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Fuzzy Integrated Inventory Model with Fuzzy Parameters

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Abstract

One of the important issue in trade credit is the integrated inventory model. In this paper, we have studied a Integrated inventory problem under uncertain cost and demand information. The aim of this paper is to find the optimum economic order quantity and total joint relevant inventory cost where input parameters annual demand, production cost, purchase cost and shortage cost are fuzzy trapezoidal numbers. Fuzzy arithmetic operation of function principle is proposed and graded mean integration representation method is used for defuzzying the fuzzy total joint relevant inventory cost.

Keywords: Fuzzy inventory, Fuzzy cost, Buyer, Vendor, Joint Economic Order Quantity, Function Principle, Graded Mean Integration Representation.

Introduction

This model consider a situation where a single vendor and buyer co-ordinate their production and inventory policies for the item that is produced and supplied by the vendor. The different strategies have been proposed in the literature for the problem. One where the delivery quantity to the buyer at each replenishment is identical and another strategy where at each delivery, all the inventory available with the vendor is supplied to the buyer.

Most of the inventory models in the previous studies only aimed at the determination of the optimum solutions that minimized cost or maximized profit from the buyer's or vendor's side.Normally in traditional inventory managements, the economic lot size is considered for a vendor and a buyer independently ie. each one find their own optimal quantity. However in the modern global competitive market, the buyer and vendor should be treated as strategic partners in the supply chain with long term cooperation relationship. As a result, the Economic lot size of buyer may not result in an optimal policy for the vendor and vice versa.To solve this problem, recently a number of studies have considered the buyer and vendor as a unit to find the optimum economic order quantity in achieving the minimum total cost.

Integrated inventory model for single supplier and a single customer was first introduced by Goyal. He derived the minimum joint variable cost for the supplier and the customer (1). The joint economic lot size model for a single vendor and a single customer was introduced by Banerjee [2]. He obtained the minimum joint relevant cost for both buyer and vendor at the same time with the assumption that the vendor makes the production setup everytime. The buyer places an order and supplies on a lot for lot basis. Goyal modified Banergjee [2] model on the assumption that vendor may possibly product a lotsize that may supply an integer number of orders to the buyer [3].

In the crisp inventory models, all the parameters in the total cost are known and have definite values without ambiguity as well as the real variable of the total cost is positive. But in the reality, it is not so sure. One or more components of an inventory model often appear to be vague and imprecise and hence for getting realistic models all such components are to be represented by fuzzysets. Recently fuzzy concepts have introduced in EOQ models. Park developed a fuzzy EOQ model using Extension Principle [8]. Wu & Yao [12] fuzzied both order quantity and shortage quantity as triangular fuzzy numbers and got the centroid of fuzzy total cost. First time Mahata et al. investigated the Joint Economic lotsize model for both buyer and vendor in fuzzy sense. Recently Mahata Et al. [13] is investigated as an integrated inventory model with the assumption of fuzzy order quantity and fuzzy shortage quantity. In this membership function of fuzzy total cost and its centroid is obtained.

In this paper we consider Integrated Inventory model with fuzzy input parameters. Here demand and cost are represented as a trapezoidal fuzzy number. Chen's functional principle is proposed for arithmetic operation of fuzzy number and Lagrangian method is used for optimization. Graded mean integration if used for defuzzying fuzzy total joint relevant inventory cost.

In section 1, discuss crisp integrated inventory model. Section 2 deals with basic concepts of fuzzy sets ; fuzzy numbers and functional principle. Section 3 deals with two fuzzy integrated inventory model with different situation. To check the validity of the proposed method, numerical example is solved in section 4. Section 5 concludes the finding and discussion of the proposed work.

Assumptions and Notations

Assumptions

- The demand rate and product rate are deterministic.
- Manufacturing setup cost, ordering cost, unit inventory holding cost for the vendor and buyer are known.
- Single vendor and single buyer are considered.
- There is a single product.
- Shortage is allowed for buyer and fully backordered.
- The vendor makes the production setup every time the buyer places an order and supplies on a lot for lot basis.

Notations

D	Annual constant demand.
P	Vendor's annual cost rate of production.
C_v	The unit production cost.
C_p	The unit purchase cost paid by the purchased.
A	The purchaser's ordering cost per order.
S	The vendor's setup cost per setup.
r	The annual inventory carrying cost per dollar invested in stocks.
π	The shortage cost per unit quantity per year.
q	The order quantity.
b	The shortage quantity.

are all crisp real numbers.

Integrated Inventory Model, joint relevant cost by considering shortage is as follow

$$F(q, b) = \frac{D}{q}(s + A) + \frac{q}{2} \,.\, r \,.\left(\frac{D}{p}C_v + C_p\right) + \frac{\left(rCp + \pi\right)\,.\,b^2}{2q} - rC_p.b \qquad (1)$$

The objective is to find the optimal shortage quantity and optimal order quantity which minimize the joint relevant total cost. The necessary condition for minimum $\frac{\partial F}{\partial b} = 0$ and $\frac{\partial F}{\partial q} = 0$ implies that

$$q^* = \sqrt{\frac{2D(S + A)\left(rC_p + \pi\right)}{r\left(\frac{D}{p}C_r + C_p\right)\left(rC_p + \pi\right) - (rC_p)^2}} \text{ and } \frac{\partial^2 F}{\partial b^2} > 0$$

Lagrangian Optimization Method

The techniques for identify the stationary points of a nonlinear programming problem subject to inequality constraints in based on the Lagrangian method. The Kaush-Kuhn-Tucker conditions are necessary and sufficient conditions for minimization problem if both the objective function and solution space are convex. In the case of minimization problem with non-negative constraints, the solution space is convex if the constraint function is concave and the Lagrangian Multipliers are non-negative. In such case, the Lagrangian function must be convex and the resulting stationary points yields a global constrained minimum.We adopt the extended Lagrangian method to solve the nonlinear programming problem with inequality constraints. This method is described in several standard books, Taha [18] is one of the lastest reference. The general idea of extended Lagrangian procedure is that if the unconstrained optimum problem does not satisfy all the constraints, the constrained optimum must occur at the boundary point of the solution space. This means that atleast one constraint must be satisfied in equation form.

In this case, the optimal point the Karush – Kuhn – Tucker necessary conditions indicate that the negative of the gradient of the objective function (represent the direction of steepest descent) must be expressible as a positive linear combination (the coefficients are the Lagrangian Multipliers) of the gradient of active constraints. The best to be hoped for using the extended Lagrangian Method is a good feasible solution. If the problem possess a unique constrained optimum, the procedure can be rectified to locate the global optimum.

Suppose that the problem is given by
Minimize y = f(x)
Subject to $g_i(x) \geq 0$, i = 1, 2, . . . m.

The nonnegativity constraints $x \geq 0$, if any, are included in the m constraints. Then, the procedure of Extension of the Lagrangian Method involves the following steps :

Step1: Solve the unconstrained problem
Minimize y = f(x)

If the resulting optimum satisfies all the constraints stop because all constraints are redundant. Otherwise, set K = 1 and go to step 2.

Step2: Activate any K constraint (ie. convert them into equality) and optimize f(x) subject to the K active constraints by the Lagrangian Method. If the resulting solution is feasible with respect to the remaining constraints stop ; it is a local optimum. Otherwise, activate another set of K constraints and repeat the step. If all sets of active constraints taken K at a time are considered without encountering a feasible solution, go to step 3.

Step3: If K = m, Stop : no feasible solution exists. Otherwise, set K = K + 1 and go to step 2.

Fuzzy Set

In a universe of discourse X, a fuzzy subset $\tilde{A}$ on X is defined by the membership function $\mu_{\tilde{A}}(x)$ which maps each element x in X to a real number in the interval [0, 1]. $\mu_{\tilde{A}}(x)$ denotes the grade or degree of membership and it is usually denoted as $\mu_{\tilde{A}} : X \to [0,1]$.A fuzzy set is said to be normal if the largest grade obtained by any element in that set is 1. That is there must exist atleast one x for which $\mu_{\tilde{A}}(X) = 1$..

The support of $\tilde{A}$ is defined as the crisp set that contains all elements of X that have non zero membershipgrades. A fuzzy set $\tilde{A}$ on X is convex iff $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right)$ for all $x_1, x_2 \in X$ and for $\lambda \in [0, 1]$, where min denotes the minimum operates.

Trapezoidal Fuzzy Number

The fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that $a_1 < a_2 < a_3 < a_4$ with membership function, representing a trapezoid of the form

$$\begin{aligned} \mu_{\tilde{A}}(x) \quad &= \frac{x - a_1}{a_2 - a_1}, \; a_1 \le x \le a_2, \\ &= 1, \qquad a_2 \le x \le a_3, \\ &= \frac{x - a_4}{a_3 - a_4}, \; a_3 \le x \le a_4, \\ &= 0, \text{ Otherwise.} \end{aligned}$$

where a_1, a_2, a_3 and a_4 are the lower limit, lower mode, upper mode and upper limit respectively of the fuzzy number $\tilde{A}$.

Fuzzy arithmetical operations under function principle

Function principle is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We describe some fuzzy arithmetical operations under Function Principle as follows Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then,

i. Addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are any real numbers.

ii. Multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \otimes \tilde{B} = (c_1, c_2, c_3, c_4)$$

where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$,

$T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$

$c_1 = \min T$, $c_2 = \min T_1$, $c_3 = \max T_1$, $c_4 = \max T$.

If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non zero positive real numbers, then

$$\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$$

iii. $\tilde{B} = (-b_4, -b_3, -b_2, -b_1)$

then the subtraction of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are any real numbers.

iv. $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ where b_1, b_2, b_3 and b_4 are all positive real numbers.

If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 are all non zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} \right)$$

v. Scalar Multiplication

Let α be any real number. Then

for $\alpha \geq 0$, $\alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$

for $\alpha < 0$, $\alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

Graded Mean Integration Representation Method

In order to draw ultimate conclusions for decision making, the fuzzy results are to be converted into crisp values. The method of extraction of crisp results from the fuzzy models is known as defuzzification. In this paper we adopt the graded mean integration representation introduced by Chen and Hseih for defuzzification.Chen and Hsieh introduced Graded Mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. For the fuzzy number $\tilde{A}$. Let L^{-1} and R^{-1} denote the inverse function of L and R, respectively. The graded mean h-level value of $\tilde{A}$ is $\frac{1}{2}\left[h\left(L^{-1}(h) + R^{-1}(h)\right)\right]$. Then the graded mean Integration Representation of $\tilde{A}$ is given by

$$P(\tilde{A}) = \frac{1}{\int_0^1 h\,dh} \int_0^1 h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh$$

For the trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ $h^{-1}(h) = a_1 + (a_2 - a_1)h$ and $R^{-1}(h) = a_4 - (a_4 - a_3)h$. The graded mean representation of trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is given by

$$P(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

For the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$

$$P(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$$

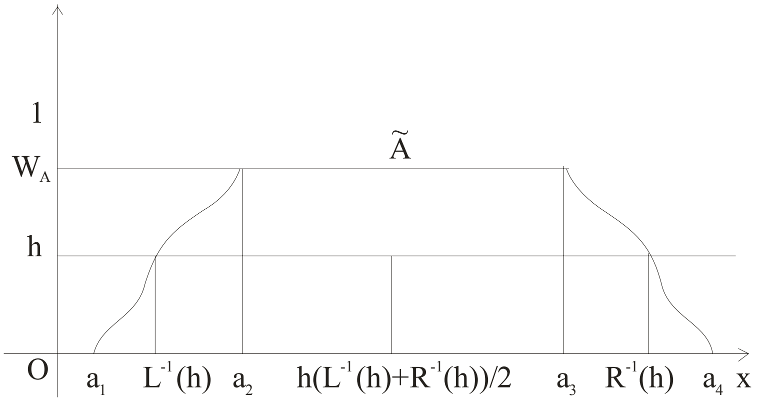

Figure1: The graded mean h-level value of generalized fuzzy number $\tilde{\mathbf{A}} = (a_1, a_2, a_3, a_4, W_A)_{LR}$.

Fuzzy integrated inventory model for crisp order quantity and crisp shortage quantity

In this section we consider model in Section (2). Among these parameters we consider only the four points D, C_p, C_r and π as fuzzy and they are represented by non-negative trapezoidal fuzzy numbers, as follows

$$\tilde{D} = (d_1, d_2, d_3, d_4)$$
$$\tilde{C}_p = (C_{p1}, C_{p2}, C_{p3}, C_{p4})$$
$$\tilde{C}_v = (C_{v1}, C_{v2}, C_{v3}, C_{v4}) \text{ and}$$
$$\tilde{\pi} = (\pi_1, \pi_2, \pi_3, \pi_4)$$

Fuzzy total joint relevant cost is given by

$$\tilde{F}(q, b) = [(\tilde{D} \oslash q) \otimes (S \oplus A)] \oplus \left[\frac{q}{2} \otimes r((\tilde{D} \oslash p) \otimes \tilde{C}_v \oplus \tilde{C}_p)\right]$$
$$\oplus \left[\left(\left((r \otimes \tilde{C}_p) \oplus \tilde{\pi}\right) \otimes \frac{b^2}{2}\right) \oslash q\right] \ominus \left[r \otimes \tilde{C}_p \otimes b\right] \qquad (1)$$

where $\oslash, \otimes, \ominus$ and $\oplus$ are the fuzzy arithmetical operation under function principle.

First, we get the fuzzy total joint relevant inventory cost $\tilde{F}(q, b)$ by using (2) – (5) in (1) as a trapezoidal fuzzy number.

$$\tilde{F}(q, b) = (F_1, F_2, F_3, F_4) \qquad (6)$$

where

$$F_1 = \frac{d_1(S + A)}{q} + \frac{qr}{2}\left(\frac{d_1 Cv_1}{P} + Cp_1\right) + \frac{(rCp_1 + \pi_1)b^2}{2q} - rCp_4 b$$

$$F_2 = \frac{d_2(S + A)}{q} + \frac{qr}{2}\left(\frac{d_2 Cv_2}{P} + Cp_2\right) + \frac{(rCp_2 + \pi_2)b^2}{2q} - rCp_3 b$$

$$F_3 = \frac{d_3(S+A)}{q} + \frac{qr}{2}\left(\frac{d_3Cv_3}{P} + Cp_3\right) + \frac{(rCp_3 + \pi_3)b^2}{2q} - rCp_2b$$

$$F_4 = \frac{d_4(S+A)}{q} + \frac{qr}{2}\left(\frac{d_4Cv_4}{P} + Cp_4\right) + \frac{(rCp_4 + \pi_4)b^2}{2q} - rCp_1b$$

Second, we defuzzify the fuzzy total joint relevant inventory cost $\tilde{F}(q, b)$ by Graded Mean Integration Value of the Fuzzy Number in (6) as

$$P(\tilde{F}) = \frac{1}{6}\left[F_1 + 2F_2 + 2F_3 + F_4\right]$$

$$= \frac{1}{6}\left[\frac{d_1(S+A)}{q} + \frac{qr}{2}\left(\frac{d_1Cr_1}{P} + Cp_1\right) + \frac{(rCp_1 + \pi_1)b^2}{2q} - rCp_4b\right]$$

$$+\frac{2}{6}\left[\frac{d_2(S+A)}{q} + \frac{qr}{2}\left(\frac{d_2Cr_2}{P} + Cp_2\right) + \frac{(rCp_2 + \pi_2)b^2}{2q} - rCp_3b\right]$$

$$+\frac{2}{6}\left[\frac{d_3(S+A)}{q} + \frac{qr}{2}\left(\frac{d_3Cr_3}{P} + Cp_3\right) + \frac{(rCp_3 + \pi_3)b^2}{2q} - rCp_2b\right]$$

$$+\frac{1}{6}\left[\frac{d_4(S+A)}{q} + \frac{qr}{2}\left(\frac{d_4Cr_4}{P} + Cp_4\right) + \frac{(rCp_4 + \pi_4)b^2}{2q} - rCp_1b\right] \quad (7)$$

Third we can get the optimal order quantity and q^* and optimal shortage quantity b^* when $P(\tilde{F})$ in minimization.

Optimal Shortage Quantity b^*

Derivative of $P(\tilde{F})$ with respect to b is

$$\frac{\partial P(\tilde{F})}{\partial b} = \frac{1}{6}\left[\frac{(rCp_1 + \pi_1)b}{q} - rCp_4\right] + \frac{2}{6}\left[\frac{(rCp_2 + \pi_2)b}{q} - rCp_3\right]$$

$$+ \frac{2}{6}\left[\frac{(rCp_3 + \pi_3)b}{q} - rCp_2\right] + \frac{1}{6}\left[\frac{(rCp_4 + \pi_4)b}{q} - rCp_1\right] \quad (8)$$

Let $\frac{\partial P(\tilde{F})}{\partial b} = 0$, we find the optimal shortage quantity b^*

$$b^* = \frac{qr\left[Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4\right]}{\left[(rCp_1 + \pi_1) + 2(rCp_2 + \pi_2) + 2(rCp_3 + \pi_3) + (rCp_4 + \pi_4)\right]} \quad (9)$$

Optimal Order Quantity q^*

Consider $P(\tilde{F}) = \frac{1}{6q}\left[d_1(S+A) + 2d_2(S+A) + 2d_3(S+A) + d_4(S+A)\right]$

$$+\frac{qr}{12}\left[\left(\frac{d_1Cv_1}{P}+Cp_1\right)+2\left(\frac{d_2Cv_2}{P}+Cp_2\right)+2\left(\frac{d_3Cv_3}{P}+Cp_3\right)+\left(\frac{d_4Cv_4}{P}+Cp_4\right)\right]$$

$$+\frac{b^2}{12q}\left[(rCp_1+\pi_1)+2(rCp_2+\pi_2)+2(rCp_3+\pi_3)+(rCp_4+\pi_4)\right]$$

$$-\frac{rb}{6}\left[Cp_4+2Cp_3+2Cp_2+Cp_1\right]$$

Derivative of P($\tilde{F}$) with respect q is

$$\frac{\partial P(\tilde{F})}{\partial q}=\frac{-1}{6q^2}\left[(S+A)\,(d_1+2d_2+2d_3+d_4)\right]$$

$$+\frac{r}{12}\left[\left(\frac{d_1Cv_1}{P}+Cp_1\right)+2\left(\frac{d_2Cv_2}{P}+Cp_2\right)+2\left(\frac{d_3Cv_3}{P}+Cp_3\right)+\left(\frac{d_4Cv_4}{P}+Cp_4\right)\right]$$

$$+\frac{1}{12}\left[(rCp_1+\pi_1)+2(rCp_2+\pi_2)+2(rCp_3+\pi_3)+(rCp_4+\pi_4)\right]\left[-\frac{b^2}{q^2}+\frac{2b}{q}.\frac{\partial b}{\partial q}\right]$$

$$-\frac{r}{6}\left[Cp_4+2Cp_3+2Cp_2+Cp_1\right].\frac{\partial b}{\partial q}$$

On substituting the values of b and $\frac{\partial b}{\partial q}$, we get

$$=-\frac{1}{6q^2}\left[(S+A)\,(d_1+2d_2+2d_3+d_4)\right]$$

$$+\frac{r}{12}\left[\left(\frac{d_1Cv_1+2d_2Cv_2+2d_3Cv_3+d_4Cv_4}{P}\right)+\left(Cp_1+2Cp_2+2Cp_3+Cp_4\right)\right]$$

$$-\frac{1}{12q^2}\left\{\left(\frac{q^2r^2\,(Cp_4+2Cp_3+2Cp_2+Cp_1)^2}{\left[r(Cp_1+2Cp_2+2Cp_3+Cp_4)+(\pi_1+2\pi_2+2\pi_3+\pi_4)\right]^2}\right)\right.$$

$$\left.\text{x}\left(r(Cp_1+2Cp_2+2Cp_3+Cp_4)+(\pi_1+2\pi_2+2\pi_3+\pi_4)\right)\right\}$$

$$+\frac{1}{6q}\left(\frac{qr^2\left(Cp_1+2Cp_2+2Cp_3+Cp_4\right)^2}{\left[r\left(Cp_1+2Cp_2+2Cp_3+Cp_4\right)+(\pi_1+2\pi_2+2\pi_3+\pi_4)\right]}\right)$$

$$-\frac{r}{6}\frac{\left[Cp_4+2Cp_3+2Cp_2+Cp_1\right].r\left[Cp_4+2Cp_3+2Cp_2+Cp_1\right]}{\left[r(Cp_1+2Cp_2+2Cp_3+Cp_4)+(\pi_1+2\pi_2+2\pi_3+\pi_4)\right]}$$

In order to find the value of q which minimizes P($\tilde{F}$), we equate the derivative P($\tilde{F}$) with respect to q to zero

ie. $\frac{\partial P(\tilde{F})}{\partial b}=0$

$$\Rightarrow \frac{r}{12}\left[\frac{(d_1Cv_1 + 2d_2Cv_2 + 2d_3Cv_3 + d_4Cv_4)}{P} + \left(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4\right)\right]$$

$$-\frac{1}{12}\left[\frac{r^2(Cp_4 + 2Cp_3 + 2Cp_3 + Cp)^2}{\left[r(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4) + (\pi_1 + 2\pi_1 + 2\pi_3 + \pi_4)\right]}\right]$$

$$= \frac{(S + A)}{6q^2}\left[d_1 + 2d_2 + 2d_3 + d_4\right]$$

which reduces to

$$q = \sqrt{\frac{2(d_1 + 2d_2 + 2d_3 + d_4)\,(S + A)\begin{bmatrix} r(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4) \\ + (\pi_1 + 2\pi_1 + 2\pi_3 + \pi_4) \end{bmatrix}}{\left\{r\left[\frac{(d_1Cv_1 + 2d_2Cv_2 + 2d_3Cv_3 + d_4Cv_4)}{P} + \left(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4\right)\right] \text{x}\begin{bmatrix} r(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4) \\ + (\pi_1 + 2\pi_1 + 2\pi_3 + \pi_4) \end{bmatrix}\right\} - \left\{r^2(Cp_4 + 2Cp_3 + 2Cp_2 + Cp_1)^2\right\}}} \quad (10)$$

with the second order derivative $\frac{\partial^2 P(\tilde{F})}{\partial q^2} > 0$ Hence we get the optimal order quantity q^*.

Fuzzy integrated inventory model with crisp order quantity and fuzzy shortage quantity

This is a replication model described in last section with the only difference that shortage quantity is regarded as fuzzy.Thus b is represented by the trapezoidal fuzzy number, $\tilde{b} = (b_1, b_2, b_3, b_4)$ with $0 < b_1 \le b_2 \le b_3 \le b_4$.

The fuzzy total joint relevant cost of this model is

$$\tilde{F}(q, \tilde{b}) = \left[(\tilde{D} \oslash q) \otimes (S \oplus A)\right] \oplus \left[\frac{q}{2} \otimes r \otimes \left((\tilde{D} \oslash p) \otimes \tilde{C}_r \oplus \tilde{C}_p\right)\right]$$

$$\oplus \left[\left((r \otimes \tilde{C}_p) \oplus \tilde{\pi}) \otimes \frac{\tilde{b} \otimes \tilde{b}}{2}\right)\right] \ominus \left[(r \otimes \tilde{C}_p \otimes \tilde{b}\right] \quad (11)$$

which reduces to the trapezoidal fuzzy number

$$\tilde{F}(q, \tilde{b}) = (F_1, F_2, F_3, F_4) \quad (12)$$

$$F_1 = \frac{d_1(S + A)}{q} + \frac{qr}{2}\left(\frac{d_1Cv_1}{P} + Cp_1\right) + \frac{(rCp_1 + \pi_1)b_1^2}{2q} - rCp_4b_4$$

$$F_2 = \frac{d_2(S + A)}{q} + \frac{qr}{2}\left(\frac{d_2Cv_2}{P} + Cp_2\right) + \frac{(rCp_2 + \pi_2)b_2^2}{2q} - rCp_3b_3$$

$$F_3 = \frac{d_3(S+A)}{q} + \frac{qr}{2}\left(\frac{d_3Cv_3}{P} + Cp_3\right) + \frac{(rCp_3+\pi_3)b_3^2}{2q} - rCp_2b_2$$

$$F_4 = \frac{d_4(S+A)}{q} + \frac{qr}{2}\left(\frac{d_4Cv_4}{P} + Cp_4\right) + \frac{(rCp_4+\pi_4)b_4^2}{2q} - rCp_1b_1$$

Then the defuzzified value of $\tilde{F}(q, \tilde{b})$ is given by

$$P\left(\tilde{F}(q,\tilde{b})\right) = \frac{1}{6}\left[\frac{d_1(S+A)}{q} + \frac{qr}{2}\left(\frac{d_1Cv_1}{P} + Cp_1\right) + \frac{(rCp_1+\pi_1)b_1^2}{2q} - rCp_4b_4\right]$$
$$+\frac{2}{6}\left[\frac{d_2(S+A)}{q} + \frac{qr}{2}\left(\frac{d_2Cv_2}{P} + Cp_2\right) + \frac{(rCp_2+\pi_2)b_2^2}{2q} - rCp_3b_3\right]$$
$$+\frac{2}{6}\left[\frac{d_3(S+A)}{q} + \frac{qr}{2}\left(\frac{d_3Cv_3}{P} + Cp_3\right) + \frac{(rCp_3+\pi_3)b_3^2}{2q} - rCp_2b_2\right]$$
$$+\frac{1}{6}\left[\frac{d_4(S+A)}{q} + \frac{qr}{2}\left(\frac{d_4Cv_4}{P} + Cp_4\right) + \frac{(rCp_4+\pi_4)b_4^2}{2q} - rCp_1b_1\right] \ldots(3)$$

In order to find the parameter which minimizes $P\left(\tilde{F}(q,\tilde{b})\right)$, consider the partial derivatives of $P\left(\tilde{F}(q,\tilde{b})\right)$ with respect to $\tilde{b} = (b_1, b_2, b_3, b_4)$ and each equated to zero.

$$\frac{\partial P(\tilde{F}(q,\tilde{b}))}{\partial b_1} = \frac{1}{6}\left[\frac{b_1(rCp_1+\pi_1)}{q} - rCp_1\right]$$

$$\frac{\partial P(\tilde{F}(q,\tilde{b}))}{\partial b_2} = \frac{2}{6}\left[\frac{b_2(rCp_2+\pi_2)}{q} - rCp_2\right]$$

$$\frac{\partial P(\tilde{F}(q,\tilde{b}))}{\partial b_3} = \frac{2}{6}\left[\frac{b_3(rCp_3+\pi_3)}{q} - rCp_3\right]$$

and

$$\frac{\partial P(\tilde{F}(q,\tilde{b}))}{\partial b_4} = \frac{1}{6}\left[\frac{b_4(rCp_4+\pi_4)}{q} - rCp_4\right]$$

By solving we get,

$$b_1 = \frac{q(rCp_1)}{rCp_1+\pi_1},\; b_2 = \frac{q(rCp_2)}{rCp_2+\pi_2},\; b_3 = \frac{q(rCp_3)}{rCp_3+\pi_3} \text{ and } b_4 = \frac{q(rCp_4)}{rCp_4+\pi_4}$$

From the above, we see that $b_1 > b_2 > b_3 > b_4$ hence it does not satisfy the constraint to $0 < b_1 \leq b_2 \leq b_3 \leq b_4$. Hence we adopt the Lagrangian Method to find the solutions of b_1, b_2, b_3 and b_4.For this, we convert the inequality constraints $b_2 - b_1 \geq 0$ into equality constraint $b_2 - b_1 = 0$ and then minimize $P\left(\tilde{F}(q,\tilde{b})\right)$ subject to $b_2 - b_1 =$

0. We have the Lagrangian function as

$L(b_1, b_2, b_3, b_4, \lambda) = P\left(\tilde{F}(q, \tilde{b})\right) = \lambda(b_2 - b_1)$ where λ is the Lagrangian Multiplier.

Taking the partial derivatives of $L(b_1, b_2, b_3, b_4, \lambda)$ with respect to b_1, b_2, b_3, b_4 and λ and equate to zero, to find the minimization of $L(b_1, b_2, b_3, b_4, \lambda)$. Then

$$\frac{\partial L}{\partial b_1} = \frac{1}{6}\left[\frac{b_1(rCp_1 + \pi_1)}{q} - rCp_1\right] + \lambda$$

$$\frac{\partial L}{\partial b_2} = \frac{2}{6}\left[\frac{b_2(rCp_2 + \pi_2)}{q} - rCp_2\right] - \lambda$$

$$\frac{\partial L}{\partial b_3} = \frac{2}{6}\left[\frac{b_3(rCp_3 + \pi_3)}{q} - rCp_3\right]$$

$$\frac{\partial L}{\partial b_4} = \frac{1}{6}\left[\frac{b_4(rCp_4 + \pi_4)}{q} - rCp_4\right]$$

$$\frac{\partial L}{\partial \lambda} = b_1 - b_2$$

Let all the above partial derivatives equal to zero and solve b_1, b_2, b_3 and b_4 then

$$b_1 = b_2 = \frac{rCp_1 + 2rCp_2}{\frac{(rCp_1 + \pi_1)}{q} + \frac{2(rCp_2 + \pi_2)}{q}} = \frac{q(rCp_1 + 2rCp_2)}{(rCp_1 + \pi_1) + 2(rCp_2 + \pi_2)}$$

$$b_3 = \frac{q(rCp_3)}{rCp_3 + \pi_3} \text{ and } b_4 = \frac{q(rCp_4)}{rCp_4 + \pi_4}$$

Since the above results show that $b_4 < b_3$, it does not satisfy the constraint $0 < b_1 \le b_2 \le b_3 \le b_4$. $\therefore$ it is not a local optimum.We get the same result if we select anyone of the other inequality constraints. Convert the inequality constraints $b_2 - b_1 \ge 0$ and $b_3 - b_2 \ge 0$ into equality constraints $b_2 - b_1 = 0$ and $b_3 - b_2 = 0$. We optimize $P\left(\tilde{F}(q, \tilde{b})\right)$ subject to $b_2 - b_1 = 0$ and $b_3 - b_2 = 0$ by the Lagrangian Method.

Now the Lagrangian function with multipliers λ_1 and λ_2 as

$L(b_1, b_2, b_3, b_4, \lambda_1, \lambda_2) = P\left(\tilde{F}(q, \tilde{b})\right) - \lambda_1(b_2 - b_1) - \lambda_2(b_3 - b_2)$

The solution can be obtained by setting the derivatives of $L(b_1, b_2, b_3, b_4, \lambda_1, \lambda_2)$ with respect to $b_1, b_2, b_3, b_4, \lambda_1$ and λ_2 are equating to zero we get,

$$\frac{\partial L}{\partial b_1} = \frac{1}{6}\left[\frac{b_1(rCp_1 + \pi_1)}{q} - rCp_1\right] + \lambda_1$$

$$\frac{\partial L}{\partial b_2} = \frac{2}{6}\left[\frac{b_2(rCp_2 + \pi_2)}{q} - rCp_2\right] - \lambda_1 + \lambda_2$$

$$\frac{\partial L}{\partial b_3} = \frac{2}{6}\left[\frac{b_3(rCp_3 + \pi_3)}{q} - rCp_3\right] - \lambda_2$$

$$\frac{\partial L}{\partial b_4} = \frac{1}{6}\left[\frac{b_4(rCp_4 + \pi_4)}{q} - rCp_4\right]$$

$$\frac{\partial L}{\partial \lambda_1} = b_1 - b_2 \qquad \frac{\partial L}{\partial \lambda_2} = b_2 - b_3$$

$$b_1 = b_2 = b_3 = \frac{rCp_1 + 2Cp_2 + 2Cp_3}{\frac{\left(rCp_1 + \pi_1\right)}{q} + \frac{2\left(rCp_2 + \pi_2\right)}{q} + \frac{2\left(rCp_3 + \pi_3\right)}{q}}$$

$$= \frac{q\left(rCp_1 + 2Cp_2 + 2Cp_3\right)}{\left(rCp_1 + \pi_1\right) + 2\left(rCp_2 + \pi_2\right) + 2\left(rCp_3 + \pi_3\right)}$$

$$b_4 = \frac{q\left(rCp_4\right)}{rCp_4 + \pi_4}$$

From the above equation we observe that $b_1 = b_2 = b_3 > b_4$. This states that above one is not the local optimum. Since it does not satisfy the constraints $0 < b_1 \leq b_2 \leq b_3 \leq b_4$. We get the same result if we repeat by selecting any two of the inequality constraints. Hence the inequality constraint $b_2 - b_1 \geq 0$, $b_3 - b_2 \geq 0$ and $b_4 - b_3 \geq 0$ are converting into equalities $b_2 - b_1 = 0$, $b_3 - b_2 = 0$ and $b_4 - b_3 = 0$. The Lagrangian function with λ_1, λ_2, λ_3 multiplies in of

$$L(b_1, b_2, b_3, b_4, \lambda_1, \lambda_2, \lambda_3) = P\left(\tilde{F}(\tilde{q}, \tilde{b})\right) - \lambda_1(b_2 - b_1) - \lambda_2(b_3 - b_2) - \lambda_3(b_4 - b_3)$$

In order to find the minimization of $L(b_1, b_2, b_3, b_4, \lambda_1, \lambda_2, \lambda_3)$. We take the partial derivatives of $L(b_1, b_2, b_3, b_4, \lambda_1, \lambda_2, \lambda_3)$ with respect to b_1, b_2, b_3, b_4, λ_1, λ_2 and λ_3. We obtain

$$\frac{\partial L}{\partial b_1} = \frac{1}{6}\left[\frac{b_1(rCp_1 + \pi_1)}{q} - rCp_1\right] + \lambda_1$$

$$\frac{\partial L}{\partial b_2} = \frac{2}{6}\left[\frac{b_2(rCp_2 + \pi_2)}{q} - rCp_2\right] - \lambda_1 + \lambda_2$$

$$\frac{\partial L}{\partial b_3} = \frac{2}{6}\left[\frac{b_3(rCp_3 + \pi_3)}{q} - rCp_3\right] - \lambda_2 + \lambda_3$$

$$\frac{\partial L}{\partial b_4} = \frac{1}{6}\left[\frac{b_4(rCp_4 + \pi_4)}{q} - rCp_4\right] + \lambda_3$$

$$\frac{\partial L}{\partial \lambda_1} = b_1 - b_2 \qquad \frac{\partial L}{\partial \lambda_2} = b_2 - b_3 \qquad \frac{\partial L}{\partial \lambda_3} = b_3 - b_4$$

$$b = b_1 = b_2 = b_3 = b_4 = \frac{rCp_1 + 2Cp_2 + 2Cp_3 + rCp_4}{\frac{(rCp_1 + \pi_1)}{q} + \frac{2(rCp_2 + \pi_2)}{q} + \frac{2(rCp_3 + \pi_3)}{q} + \frac{(rCp_4 + \pi_4)}{q}}$$

$$= \frac{qr(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4)}{(rCp_1 + \pi_1) + 2(rCp_2 + \pi_2) + 2(rCp_3 + \pi_3) + (rCp_4 + \pi_4)}$$

Because the above solution $\tilde{b} = (b_1, b_2, b_3, b_4)$ satisfies all inequality constraints, the procedure terminates with $\tilde{b}$ as a local optimum solution to the problem.Furthermore, the above local optimum solution in the only one feasible solution, So $\tilde{b} = (b_1^*, b_2^*, b_3^*, b_4^*)$ in the optimum fuzzy shortage quantity of this inventory model according to extension of the Lagrangean Method..Since $b_1^* = b_2^* = b_3^* = b_4^* = b^*$ implies $\tilde{b}^* = (b^*, b^*, b^*, b^*)$

$$\text{where } b^* = \frac{qr[Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4]}{(rCp_1 + \pi_1) + 2(rCp_2 + \pi_2) + 2(rCp_3 + \pi_3) + (rCp_4 + \pi_4)} \qquad (15)$$

Since $b_1 = b_2 = b_3 = b_4 = b^*$, eqn. (13) reduces to

$$P(\tilde{F}(\tilde{q}, \tilde{b})) = \frac{1}{6}[F_1 + 2F_2 + 2F_3 + F_4]$$

$$= \frac{1}{6}\left[\frac{d_1(S + A)}{q} + \frac{qr}{2}\left(\frac{d_1Cr_1}{P} + Cp_1\right) + \frac{(rCp_1 + \pi_1)b^2}{2q} - rCp_4b\right]$$

$$+ \frac{2}{6}\left[\frac{d_2(S + A)}{q} + \frac{qr}{2}\left(\frac{d_2Cr_2}{P} + Cp_2\right) + \frac{(rCp_2 + \pi_2)b^2}{2q} - rCp_3b\right]$$

$$+ \frac{2}{6}\left[\frac{d_3(S + A)}{q} + \frac{qr}{2}\left(\frac{d_3Cr_3}{P} + Cp_3\right) + \frac{(rCp_3 + \pi_3)b^2}{2q} - rCp_2b\right]$$

$$+ \frac{1}{6}\left[\frac{d_4(S + A)}{q} + \frac{qr}{2}\left(\frac{d_4Cr_4}{P} + Cp_4\right) + \frac{(rCp_4 + \pi_4)b^2}{2q} - rCp_1b\right] \ldots(16)$$

In order to find the optimal order quantity, we minimize $P(\tilde{F})$. Taking partial derivatives of $P(\tilde{F})$ with respect to q and equate to zero. Derivative of $P(\tilde{F}(\tilde{q}, \tilde{b}))$ with respect to q and on substituting the valuer of b and $\frac{\partial b}{\partial q}$, we get

$$\frac{\partial P(\tilde{F}(q, \tilde{b}))}{\partial q} = \frac{r}{12}\left[\frac{(d_1Cv_1 + 2d_2Cv_2 + 2d_3Cv_3 + 2d_4Cv_4)}{P} + (Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4)\right]$$

$$-\frac{1}{12}\left[\frac{r^2(Cp_2+2Cp_2+2Cp_2+Cp_2)^2}{[r(Cp_1+2Cp_2+2Cp_3+Cp_4)+(\pi_1+2\pi_1+2\pi_3+\pi_4)]}\right]$$

$$-\frac{(S+A)}{6q^2}\left[d_1+2d_2+2d_3+d_4\right]$$

In order to find the value of q, we equate the derivation of $P\left(\tilde{F}(q,\tilde{b})\right)$ with respect to q to zero, ie. $\frac{\partial P(\tilde{F}(q,\tilde{b}))}{\partial q}=0.$

$$q^*=\sqrt{\frac{2(d_1+2d_2+2d_3+d_4)\,(S+A)\begin{bmatrix} r(Cp_1+2Cp_2+2Cp_3+Cp_4) \\ +(\pi_1+2\pi_2+2\pi_3+\pi_4)\end{bmatrix}}{r\left[\frac{(d_1Cv_1+2d_2Cv_2+2d_3Cv_3+d_4Cv_4)}{P}+\left(Cp_1+2Cp_2+2Cp_3+Cp_4\right)\right] \\ x\begin{bmatrix} r_1(Cp_1+2Cp_2+2Cp_3+Cp_4) \\ +(\pi_1+2\pi_1+2\pi_3+\pi_4)\end{bmatrix}-\left\{r^2(Cp_4+2Cp_3+2Cp_2+Cp_1)^2\right\}}}$$

Thus we optimal order quantity, optimal shortage quantity and optimal joint relevant cost as

$$q^*=\sqrt{\frac{2(d_1+2d_2+2d_3+d_4)\,(S+A)\begin{bmatrix} r(Cp_1+2Cp_2+2Cp_3+Cp_4) \\ +(\pi_1+2\pi_1+2\pi_3+\pi_4)\end{bmatrix}}{r\left[\frac{(d_1Cv_1+2d_2Cv_2+2d_3Cv_3+d_4Cv_4)}{P}+\left(Cp_1+2Cp_2+2Cp_3+Cp_4\right)\right] \\ x\begin{bmatrix} r_1(Cp_1+2Cp_2+2Cp_3+Cp_4) \\ +(\pi_1+2\pi_1+2\pi_3+\pi_4)\end{bmatrix}-\left\{r^2(Cp_1+2Cp_2+2Cp_3+Cp_4)^2\right\}}}$$

$$b=b_1=b_2=b_3=b_4=\frac{q^*r\left[Cp_1+2Cp_2+2Cp_3+Cp_4\right]}{\left(rCp_1+\pi_1\right)+2\left(rCp_2+\pi_2\right)+2\left(rCp_3+\pi_3\right)+\left(rCp_4+\pi_4\right)}$$

$$\tilde{F}(q^*,\tilde{b}^*)=\left(F_1^*,F_2^*,F_3^*,F_4^*\right)$$

where

$$F_1^*=\frac{d_1(S+A)}{q^*}+\frac{q^*r}{2}\left(\frac{d_1Cv_1}{P}+Cp_1\right)+\frac{(rCp_1+\pi_1)b^*}{2q^*}-rCp_4b^*$$

$$F_2^*=\frac{d_2(S+A)}{q}+\frac{q^*r}{2}\left(\frac{d_2Cv_2}{P}+Cp_2\right)+\frac{(rCp_2+\pi_2)b^*}{2q^*}-rCp_3b^*$$

$$F_3^*=\frac{d_3(S+A)}{q}+\frac{q^*r}{2}\left(\frac{d_3Cv_3}{P}+Cp_3\right)+\frac{(rCp_3+\pi_3)b^*}{2q^*}-rCp_2b^*$$

$$F_4^*=\frac{d_4(S+A)}{q}+\frac{q^*r}{2}\left(\frac{d_4Cv_4}{P}+Cp_4\right)+\frac{(rCp_4+\pi_4)b^*}{2q^*}-rCp_1b^*$$

Numerical Example

Consider any problem in which the yearly demand is more or less than 1,000 units, Vendor's Annual constant rate of production is 3200 unit/year. The Purchaser's ordering cost per order is \$100. The Vendor's setup is \$400. The unit production cost is more or less than \$20. The unit purchase cost paid by the purchase is more or less than \$25. The annual inventory carrying cost per dollar invested in stocks is \$0.2 and the shortage cost per unit quantity per year is more or less than \$10. Determine the optimum order quantity and optimum shortage quantity?

Here we represent the case of vague value, "more or less than Y" as the type of trapezoidal fuzzy number.Suppose Fuzzy yearly demand is "more or less than1000"

$$\tilde{D} = (d_1, d_2, d_3, d_4) = (900, 950, 1050, 1100)$$

Fuzzy production cost is"more or less than \$ 20"

$$\tilde{C}v = (Cv_1, Cv_2, Cv_3, Cv_4) = (18, 19, 21, 22)$$

Fuzzy purchasing cost is"more or less than \$ 25"

$$\tilde{C}p = (Cp_1, Cp_2, Cp_3, Cp_4) = (18, 23, 27, 32)$$

Fuzzy shortage cost = "more or less than \$ 10"

$$\pi = (\pi_1, \pi_2, \pi_3, \pi_4) = (8, 9, 11, 12)$$

The Optimum cost b^* is given by

$$b^* = \frac{q^* r\left[Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4\right]}{\left(rCp_1 + \pi_1\right) + 2\left(rCp_2 + \pi_2\right) + 2\left(rCp_3 + \pi_3\right) + \left(rCp_4 + \pi_4\right)}$$

The Optimum ordering cost is given by

$$q^* = \sqrt{\frac{2(d_1 + 2d_2 + 2d_3 + d_4)\,(S + A)\begin{bmatrix} r(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4) \\ + (\pi_1 + 2\pi_1 + 2\pi_3 + \pi_4) \end{bmatrix}}{\begin{array}{c} r\left[\dfrac{(d_1Cv_1 + 2d_2Cv_2 + 2d_3Cv_3 + d_4Cv_4)}{P} + \left(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4\right)\right] \\ x\begin{bmatrix} r_1(Cp_1 + 2Cp_2 + 2Cp_3 + Cp_4) \\ + (\pi_1 + 2\pi_1 + 2\pi_3 + \pi_4) \end{bmatrix} - \left\{r^2(Cp_4 + 2Cp_3 + 2Cp_2 + Cp_1)^2\right\} \end{array}}}$$

$\therefore$ We get $q^* = 466.78$ units $\tilde{b} = (155.59, 155.59, 155.59, 155.59)$ $\tilde{F}^*(q^*, b^*) = (1345.584, 1866.974, 2416.23, 2941.991)$

Conclusion

This paper proposed two fuzzy models for integrated inventory problem. In the first model, demand production cost, purchase cost, shortage cost are represented by fuzzy number while shortage quantity is treated as a fixed constant. In the second model shortage quantity is also represented as a fuzzy number. For each fuzzy model, a method of defuzzification, graded mean integration representation is employed to find the estimate of total joint inventory cost in the fuzzy sense and then corresponding optimal order lotsize is derived to maximize the total profit. Besides, we show that the proposed fuzzy models can be reduced to the crisp model and optimal order lot size in the fuzzy sense can be reduced to the classical EOQ formula. Numerical examples are carried out to investigate the behavior of our proposed models and the results are compared with those obtained from crisp model. Fuzzy inventory models are realistic and it provides more information for manager and manufacturer.

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