N}(\lambda_i))] = 0$ and hence $1-\text{cl}(\wedge_{i=1}^{N}(\lambda_i))] = 0$, implies that $\text{cl}(\wedge_{i=1}^{N}(\lambda_i)) = 1$,where (λ_i)' s are fuzzy dense and fuzzy G_δ -sets in (X,T).Hence (X,T) is a fuzzy Volterra space.

Proposition 3.5 : If the fuzzy topological space (X,T) is a fuzzy Baire andfuzzy submaximal space , then (X,T) is a fuzzy Volterra space.

Proof: Let (λ_i)'s(i = 1 to N)be fuzzy dense and fuzzy G_δ-sets in (X,T).Since (X, T) is a fuzzy submaximal space,the fuzzy dense sets (λ_i)'s are fuzzy open setsin(X,T). Then (λ_i)' s (i = 1 toN) are fuzzy dense and fuzzy open sets in (X,T). Then, proceeding as in the proof of 3.4, we have$\text{cl}(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ -sets in (X,T).Hence (X,T) is a fuzzy Volterra space.

Theorem 3.4 [14] : In a fuzzy topological space (X,T), a fuzzy set λ is a fuzzy σ –nowhere dense set if and only if $1-\lambda$ is a fuzzy dense and fuzzy G_δ -set.

Proposition 3.6: If each fuzzy σ -first category set is a fuzzy σ - nowhere dense set in a fuzzy topological space (X,T), then (X, T) is a fuzzy Volterra space.

Proof: Let (λ_i)'s (i = 1 to N) be fuzzy dense and fuzzy G_δ-sets in (X,T).Then,by theorem3.4, $(1-\lambda_i)$'s are fuzzy σ -nowheredense sets in (X,T). Let $\mu_\alpha(\alpha = 1 \text{ to } \infty)$ be fuzzy σ - nowhere dense sets in (X,T) in which the first N fuzzy σ –nowhere densesetsbe$(1-\lambda_i)$'s(i =1 toN).Now$\vee_{\alpha=1}^{\infty}(\mu_\alpha)$ is a fuzzyσ-first category set in (X,T). By hypothesis,$\vee_{\alpha=1}^{\infty}(\mu_\alpha)$ is a fuzzyσ- nowhere dense set in(X,T).Then,$\vee_{\alpha=1}^{\infty}(\mu_\alpha)$ is a fuzzyF_σ- set in (X,T) such that $\text{int}[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$. Now $\text{int}[\vee_{i=1}^{N}(1-\lambda_i)] \leq \text{int}[\vee_{\alpha=1}^{\infty}(\mu_\alpha)]$, implies that $\text{int}[\vee_{i=1}^{N}(1-\lambda_i)] \leq 0$. That is, $\text{int}[\vee_{i=1}^{N}(1-\lambda_i)] = 0$. Then, int$[1-(\wedge_{i=1}^{N}(\lambda_i))] = 0$andhence$[1-cl(\wedge_{i=1}^{N}(\lambda_i))] = 0$. Thus, $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ -sets in (X,T), implies that (X,T) is a fuzzy Volterraspace.

Functions and Fuzzy Volterra Spaces

In this section, by means of fuzzy functions, the conditions under which fuzzy topological spacesbecomefuzzyVolterraspaces, are studied.

Definition 4.1[1] :A function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called fuzzy semi- continuous if f^{-1} (λ) is fuzzy semi-open in (X,T) for each fuzzy open set λ in (Y,S) .

Definition 4.2 [13]: A function f : (X,T)→(Y,S) from a fuzzy topological space (X,T) intoanother fuzzy topological space (Y,S) is called somewhat fuzzy continuous if$\lambda \in$ S and $f^{-1}(\lambda) \neq 0$ implies that there exist a fuzzy open set μ in (X ,T) such that $\mu \neq 0$ and$\mu \leq f^{-1}(\lambda)$.

Definition 4.3 [13]: A function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is called somewhat fuzzy open if $\lambda \in$ T and $\lambda \neq 0$ implies that there exists a fuzzy open set μ in (Y,S) such that $\mu \neq 0$ and $\mu \leq f(\lambda)$.

Theorem 4.1 [11]: If the function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is fuzzy semi-continuous and somewhat fuzzy open and somewhat fuzzy continuous function and if (X,T) is a fuzzy Baire space (Y,S), then(Y,S)is a fuzzy Baire space.

Proposition 4.1 : If the function f : (X,T) → (Y,S) from a fuzzy Baire space (X,T) into a fuzzy P-space (Y,S) is a fuzzy semi-continuous,somewhat fuzzy open and somewhat fuzzy continuous function, then(Y,S)is a fuzzy Volterra space.

Proof: Let f be a fuzzy semi-continuous, somewhat fuzzy open and somewhat fuzzy continuous function from a fuzzy Baire space (X,T) into a fuzzy P-space (Y,S) . By theorem 4.1, (Y,S)is a fuzzy Baire space. Hence (Y,S)is a fuzzy Baire and fuzzy P-space. Then, by proposition 3.4, (Y,S)is a fuzzy Volterra space.

Proposition 4.2 : If the function f : (X,T) → (Y,S) from a fuzzy Baire space (X,T) into a fuzzy submaximal space (Y,S) is a fuzzy semi-continuous, somewhat fuzzy open and somewhat fuzzy continuous function, then(Y,S)is a fuzzy Volterra space.

Proof: Let f be a fuzzy semi-continuous, somewhat fuzzy open and somewhat fuzzy continuous function from a fuzzy Baire space (X,T) into a fuzzy submaximal space

(Y,S) . By theorem 4.1, (Y,S)is a fuzzy Baire space. Hence (Y,S)is a fuzzy Baire and fuzzy submaximal space. Then, byproposition 3.5, (Y,S)is a fuzzy Volterraspace.

Theorem 4.2 [11]: If the function f :(X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) is a somewhat fuzzy continuous, somewhat fuzzy open and 1−1 and onto function and if (Y,S) is a fuzzy Baire space, then(X,T)is a fuzzy Baire space.

Proposition 4.3 : If the function f : (X,T) → (Y,S) from a fuzzy P-space (X,T) into a fuzzyBaire (Y,S) is a somewhat fuzzy continuous, somewhat fuzzy open and 1−1 and onto function , then(X,T) is a fuzzy Volterra space.

Proof: Let f be a somewhat fuzzy continuous, somewhat fuzzy open and 1−1 and onto function from a fuzzy P-space (X,T) into a fuzzy Baire space (Y,S) . By theorem 4.2, (X,T)is a fuzzy Baire space. Hence (X,T)is a fuzzy Baire and fuzzy P-space. Then, by proposition 3.4, (X,T)is a fuzzy Volterra space.

Proposition 4.4 : If the function f : (X,T) → (Y,S) from a fuzzy submaximal space (X,T) into a fuzzy Baire (Y,S) is a somewhat fuzzy continuous, somewhat fuzzy open and 1−1 and onto function , then(X,T) is a fuzzy Volterra space.

Proof: Let f be a somewhat fuzzy continuous,somewhat fuzzy open and 1−1 and onto function from a fuzzy submaximal space (X,T) into a fuzzy Baire space (Y,S) . By theorem 4.2, (X,T)is a fuzzy Baire space. Hence (X,T)is a fuzzy Baire and fuzzy submaximal space. Then, by proposition 3.5, (X,T)is a fuzzy Volterra space.

Theorem 4.3 [11]: Let the function f : (X,T) → (Y,S) from a fuzzy topological space (X,T) into another fuzzy topological space (Y,S) be fuzzy continuous, fuzzy open, 1 -1 and onto function. If (X,T) is a fuzzyBaireSpace, then (Y,S) is a fuzzy Bairespace.

Proposition 4.5 : If the function f : (X,T) → (Y,S) from a fuzzy Baire space (X,T) into a fuzzy P-space (Y,S) , is fuzzy continuous, fuzzy open, 1 -1 and onto function, then(Y,S)is a fuzzy Volterra space.

Proof: Let f be a fuzzy continuous, fuzzy open, 1 -1 and onto function from a fuzzy Baire space (X,T) into a fuzzy P-space (Y,S) . By theorem 4.3, (Y,S)is a fuzzy Baire space. Hence (Y,S)is a fuzzy Baire and fuzzy P-space. Then, by proposition 3.4, (Y,S)is a fuzzy Volterra space.

Proposition 4.6 : If the function f : (X,T) → (Y,S) from a fuzzy Baire space (X,T) into a fuzzy submaximal space (Y,S) , is fuzzy continuous, fuzzy open, 1 -1 and onto function, then(Y,S)is a fuzzy Volterra space.

Proof: Let f be a fuzzy continuous, fuzzy open, 1 -1 and onto function from a fuzzy Baire space (X,T) onto a fuzzy submaximal space (Y,S) . By theorem 4.3, (Y,S)is a fuzzy Baire space. Hence (Y,S)is a fuzzy Baire and fuzzy submaximal space. Then, by proposition 3.5, (Y,S)is a fuzzy Volterra space.

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