

## Common Fixed Point Theorem for Pair of Subcompatible Maps in Fuzzy Metric Space

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### Abstract

Depending upon the fuzzy concepts of implicit relation, coincidence point, subcompatible maps and subsequentially continuous maps, in this paper, we have established the existence of unique common fixed point for four self mappings having implicit relation, coincidence point, subcompatible maps and subsequentially continuous maps in a fuzzy metric space. Also the common fixed points for a sequence of mappings have been established under these conditions. In the last section, we have established a near-contractive common fixed point theorem, which was initiated in the paper [9].

**Keywords:** Fuzzy Metric Space, Fixed Point, Common Fixed Point, Coincidence point, Subcompatible maps, Subsequentially continuous maps, Implicit Relation, A near-contractive common fixed point.

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### Introduction

Ever since the concept of fuzzy sets was introduced by Zadeh [8] in 1965 to describe the situation in which data are imprecise or vague or uncertain. Many authors [13, 14, 17] etc. have extensively developed the theory of fuzzy sets due to a wide range of application in the field of population dynamics, chaos control, computer programming, medicine, etc. Kramosil and Michalek [11] introduced the concept of fuzzy metric spaces (briefly, FM-spaces) in 1975, which opened an avenue for further development of analysis in such spaces. Later on it is modified that a few concepts of mathematical analysis have been developed by George and Veeramani [1, 2] and also they have developed the fixed point theorem in fuzzy metric space [12]. In fuzzy metric space, the notion of compatible maps under the name of asymptotically commuting maps was introduced in the paper [14] and then in the paper [4], the

notion of weak compatibility has been studied in fuzzy metric space. However, the study of common fixed points of non-compatible maps is of great interest, which has been initiated by Pant. With the help of the property (E.A.), which was introduced in the paper [10], Pant and Pant [17] studied the common fixed points of a pair of non-compatible maps in fuzzy metric space.

In fact, there have been also attempts to “fuzzify” various mathematical concepts. The concepts of subcompatible maps, subsequential maps were introduced in the paper [6] and later on these were fuzzified in the paper [13]. The concepts of subcompatibility and subsequential continuity, which are respectively weaker than occasionally weak compatibility [7] and reciprocal continuity, depending upon these fuzzy concepts, in this paper, we have established the existence of unique common fixed point for four self mappings having implicit relation, coincidence point, subcompatible maps and subsequentially continuous maps in fuzzy metric space. Also the common fixed points for a sequence of mappings have been established under these conditions. In the last section, we have established a near-contractive common fixed point theorem, which was initiated in the paper [9].

## Preliminaries

We quote some definitions and statements of a few theorems which will be needed in the sequel.

**Definition 2.1** [3] A binary operation  $*$  :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is continuous  $t$ -norm if  $*$  satisfies the following conditions :

- (i)  $*$  is commutative and associative,
- (ii)  $*$  is continuous,
- (iii)  $a * 1 = a \forall a \in [0, 1]$ ,
- (iv)  $a * b \leq c * d$  whenever  $a \leq c, b \leq d$  and  $a, b, c, d \in [0, 1]$ .

**Result 2.2** [5] (a) For any  $r_1, r_2 \in (0, 1)$  with  $r_1 > r_2$ , there exist  $r_3 \in (0, 1)$  such that  $r_1 * r_3 > r_2$ ,

(b) For any  $r_5 \in (0, 1)$ , there exist  $r_6 \in (0, 1)$  such that  $r_6 * r_6 \geq r_5$ .

**Definition 2.3** [1] The 3-tuple  $(X, \mu, *)$  is called a **fuzzy metric space** if  $X$  is an arbitrary non-empty set,  $*$  is a continuous  $t$ -norm and  $\mu$  is a fuzzy set in  $X^2 \times (0, \infty)$  satisfying the following conditions :

- (i)  $\mu(x, y, t) > 0$  ;
- (ii)  $\mu(x, y, t) = 0$  if and only if  $x = y$  ;
- (iii)  $\mu(x, y, t) = \mu(y, x, t)$  ;

$$(iv) \mu(x, y, s) * \mu(y, z, t) \leq \mu(x, z, s+t);$$

(v)  $\mu(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$  is continuous; for all  $x, y, z \in X$  and  $t, s > 0$ .

**Definition 2.4** [15] Let  $(X, \mu, *)$  be a fuzzy metric space. A sequence  $\{x_n\}_n$  in  $X$  is called **Cauchy sequence** if and only if

$$\lim_{n \rightarrow \infty} \mu(x_n, x_{n+p}, t) = 1 \text{ for each } t > 0 \text{ and } p = 1, 2, 3, \dots$$

A sequence  $\{x_n\}_n$  in  $X$  is said to **converge** to  $x \in X$  if and only if

$$\lim_{n \rightarrow \infty} \mu(x_n, x, t) = 1 \text{ for each } t > 0.$$

A fuzzy metric space  $(X, \mu, *)$  is said to be **complete** if and only if every Cauchy sequence in  $X$  is convergent in  $X$ .

**Definition 2.5** [14] Let  $A$  and  $B$  be maps from an fuzzy metric space  $(X, \mu, *)$  into itself. The maps  $A$  and  $B$  are said to be **compatible** (or **asymptotically commuting**), if

$$\lim_{n \rightarrow \infty} \mu(ABx_n, BAx_n, t) = 1 \quad \forall t > 0,$$

whenever  $\{x_n\}_n$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z$  for some  $z \in X$ .

**Definition 2.6** [13] Let  $(X, \mu, *)$  be a fuzzy metric space.  $A$  and  $S$  be self maps on  $X$ . A point  $x$  in  $X$  is called a **coincidence point** of  $A$  and  $S$  iff  $Ax = Sx$ .

In this case  $w = Ax = Sx$  is called a **point of coincidence** of  $A$  and  $S$ .

**Definition 2.7** [13] Let  $(X, \mu, *)$  be fuzzy metric space. Self maps  $A$  and  $S$  on  $X$  are said to be **subsequentially continuous** if and only if there exist a sequence  $\{x_n\}_n$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z, \quad z \in X \text{ and satisfy}$$

$$\lim_{n \rightarrow \infty} ASx_n = Az, \quad \lim_{n \rightarrow \infty} SAx_n = Sz.$$

Clearly, if  $A$  and  $S$  are continuous or reciprocally continuous then they are obviously subsequentially continuous.

**Definition 2.8** [13] Let  $(X, \mu, *)$  be fuzzy metric space. Self maps  $A$  and  $S$  on  $X$  are said to be **Subcompatible** if and only if there exists a sequence  $\{x_n\}_n$  in  $X$  such

that  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$ ,  $z \in X$  and satisfy

$$\lim_{n \rightarrow \infty} \mu(ASx_n, SAx_n, t) = 1.$$

**Lemma 2.9** [16] Let  $(X, \mu, *)$  be fuzzy metric space. If  $x_n \rightarrow x$  and  $y_n \rightarrow y$  in  $(X, \mu, *)$  then  $\mu(x_n, y_n, t) \rightarrow \mu(x, y, t)$  as  $n \rightarrow \infty$  for all  $t > 0$  in  $\mathbb{R}$ .

### A General Common Fixed Point Theorem

**Theorem 3.1** Let  $f, g, h$  and  $k$  be four self maps of a fuzzy metric space  $(X, \mu, *)$ . If the pairs  $\{f, h\}$  and  $\{g, k\}$  are subcompatible and subsequentially continuous, then

- $f$  and  $h$  have a coincidence point,
- $g$  and  $k$  have a coincidence point.

Further, let  $\varphi: I^6 \rightarrow I$  be continuous function satisfying the following condition:

$$(\varphi_1): \varphi(u(t), u(t), 1, 1, u(t), u(t)) > 1, \quad u(t) \in [0, 1)$$

We suppose that  $\{f, h\}$  and  $\{g, k\}$  satisfy, for all  $x$  and  $y$  in  $X$ ,  
 $(\varphi_2): \varphi(\mu(fx, gy, t), \mu(hx, ky, t), \mu(fx, hx, t), \mu(gy, ky, t), \mu(hx, gy, t), \mu(ky, fx, t)) \leq 1$   
 Then  $f, g, h$  and  $k$  have a unique common fixed point.

**Proof.** Since the pairs  $\{f, h\}$  and  $\{g, k\}$  are subcompatible and subsequentially continuous, there exists two sequences  $\{x_n\}_n$  and  $\{y_n\}_n$  in  $X$  such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} hx_n = l$$

for some  $l \in X$  and which satisfy

$$\lim_{n \rightarrow \infty} \mu(f h x_n, h f x_n, t) = 1 \Rightarrow \mu(f l, h l, t) = 1 \Rightarrow f l = h l$$

that is,  $l$  is a coincidence point of  $f$  and  $h$ . Also, we have

$$\lim_{n \rightarrow \infty} g y_n = \lim_{n \rightarrow \infty} k y_n = l'$$

for some  $l' \in X$  and which satisfy

$$\lim_{n \rightarrow \infty} \mu(g k y_n, k g y_n, t) = 1 \Rightarrow \mu(g l', k l', t) = 1 \Rightarrow g l' = k l'$$

that is,  $l'$  is a coincidence point of  $g$  and  $k$ .

Now, we prove that  $l = l'$ . Putting  $x = x_n$  and  $y = y_n$  in  $(\varphi_2)$ , we have  $\varphi(\mu(f x_n, g y_n, t), \mu(h x_n, k y_n, t), \mu(f x_n, h x_n, t), \mu(g y_n, k y_n, t), \mu(h x_n, g y_n, t), \mu(k y_n, f x_n, t)) \leq 1$ .

Taking the limit as  $n \rightarrow \infty$ , we have

$$\begin{aligned} &\varphi(\mu(l, l', t), \mu(l, l', t), \mu(l, l, t), \mu(l', l', t), \mu(l, l', t), \mu(l, l', t)) \leq 1 \\ \Rightarrow &\varphi(\mu(l, l', t), \mu(l, l', t), 1, 1, \mu(l, l', t), \mu(l, l', t)) \leq 1 \end{aligned}$$

which contradicts  $(\varphi_1)$  if  $l \neq l'$  and this implies that

$$\mu(l, l', t) = 1 \Rightarrow l = l'.$$

Also we claim that  $f l = l$ . If  $f l \neq l$ , using  $(\varphi_2)$  we have

$$\begin{aligned} &\varphi(\mu(f l, g y_n, t), \mu(h l, k y_n, t), \mu(f l, h l, t), \mu(g y_n, k y_n, t), \\ &\mu(h l, g y_n, t), \mu(k y_n, f l, t)) \leq 1. \end{aligned}$$

Taking the limit as  $n \rightarrow \infty$

$$\begin{aligned} &\varphi(\mu(f l, l, t), \mu(h l, l, t), \mu(f l, h l, t), \mu(g l, k l, t), \mu(h l, l, t), \mu(l, f l, t)) \leq 1 \\ \Rightarrow &\varphi(\mu(f l, l, t), \mu(h l, l, t), 1, 1, \mu(h l, l, t), \mu(l, f l, t)) \leq 1 \end{aligned}$$

which is a contradiction of  $(\varphi_1)$ , then we get

$$\mu(f l, l, t) = 1 \Rightarrow f l = l \Rightarrow l = f l = h l$$

Again, suppose that  $gl \neq l$ . Putting  $x = y = l$  in  $(\varphi_2)$

$$\begin{aligned} & \varphi(\mu(fl, gl, t), \mu(hl, kl, t), \mu(fl, hl, t), \mu(gl, kl, t), \mu(hl, gl, t), \mu(kl, fl, t)) \leq 1 \\ \Rightarrow & \varphi(\mu(fl, gl, t), \mu(hl, kl, t), 1, 1, \mu(hl, gl, t), \mu(kl, fl, t)) \leq 1 \end{aligned}$$

which contradicts  $(\varphi_1)$ . Thus  $l = gl = kl$ . Therefore  $l = fl = gl = hl = kl$  i.e,  $l = l'$  is common fixed point of  $f, g, h$  and  $k$ .

Finally, suppose that there exists another common fixed point  $z$  of  $f, g, h$  and  $k$  such that  $z \neq l$ . Then by inequality  $(\varphi_2)$  we get

$$\begin{aligned} & \varphi(\mu(fl, gz, t), \mu(hl, kz, t), \mu(fl, hl, t), \mu(gz, kz, t), \\ & \mu(hl, gz, t), \mu(kz, fl, t)) \leq 1 \\ \Rightarrow & \varphi(\mu(l, z, t), \mu(l, z, t), 1, 1, \mu(l, z, t), \mu(l, z, t)) \leq 1 \end{aligned}$$

which is a contradiction of  $(\varphi_1)$  then we get  $z = l$ .

**Corollary 3.2:** Let  $f, g, h$  be three self maps of a fuzzy metric space  $(X, \mu, *)$ . Suppose that the pairs  $\{f, h\}$  and  $\{g, h\}$  are subcompatible and subsequentially continuous, then

- $f$  and  $h$  have a coincidence point,
- $g$  and  $h$  have a coincidence point.

Let  $\varphi: I^6 \rightarrow I$  be continuous function satisfying the condition  $(\varphi_1)$  and  $\varphi(\mu(fx, gy, t), \mu(hx, hy, t), \mu(fx, hx, t), \mu(gy, hy, t), \mu(hx, gy, t), \mu(hy, fx, t)) \leq 1$  for all  $t > 0$  and for all  $x, y$  in  $X$ . Then  $f, g$  and  $h$  have a unique common fixed point.

**Theorem 3.3** Let  $h, k$  and  $\{f_n\}_n$  be maps of a fuzzy metric space  $(X, \mu, *)$  into itself such that the pairs  $\{f_n, h\}$  and  $\{f_{n+1}, k\}$  are subcompatible and subsequentially continuous. Then for each  $n$

- $f_n$  and  $h$  have a coincidence point,
- $f_{n+1}$  and  $k$  have a coincidence point.

Further we suppose that  $f_n, f_{n+1}, h$  and  $k$  satisfy the inequality

$$\begin{aligned} & \varphi(\mu(f_n x, f_{n+1} y, t), \mu(hx, ky, t), \mu(f_n x, hx, t), \mu(f_{n+1} y, ky, t), \\ & \mu(hx, f_{n+1} y, t), \mu(ky, f_n x, t)) \leq 1 \end{aligned}$$

for all  $x$  and  $y$  in  $X$ , for every  $n \in \mathbb{N}$ , where  $\varphi$  is as in Theorem(3.1). Then  $h, k$  and  $\{f_n\}_n$  have a unique common fixed point.

**Proof.** By letting  $n = 1$  we get the assumptions of Theorem(3.1) for  $h, k, f_1$  and  $f_2$ , and hence we have the unique common fixed point  $l$  of  $h, k, f_1$  and  $f_2$ . Now,  $l$  is the unique common fixed point of  $h, k, f_1$ . Otherwise, if  $l'$  is another common fixed point of  $h, k, f_1$ , then by the inequality (iii) we see for all  $t > 0$ ,

$$\begin{aligned} & \varphi(\mu(f_1 l', f_2 l, t), \mu(hl', kl, t), \mu(f_1 l', hl', t), \mu(f_2 l, kl, t), \\ & \mu(hl', f_2 l, t), \mu(kl, f_1 l', t)) \leq 1 \\ \Rightarrow & \varphi(\mu(l', l, t), \mu(l', l, t), 1, 1, \mu(l', l, t), \mu(l', l, t)) \leq 1 \end{aligned}$$

which contradicts  $(\varphi_1)$ , then  $l = l'$ . By the same manner, it can be proved that  $l$  is the unique common fixed point of  $h, k, f_2$ .

Now, letting  $n = 2$  we obtain hypotheses of Theorem(3.1) for  $h, k, f_2$  and  $f_3$  and then, they have a unique common fixed point  $l$ . Analogously,  $l$  is the unique common fixed point of  $h, k, f_2$  and  $h, k, f_3$ . Continuing by this method, we clearly see that  $l$  is the required element.

**Theorem 3.4** Let  $f, g, h$  and  $k$  be four self mappings of a fuzzy metric space  $(X, \mu, *)$ . If the pairs  $\{f, h\}$  and  $\{g, k\}$  are subcompatible and subsequentially continuous, then

- $f$  and  $h$  have a coincidence point,
- $g$  and  $k$  have a coincidence point.

Let  $\varphi' : I^4 \rightarrow I$  be continuous function satisfying the following condition :

$$(\varphi'_1) : \varphi'(u(t), u(t), u(t), u(t)) > 1, \quad u(t) \in [0, 1)$$

We further suppose that  $\{f, h\}$  and  $\{g, k\}$  satisfy, for all  $x$  and  $y$  in  $X$ ,

$$(\varphi'_2): \varphi'(\mu(fx, gy, t), \mu(hx, ky, t), \mu(hx, gy, t), \mu(ky, fx, t)) \leq 1$$

Then  $f, g, h$  and  $k$  have a unique common fixed point.

**Proof.** The proof of (i) and (ii) directly follows from the proof of theorem(3.1). As in the theorem(3.1), suppose that

$$\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} h x_n = l, \quad \lim_{n \rightarrow \infty} g y_n = \lim_{n \rightarrow \infty} k y_n = l'$$

where  $l$  is a coincidence point of  $f$  and  $h$ , and  $l'$  is a coincidence point of  $g$  and  $k$ .

Putting  $x = x_n$  and  $y = y_n$  in  $(\varphi'_2)$ , we have

$$\varphi'(\mu(fx_n, gy_n, t), \mu(hx_n, ky_n, t), \mu(hx_n, gy_n, t), \mu(ky_n, fx_n, t)) \leq 1$$

taking the limit as  $n \rightarrow \infty$

$$\varphi'(\mu(l, l', t), \mu(l, l', t), \mu(l, l', t), \mu(l, l', t)) \leq 1$$

which contradicts  $(\varphi'_1)$  if  $0 \leq \mu(l, l', t) < 1$ , then we get

$$\mu(l, l', t) = 1 \Rightarrow l = l'$$

Also, we claim that  $fl = l$ . If  $fl \neq l$  using  $(\varphi'_2)$  we have

$$\varphi'(\mu(fl, gy_n, t), \mu(hl, ky_n, t), \mu(hl, gy_n, t), \mu(ky_n, fl, t)) \leq 1$$

taking the limit as  $n \rightarrow \infty$

$$\varphi'(\mu(fl, l, t), \mu(hl, l, t), \mu(hl, l, t), \mu(l, fl, t)) \leq 1$$

which contradicts  $(\varphi'_1)$ , then we get

$$\mu(fl, l, t) = 1 \Rightarrow fl = l \Rightarrow l = fl = hl$$

Similarly, we have  $l = gl = kl$ . Therefore  $l = fl = hl = gl = kl$ .

i.e,  $l = l'$  is a common fixed point of  $f, g, h$  and  $k$ .

Suppose that there exists another common fixed point  $z$  of  $f, g, h$  and  $k$  such

that  $z \neq l$ . Then by inequality  $(\varphi'_2)$  we get

$$\begin{aligned} \varphi'(\mu(fl, gz, t), \mu(hl, kz, t), \mu(hl, gz, t), \mu(kz, fl, t)) &\leq 1 \\ \Rightarrow \varphi'(\mu(l, z, t), \mu(l, z, t), \mu(l, z, t), \mu(z, l, t)) &\leq 1 \end{aligned}$$

which is a contradiction of  $(\varphi'_1)$ , then we get  $l = z$ . This completes the proof

### A Near-Contractive Common Fixed Point Theorem

**Theorem 4.1** Let  $(X, \mu, *)$  be fuzzy metric space,  $f, g, h$  and  $k$  be maps from  $X$  in to itself. If the pairs  $\{f, h\}$  and  $\{g, k\}$  are subcompatible and subsequentially continuous, then

- $f$  and  $h$  have a coincidence point,
- $g$  and  $k$  have a coincidence point.

Let  $\phi$  be continuous function of  $[0, 1]$  into itself and satisfying inequality

$$\begin{aligned} \phi(\mu(fx, gy, t)) &\geq a(\mu(hx, ky, t))\phi(\mu(hx, ky, t)) \\ &+ b(\mu(hx, ky, t)) \min\{\phi(\mu(hx, gy, t)), \phi(\mu(ky, fx, t))\} \end{aligned}$$

for all  $x$  and  $y$  in  $X$ , where  $a, b : [0, 1] \rightarrow [0, 1]$  are continuous and satisfying the condition  $a(t) + b(t) > 1 \forall t > 0$ .

Then  $f, g, h$  and  $k$  have a unique common fixed point.

**Proof.** The proof of (i) and (ii) is similar to proof of (i) and (ii) of theorem(3.1). As in the theorem (3.1), suppose that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} hx_n = l, \quad \lim_{n \rightarrow \infty} gy_n = \lim_{n \rightarrow \infty} ky_n = l'$$

where  $l$  is a coincidence point of  $f$  and  $h$ , and  $l'$  is a coincidence point of  $g$  and  $k$ .

Suppose that  $l \neq l'$ , then  $\mu(l, l', t) < 1$  for all  $t > 0$ , that is, the double inequality:  $0 < \mu(l, l', t) < 1$  holds.

Now putting  $x = x_n$  and  $y = y_n$  in (iii), we have

$$\phi(\mu(fx_n, gy_n, t)) \geq a(\mu(hx_n, ky_n, t))\phi(\mu(hx_n, ky_n, t))$$

$$+ b(\mu(hx_n, ky_n, t)) \min\left\{\phi(\mu(hx_n, gy_n, t)), \phi(\mu(ky_n, fx_n, t))\right\}$$

Taking limit as  $n \rightarrow \infty$ , we get

$$\begin{aligned}\phi(\mu(l, l', t)) &\geq [a(\mu(l, l', t)) + b(\mu(l, l', t))] \phi(\mu(l, l', t)) \\ &> \phi(\mu(l, l', t))\end{aligned}$$

which is a contradiction. Therefore, it must be the case that  $l = l'$ .

Next, if  $fl \neq l$ , the use of condition (iii) gives

$$\begin{aligned}\phi(\mu(fl, gy_n, t)) &\geq a(\mu(hl, ky_n, t)) \phi(\mu(hl, ky_n, t)) \\ &+ b(\mu(hl, ky_n, t)) \min\left\{\phi(\mu(hl, gy_n, t)), \phi(\mu(ky_n, fl, t))\right\}\end{aligned}$$

By property of  $\phi$ ,  $a$  and  $b$ , we get

$$\begin{aligned}\phi(\mu(fl, l, t)) &\geq [a(\mu(fl, l, t)) + b(\mu(fl, l, t))] \phi(\mu(fl, l, t)) \\ &> \phi(\mu(fl, l, t))\end{aligned}$$

this contradiction implies that  $fl = l$  and hence  $l = fl = hl$ .

Similarly, we have  $l = gl = kl$ . Now, assume that there exists another common fixed point  $z$  of  $f, g, h$  and  $k$  such that  $z \neq l$ . By inequality (iii) and properties of functions  $\phi, a$  and  $b$ , we obtain

$$\begin{aligned}\phi(\mu(l, z, t)) &= \phi(\mu(fl, gz, t)) \geq a(\mu(hl, kz, t)) \phi(\mu(hl, kz, t)) \\ &+ b(\mu(hl, kz, t)) \min\left\{\phi(\mu(hl, gz, t)), \phi(\mu(kz, fl, t))\right\} \\ &= [a(\mu(l, z, t)) + b(\mu(l, z, t))] \phi(\mu(l, z, t)) > \phi(\mu(l, z, t))\end{aligned}$$

this contradiction implies that  $z = l$ .

**Corollary 4.2** Let  $h, k$  and  $\{f_n\}_n$  be maps of a fuzzy metric space  $(X, \mu, *)$  into itself such that the pairs  $\{f_n, h\}$  and  $\{f_{n+1}, k\}$  are subcompatible and subsequentially continuous, Then for each  $n$

$f_n$  and  $h$  have a coincidence point,

$f_{n+1}$  and  $k$  have a coincidence point.

Let  $\phi$  be continuous function of  $[0, 1]$  in to itself such that  $\phi(t) = 1$  iff  $t = 1$  and satisfying inequality

$$\phi(\mu(f_n x, f_{n+1} y, t)) \geq a(\mu(hx, ky, t))\phi(\mu(hx, ky, t)) + b(\mu(hx, ky, t)) \min\{\phi(\mu(hx, f_{n+1} y, t)), \phi(\mu(ky, f_n x, t))\}$$

for all  $x$  and  $y$  in  $X$ , where  $a, b : [0, 1] \rightarrow [0, 1]$  are continuous and satisfying the condition  $a(t) + b(t) > 1 \quad \forall t > 0$ . Then  $h, k$  and  $\{f_n\}_n$  have a unique common fixed point.

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