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New Exact Solution of The (4+1)-Dimensional Fractional Boiti-Leon-Manna-Pempinelli Equation by The $\left(m+\frac{G'}{G}\right)$ Expansion Method

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Abstract

In this paper,we introduce a new the (4+1)-dimensional fractional Boiti-Leon-Manna-Pempinelli equation. The fractional $\left(m+\frac{G'}{G}\right)$ extension method is successfully applied to establish the exact solutions for the the (4+1)-dimensional fractional Boiti-Leon-Manna-Pempinelli equation. The fractional derivative version of Yang modified, linked with fractional complex transform is employed to reduce fractional differential equations into the corresponding ordinary differential equations. The results show that the new exact solution are precisely obtained and the efficiency of the methods is demonstrated.

Keywords: The $\left(m+\frac{G'}{G}\right)$ extension method, exact solutions, the (4+1)-dimensional fractional Boiti-Leon-Manna-Pempinelli equation.

1. INTRODUCTION

The percaption of a derivative of order any real number is longer a stunning faet but quite the contrary. Its contribution has allowed and still allows us to glimpse natural phenomena which surround us in a different way and to model them by differential equation or systems of non-integer order and which better reflect the passage from real to know. Fractional differential equations (FDEs) arise in numerous problems of physics biology systems identification, chemistry, finance and fractional dynamics [1-15]. Many researchers have used diverse methods to get exact solutions, such as, the functional variable method, Lie symmetries, Riccati-sub equation method, first integral method, Improved fractional sub-equation method, the Exp-function method and so on. [1-20]

The $(2+1)$-dimensional NLEEs

$$u_{yt} - u_{xxxy} - u_{xx}u_y - u_x u_{xy} = 0, \tag{1}$$

which has been proposed by Boiti et al [16].

In 2012 M.T. Darvishi et al. extended Eq.(1) which is called the (2+1)-dimensional Boiti-Leon-Manna-Pempinelli equation [17]

$$u_{yt} - u_{xxxy} - 3u_{xy}u_x - 3u_{xx}u_y = 0. \tag{2}$$

In 2017, the (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation

$$u_{yt} + u_{zt} + u_{xxxy} + u_{xxxz} - 3u_x(u_{xy} + u_{xz}) - 3u_{xx}(u_y + u_z) \quad = 0, \tag{3}$$

which was derived by S. M. Mabrouk and A. S. Rashed [18].

In 2019 G. Xu and A. wazwaz [19] introduced the (4+1)-dimensional Boiti–Leon–Manna–Pempinelli equation

$$\begin{aligned} &u_{yt} + u_{zt} + u_{st} + au_{xxxy} + au_{xxxz} + au_{xxxs} \\ &+bu_x(u_{xy} + u_{xz} + u_{xs}) + bu_{xx}(u_y + u_z + u_s) = 0. \end{aligned} \tag{4}$$

In this paper we propose a new the (4+1)-dimensional fractional Boiti-Leon-Manna-Pempinelli equation with the form

$$\begin{aligned} &D_t^\alpha D_y^\alpha u + D_t^\alpha D_z^\alpha u + D_t^\alpha D_s^\alpha u + aD_y^\alpha D_x^{3\alpha} u + aD_z^\alpha D_x^{3\alpha} u + aD_s^\alpha D_x^{3\alpha} u \\ &+bD_x^\alpha u(D_y^\alpha D_x^\alpha u + D_z^\alpha D_x^\alpha u_+ D_s^\alpha D_x^\alpha u) + bD_x^{2\alpha} u(D_y^\alpha u + D_z^\alpha u + D_s^\alpha u) = 0, \end{aligned} \tag{5}$$

where x, y, z, s and t are variables, a and b are constant parameters, $\alpha \in (0,1)$.

The formal definitions of local fractional derivative have been explained in Section 2. The basic algorithms of the fractional $\left(m + \frac{G'}{G}\right)$ extension method have been described in Section 3. The application of proposed methods for getting new exact solution of a new (4+1)-dimensional fractional Boiti-Leon-Manna-Pempinelli equation in Section 4. Section 5, presents the conclusions of the article.

2. LOCAL FRACTIONAL DERIVATIVE AND ITS PROPERTIES

The summary of Local fractional derivative of order α which is used further in this paper is defined by the following expression [22-23]:

$$D^\alpha f(t_0) = \frac{d^\alpha f(t)}{dt^\alpha} = \lim_{t \to t_0} \frac{\Delta^\alpha (f(t) - f(t_0))}{(t - t_0)^\alpha}, \qquad 0 < \alpha \leq 1 \tag{6}$$

in which $\Delta^\alpha(f(t)-f(t_0))$ is the Δ^α function defined by

$$\Delta^\alpha(f(t)-f(t_0)) \cong \gamma(1+\alpha)(f(t)-f(t_0)). \tag{7}$$

Some important properties of the local derivative famous formula can be listed as follows

$$\begin{aligned}
D^\alpha(f_1(t) \pm f_2(t)) &= D^\alpha(f_1(t)) \pm D^\alpha(f_2(t)), && (8)\\
D^\alpha(cf(t)) &= cD^\alpha(f(t)), \qquad c = constant\\
D^\alpha t^\beta &= \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} t^{\beta-\alpha}, \qquad \beta \geq \alpha > 0\\
D^\alpha(f(g(t))) &= D^\alpha(g(t))g^{(')}(t). && (9)
\end{aligned}$$

3. THE $\left(M + \frac{G'}{G}\right)$ EXPANSION METHOD

Consider the general nonlinear PDE:

$$P(U, U_x, U_t, U_{xx}, U_{tt}, U_{xt}...) = 0, \tag{10}$$

where $u = u(x,t)$ is an unkhown function, P is a polynomial in $u(x,t)$ and the subscripts for the partial derivatives.

Step 1: The traveling wave variable ansatz

$$U(x, y, z, s, t) = u(\xi) \quad and \quad \xi = kx + ly + pz + qs + \omega t, \tag{11}$$

where k, l, p, q and ω are nonzero constants. Now using transformation (11) in to Eq. (10), the following ordinary differential equation (ODE):

$$N(u, u', u'', ...) = 0. \tag{12}$$

Step 2: Assuming that by means of $\left(m + \frac{G'}{G}\right)$ expansion method [21]. The traveling wave solution of Eq (12) has in the following form:

$$\begin{aligned}
u(\xi) &= \sum_{i=-M}^{M} a_i(m+F)^i\\
&= a_{-M}(m+F)^{-M} + ... + ma_0 + a_1(m+F) + ... + a_M(m+F)^M, && (13)
\end{aligned}$$

where a_i, $i = 0, 1, ..., n$ and m are nonzero constants. According to the principles of balance, we find the value of n. In this manuscript, we define F to be a function as:

$$F = \frac{G'(\xi)}{G(\xi)}, \tag{14}$$

where $G(\xi)$ satisfy $G^{''} + (\lambda + 2m)G^{'} + \mu G = 0$.

Step 3: To determine the positive integer M, take a homogeneous balance between the highest-order nonlinear terms and the derivatives of the highest order linear terms appearing in Eq. (12).

Step 4: By substituting Eq (13) and M in to (12) and equating the coefficient of $(m + F)^n(n = 0, \pm 1, \pm 2, ...)$ in to zero the system of algebraic equations can be genrated. By solving the system, the $a_i(i = 0, \pm 1, \pm 2, ...M)$ and remaining constants can be found. We get the exact solutions of Eq. (10)

4. APPLICATION OF THE $\left(M + \frac{G'}{G}\right)$EXPANSION METHOD FOR THE SOLUTION OF THE TIME-FRACTIONAL (4+1)-DIMENSIONAL BLMP EQUATION

In this section, we have applied $\left(m + \frac{G'}{G}\right)$ expansion method for getting the exact solutions for (4+1)-dimensional Boiti-Leon-Manna-Pempinelli equation.

Consider the (4+1)-dimensional fractional Boiti-Leon-Manna-Pempinelli equation

$$\begin{aligned} &D_t^\alpha D_y^\alpha u + D_t^\alpha D_z^\alpha u + D_t^\alpha D_s^\alpha u + aD_y^\alpha D_x^{3\alpha} u + aD_z^\alpha D_x^{3\alpha} u + aD_s^\alpha D_x^{3\alpha} u \\ &+bD_x^\alpha u(D_y^\alpha D_x^\alpha u + D_z^\alpha D_x^\alpha u_+ D_s^\alpha D_x^\alpha u) + bD_x^{2\alpha} u(D_y^\alpha u + D_z^\alpha u + D_s^\alpha u) = 0 \end{aligned} \quad (15)$$

We introduce the following transformations :

$$\begin{aligned} &u(x, y, z, s, t) = u(\xi), \\ &\xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ly^\alpha}{\Gamma(1+\alpha)} + \frac{pz^\alpha}{\Gamma(1+\alpha)} + \frac{qs^\alpha}{\Gamma(1+\alpha)} + \frac{\omega t^\alpha}{\Gamma(1+\alpha)}, \end{aligned} \quad (16)$$

where k, l, p, q, ω are constant. Substituting Eq.(16) into Eq.(15), we obtain

$$(l + p + q) \left(\omega u'' (\xi) + a \cdot k^3 u^{(IV)} (\xi) + 2 \cdot b \cdot k^2 u' (\xi) u'' (\xi)\right) = 0. \quad (17)$$

According to the $\left(m + \frac{G'}{G}\right)$ expansion method, we suppose that

$$\begin{aligned} u(\xi) &= \sum_{i=-M}^{M} a_i(m + F)^i, \\ &= a_{-M}(m + F)^{-M} + ... + ma_0 + a_1(m + F) + ... + a_M(m + F)^M, \end{aligned} \quad (18)$$

where $a_i(i = 0, \pm 1, \pm 2, ..., \pm M)$ and m are constants. Here, by equating the dominant terms from Eq.(17) $u^{(IV)}(\xi)$ and $u'(\xi)u''(\xi)$, we have $M = 1$. Therefore, by Eq.(18) we have

$$u(\xi) = a_{-1} \left((m + F)\right)^{-1} + a_0 + a_1 (m + F), \quad (19)$$

where a_{-1}, a_0, a_1, m are constant.
Substitating Eq.(19) together with Eq.(14) into Eq.(17), then equating every coefficient of $\left(m+\frac{G'}{G}\right)^{-i}\left(\frac{G'}{G}\right)^{j}$ $(i = 0,1,2,3,...)$ to zero, the system of nonlinear algebraic equations can be generated for $a_{-1}, a_0, a_1, \omega, l, p, q$ and k follows

$$\begin{aligned}\left(m+\frac{G'}{G}\right)^{0}\left(\frac{G'}{G}\right)^{0}: \quad & ak^3la_1\lambda^3\mu + 6\,ak^3la_1\lambda^2m\mu + 12ak^3la_1\lambda\,m^2\mu \\ & +8ak^3la_1m^3\mu + ak^3pa_1\lambda^3\mu + ak^3qa_1\lambda^3\mu \\ & +6ak^3pa_1\lambda^2m\mu + 6\,ak^3qa_1\lambda^2m\mu + 12ak^3pa_1\lambda\,m^2\mu \\ & +12\,ak^3qa_1\lambda m^2\mu + 8ak^3pa_1m^3\mu + 8ak^3qa_1m^3\mu \\ & +8ak^3la_1\lambda\,\mu^2 + 16ak^3la_1m\mu^2 + 8ak^3pa_1\lambda\,\mu^2 \\ & +8ak^3qa_1\lambda\,\mu^2 + 16\,ak^3pa_1m\mu^2 + 16\,ak^3qa_1m\mu^2 \\ & -2bk^2{a_1}^2l\lambda\,\mu^2 - 4bk^2{a_1}^2lm\mu^2 - 2\,bk^2{a_1}^2p\lambda\,\mu^2 \\ & -2\,bk^2{a_1}^2q\lambda\,\mu^2 - 4\,bk^2{a_1}^2pm\mu^2 - 4bk^2{a_1}^2qm\mu^2 \\ & +wla_1\lambda\,\mu + 2wla_1m\mu + wpa_1\lambda\,\mu + wqa_1\lambda\,\mu \\ & +2wpa_1m\mu + 2wqa_1m\mu = 0. \end{aligned} \tag{20}$$

$$\begin{aligned}\left(m+\frac{G'}{G}\right)^{0}\left(\frac{G'}{G}\right): \quad & ak^3l\lambda^4a_1 + 8\,ak^3l\lambda^3ma_1 + 24\,ak^3l\lambda^2m^2a_1 + 32\,ak^3l\lambda\,m^3a_1 \\ & +16ak^3lm^4a_1 + ak^3\lambda^4pa_1 + ak^3\lambda^4qa_1 + 8ak^3\lambda^3mpa_1 \\ & +8ak^3\lambda^3mqa_1 + 24\,ak^3\lambda^2m^2pa_1 + 24\,ak^3\lambda^2m^2qa_1 \\ & +32ak^3\lambda\,m^3pa_1 + 32ak^3\lambda\,m^3qa_1 + 16\,ak^3m^4pa_1 \\ & +16\,ak^3m^4qa_1 + 22\,ak^3l\lambda^2\mu\,a_1 + 88ak^3l\lambda\,m\mu\,a_1 \\ & +88\,ak^3lm^2\mu\,a_1 + 22\,ak^3\lambda^2\mu\,pa_1 + 22\,ak^3\lambda^2\mu\,qa_1 \\ & +88\,ak^3\lambda\,m\mu\,pa_1 + 88\,ak^3\lambda\,m\mu\,qa_1 + 88\,ak^3m^2\mu\,pa_1 \\ & +88ak^3m^2\mu\,qa_1 - 4\,bk^2l\lambda^2\mu{a_1}^2 - 16\,bk^2l\lambda\,m\mu\,{a_1}^2 \\ & -16\,bk^2lm^2\mu\,{a_1}^2 - 4bk^2\lambda^2\mu\,p{a_1}^2 - 4\,bk^2\lambda^2\mu\,q{a_1}^2 \\ & -16\,bk^2\lambda\,m\mu\,p{a_1}^2 - 16\,bk^2\lambda\,m\mu q{a_1}^2 - 16\,bk^2m^2\mu\,p{a_1}^2 \\ & -16\,bk^2m^2\mu\,q{a_1}^2 + 16\,ak^3l\mu^2a_1 + 16\,ak^3\mu^2pa_1 \\ & +16\,ak^3\mu^2qa_1 - 4\,bk^2l\mu^2{a_1}^2 - 4\,bk^2\mu^2p{a_1}^2 - 4\,bk^2\mu^2q{a_1}^2 \\ & +l\lambda^2wa_1 + 4l\lambda\,mwa_1 + 4lm^2wa_1 + \lambda^2pwa_1 + \lambda^2qwa_1 \\ & +4\lambda mpwa_1 + 4\,\lambda\,mqwa_1 + 4\,m^2pwa_1 + 4\,m^2qwa_1 \\ & +2\,l\mu\,wa_1 + 2\mu\,pwa_1 + 2\,\mu\,qwa_1 = 0. \end{aligned} \tag{21}$$

$$
\begin{aligned}
\left(m+\frac{G'}{G}\right)^{0}\left(\frac{G'}{G}\right)^{2}: \quad & 15ak^3l\lambda^3a_1 + 90\,ak^3l\lambda^2ma_1 + 180\,ak^3l\lambda\,m^2a_1\\
& +120\,ak^3lm^3a_1 + 15ak^3\lambda^3pa_1 + 15\,ak^3\lambda^3qa_1\\
& +9ak^3\lambda^2mpa_1 + 90\,ak^3\lambda^2mqa_1 + 180ak^3\lambda\,m^2pa_1\\
& +180\,ak^3\lambda\,m^2qa_1 + 120\,ak^3m^3pa_1 + 120\,ak^3m^3qa_1\\
& -2\,bk^2l\lambda^3{a_1}^2 - 12\,bk^2l\lambda^2m{a_1}^2 - 24\,bk^2l\lambda\,m^2{a_1}^2\\
& -16\,bk^2lm^3{a_1}^2 - 2bk^2\lambda^3p{a_1}^2 - 2\,bk^2\lambda^3q{a_1}^2\\
& -12\,bk^2\lambda^2mp{a_1}^2 - 12\,bk^2\lambda^2mq{a_1}^2 - 24\,bk^2\lambda\,m^2p{a_1}^2\\
& -24\,bk^2\lambda\,m^2q{a_1}^2 - 16\,bk^2m^3p{a_1}^2 - 16\,bk^2m^3q{a_1}^2\\
& +60\,ak^3l\lambda\,\mu\,a_1 + 120\,ak^3lm\mu\,a_1 + 60\,ak^3\lambda\,\mu\,pa_1\\
& +60\,ak^3\lambda\,\mu\,qa_1 + 120\,ak^3m\mu\,pa_1 + 120\,ak^3m\mu\,qa_1\\
& -12\,bk^2l\lambda\,\mu\,{a_1}^2 - 24\,bk^2lm\mu\,{a_1}^2 - 12\,bk^2\lambda\,\mu\,p{a_1}^2\\
& -12\,bk^2\lambda\,\mu\,q{a_1}^2 - 24\,bk^2m\mu\,p{a_1}^2 - 24\,bk^2m\mu\,q{a_1}^2\\
& +3\,l\lambda\,wa_1 + 6\,lmwa_1 + 3\lambda\,pwa_1 + 3\,\lambda\,qwa_1 + 6\,mpwa_1\\
& +6\,qwa_1m15\,ak^3l\lambda^3a_1 + 90\,ak^3l\lambda^2ma_1 + 180\,ak^3l\lambda\,m^2a_1\\
& +120\,ak^3lm^3a_1 + 15ak^3\lambda^3pa_1 + 15ak^3\lambda^3qa_1\\
& +90ak^3\lambda^2mpa_1 + 90\,ak^3\lambda^2mqa_1 + 180ak^3\lambda\,m^2pa_1\\
& +180\,ak^3\lambda\,m^2qa_1 + 120\,ak^3m^3pa_1 + 120\,ak^3m^3qa_1\\
& -2\,bk^2l\lambda^3{a_1}^2 - 12\,bk^2l\lambda^2m{a_1}^2 - 24\,bk^2l\lambda\,m^2{a_1}^2\\
& -16\,bk^2lm^3{a_1}^2 - 2\,bk^2\lambda^3p{a_1}^2 - 2\,bk^2\lambda^3q{a_1}^2\\
& -12\,bk^2\lambda^2mp{a_1}^2 - 12\,bk^2\lambda^2mq{a_1}^2 - 24\,bk^2\lambda\,m^2p{a_1}^2\\
& -24\,bk^2\lambda\,m^2q{a_1}^2 - 16\,bk^2m^3p{a_1}^2 - 16\,bk^2m^3q{a_1}^2\\
& +60\,ak^3l\lambda\,\mu\,a_1 + 120\,ak^3lm\mu\,a_1 + 60\,ak^3\lambda\,\mu\,pa_1\\
& +60\,ak^3\lambda\,\mu\,qa_1 + 120\,ak^3m\mu\,pa_1 + 120\,ak^3m\mu\,qa_1\\
& -12\,bk^2l\lambda\,\mu\,{a_1}^2 - 24\,bk^2lm\mu\,{a_1}^2 - 12\,bk^2\lambda\,\mu\,p{a_1}^2\\
& -12bk^2\lambda\,\mu\,q{a_1}^2 - 24\,bk^2m\mu\,p{a_1}^2 - 24\,bk^2m\mu\,q{a_1}^2\\
& +3\,l\lambda\,wa_1 + 6\,lmwa_1 + 3\,\lambda\,pwa_1 + 3\,\lambda\,qwa_1\\
& +6\,mpwa_1 + 6\,mqwa_1 = 0. \qquad (22)
\end{aligned}
$$

$$\left(m+\frac{G'}{G}\right)^0\left(\frac{G'}{G}\right)^3: \quad \begin{aligned}&50\,ak^3l\lambda^2a_1+200\,ak^3l\lambda\,ma_1+200ak^3lm^2a_1\\&+50\,ak^3\lambda^2pa_1+50\,ak^3\lambda^2qa_1+200\,ak^3\lambda\,mpa_1\\&+200\,ak^3\lambda\,mqa_1+200\,ak^3m^2pa_1+200\,ak^3m^2qa_1\\&-8\,bk^2l\lambda^2{a_1}^2-32\,bk^2l\lambda\,m{a_1}^2-32\,bk^2lm^2{a_1}^2\\&-8\,bk^2\lambda^2p{a_1}^2-8\,bk^2\lambda^2q{a_1}^2-32\,bk^2\lambda\,mp{a_1}^2\\&-32\,bk^2\lambda\,mq{a_1}^2-32\,bk^2m^2p{a_1}^2-32\,bk^2m^2q{a_1}^2\\&+40\,ak^3l\mu\,a_1+40\,ak^3\mu\,pa_1+40\,ak^3\mu\,qa_1\\&-8bk^2l\mu\,{a_1}^2-8\,bk^2\mu\,p{a_1}^2-8\,bk^2\mu\,q{a_1}^2\\&+2\,lwa_1+2\,pwa_1+2\,qwa_1=0.\end{aligned} \tag{23}$$

$$\left(m+\frac{G'}{G}\right)^0\left(\frac{G'}{G}\right)^4: \quad \begin{aligned}&60\,ak^3l\lambda\,a_1+120\,ak^3lma_1+60\,ak^3\lambda\,pa_1\\&+60\,ak^3\lambda\,qa_1+120\,ak^3mpa_1+120\,ak^3mqa_1\\&-10\,bk^2l\lambda\,{a_1}^2-20\,bk^2lm{a_1}^2-10\,bk^2\lambda\,p{a_1}^2\\&-10\,bk^2\lambda\,q{a_1}^2-20\,bk^2mp{a_1}^2-20\,bk^2mq{a_1}^2=0.\end{aligned} \tag{24}$$

$$\left(m+\frac{G'}{G}\right)^0\left(\frac{G'}{G}\right)^5: \quad \begin{aligned}&24\,ak^3la_1+24\,ak^3pa_1+24\,ak^3qa_1-4\,bk^2l{a_1}^2\\&-4\,bk^2p{a_1}^2-4\,bk^2q{a_1}^2=0.\end{aligned} \tag{25}$$

$$\left(m+\frac{G'}{G}\right)^{-2}\left(\frac{G'}{G}\right)^0: \quad \begin{aligned}&-ak^3la_{-1}\lambda^3\mu-6\,ak^3la_{-1}\lambda^2m\mu-12\,ak^3la_{-1}m^2\lambda\,\mu\\&-8\,ak^3la_{-1}m^3\mu-ak^3pa_{-1}\lambda^3\mu-ak^3qa_{-1}\lambda^3\mu\\&-6\,ak^3pa_{-1}\lambda^2m\mu-6\,ak^3qa_{-1}\lambda^2m\mu\\&-12\,ak^3pa_{-1}m^2\lambda\,\mu-12\,ak^3qa_{-1}m^2\lambda\,\mu\\&-8\,ak^3pa_{-1}m^3\mu-8\,ak^3qa_{-1}m^3\mu-8\,ak^3la_{-1}\mu^2\lambda\\&-16\,ak^3la_{-1}\mu^2m-8\,ak^3pa_{-1}\mu^2\lambda-8\,ak^3qa_{-1}\mu^2\lambda\\&-16\,ak^3pa_{-1}\mu^2m-16\,ak^3qa_{-1}\mu^2m+4\,bk^2a_{-1}a_1l\mu^2\lambda\\&+8\,bk^2a_{-1}a_1l\mu^2m+4\,bk^2a_{-1}a_1p\mu^2\lambda-2\,wqa_{-1}m\mu\\&+4\,bk^2a_{-1}a_1q\mu^2\lambda+8\,bk^2a_{-1}a_1p\mu^2m\\&+8\,bk^2a_{-1}a_1q\mu^2m-wla_{-1}\lambda\,\mu-2\,wla_{-1}m\mu\\&-wpa_{-1}\lambda\,\mu-wqa_{-1}\lambda\,\mu-2\,wpa_{-1}m\mu=0.\end{aligned} \tag{26}$$

$$\left(m+\frac{G'}{G}\right)^{-2}\left(\frac{G'}{G}\right): \quad \begin{aligned}
&-ak^3l\lambda^4a_{-1}-8\,ak^3l\lambda^3ma_{-1}-24\,ak^3l\lambda^2m^2a_{-1}\\
&-32\,ak^3l\lambda\,m^3a_{-1}-16\,ak^3lm^4a_{-1}-ak^3\lambda^4pa_{-1}\\
&-ak^3\lambda^4qa_{-1}-8\,ak^3\lambda^3mpa_{-1}-8\,ak^3\lambda^3mqa_{-1}\\
&-24\,ak^3\lambda^2m^2pa_{-1}-24\,ak^3\lambda^2m^2qa_{-1}-32\,ak^3\lambda\,m^3pa_{-1}\\
&-32\,ak^3\lambda\,m^3qa_{-1}-16\,ak^3m^4pa_{-1}-16\,ak^3m^4qa_{-1}\\
&-22\,ak^3l\lambda^2\mu\,a_{-1}-88\,ak^3l\lambda\,m\mu\,a_{-1}-88\,ak^3lm^2\mu\,a_{-1}\\
&-22\,ak^3\lambda^2\mu\,pa_{-1}-22\,ak^3\lambda^2\mu\,qa_{-1}-88\,ak^3\lambda\,m\mu\,pa_{-1}\\
&-88\,ak^3\lambda\,m\mu\,qa_{-1}-88\,ak^3m^2\mu\,pa_{-1}-88\,ak^3m^2\mu\,qa_{-1}\\
&+8\,bk^2l\lambda^2\mu\,a_{-1}a_1+32\,bk^2l\lambda\,m\mu\,a_{-1}a_1+32\,bk^2lm^2\mu\,a_{-1}a_1\\
&+8\,bk^2\lambda^2\mu\,pa_{-1}a_1+8\,bk^2\lambda^2\mu\,qa_{-1}a_1+32\,bk^2\lambda\,m\mu\,pa_{-1}a_1\\
&+32\,bk^2\lambda\,m\mu\,qa_{-1}a_1+32\,bk^2m^2\mu\,pa_{-1}a_1+8\,bk^2\mu^2qa_{-1}a_1\\
&+32\,bk^2m^2\mu\,qa_{-1}a_1-16\,ak^3l\mu^2a_{-1}-16\,ak^3\mu^2pa_{-1}\\
&-16\,ak^3\mu^2qa_{-1}+8\,bk^2l\mu^2a_{-1}a_1+8\,bk^2\mu^2pa_{-1}a_1\\
&-l\lambda^2wa_{-1}-4\,l\lambda\,mwa_{-1}-4\,lm^2wa_{-1}-\lambda^2pwa_{-1}\\
&-\lambda^2qwa_{-1}-4\,\lambda\,mpwa_{-1}-4\,\lambda\,mqwa_{-1}-4\,m^2pwa_{-1}\\
&-4\,m^2qwa_{-1}-2\,l\mu\,wa_{-1}-2\,\mu\,pwa_{-1}-2\,\mu\,qwa_{-1}=0. \quad (27)
\end{aligned}$$

$$\left(m+\frac{G'}{G}\right)^{-2}\left(\frac{G'}{G}\right)^2: \quad \begin{aligned}
&-ak^3l\lambda^4a_{-1}-8\,ak^3l\lambda^3ma_{-1}-24\,ak^3l\lambda^2m^2a_{-1}\\
&-32\,ak^3l\lambda\,m^3a_{-1}-16\,ak^3lm^4a_{-1}-ak^3\lambda^4pa_{-1}\\
&-ak^3\lambda^4qa_{-1}-8\,ak^3\lambda^3mpa_{-1}-8\,ak^3\lambda^3mqa_{-1}\\
&-24\,ak^3\lambda^2m^2pa_{-1}-24\,ak^3\lambda^2m^2qa_{-1}-32\,ak^3\lambda\,m^3pa_{-1}\\
&-32\,ak^3\lambda\,m^3qa_{-1}-16\,ak^3m^4pa_{-1}-16\,ak^3m^4qa_{-1}\\
&-22\,ak^3l\lambda^2\mu\,a_{-1}-88\,ak^3l\lambda\,m\mu\,a_{-1}-88\,ak^3lm^2\mu\,a_{-1}\\
&-22\,ak^3\lambda^2\mu\,pa_{-1}-22\,ak^3\lambda^2\mu\,qa_{-1}-88\,ak^3\lambda\,m\mu\,pa_{-1}\\
&-88\,ak^3\lambda\,m\mu\,qa_{-1}-88\,ak^3m^2\mu\,pa_{-1}-88\,ak^3m^2\mu\,qa_{-1}\\
&+8\,bk^2l\lambda^2\mu\,a_{-1}a_1+32\,bk^2l\lambda\,m\mu\,a_{-1}a_1-\lambda^2qwa_{-1}\\
&+32\,bk^2lm^2\mu\,a_{-1}a_1+8\,bk^2\lambda^2\mu\,pa_{-1}a_1-l\lambda^2wa_{-1}\\
&+8\,bk^2\lambda^2\mu\,qa_{-1}a_1+32\,bk^2\lambda\,m\mu\,pa_{-1}a_1-2\,\mu\,qwa_{-1}\\
&+32\,bk^2\lambda\,m\mu\,qa_{-1}a_1+32\,bk^2m^2\mu\,pa_{-1}a_1-2\,\mu\,pwa_{-1}\\
&+32\,bk^2m^2\mu\,qa_{-1}a_1-16\,ak^3l\mu^2a_{-1}-16\,ak^3\mu^2pa_{-1}\\
&-16\,ak^3\mu^2qa_{-1}+8\,bk^2l\mu^2a_{-1}a_1+8\,bk^2\mu^2pa_{-1}a_1\\
&+8\,bk^2\mu^2qa_{-1}a_1-4\,l\lambda\,mwa_{-1}-4\,lm^2wa_{-1}\\
&-\lambda^2pwa_{-1}-4\,\lambda\,mpwa_{-1}-4\,\lambda\,mqwa_{-1}\\
&-4\,m^2pwa_{-1}-4\,m^2qwa_{-1}-2\,l\mu\,wa_{-1}=0. \quad (28)
\end{aligned}$$

$$\left(m+\frac{G'}{G}\right)^{-2}\left(\frac{G'}{G}\right)^{3}: \quad \begin{aligned}&-50\,ak^3la_{-1}\lambda^2-200\,ak^3la_{-1}\lambda\,m-200\,ak^3la_{-1}m^2\\&-50\,ak^3pa_{-1}\lambda^2-50\,ak^3qa_{-1}\lambda^2-200\,ak^3pa_{-1}\lambda\,m\\&-200\,ak^3qa_{-1}\lambda\,m-200\,ak^3pa_{-1}m^2-200\,ak^3qa_{-1}m^2\\&+16\,bk^2a_{-1}a_1l\lambda^2+64\,bk^2a_{-1}a_1l\lambda\,m+64\,bk^2a_{-1}a_1lm^2\\&+16\,bk^2a_{-1}a_1p\lambda^2+16\,bk^2a_{-1}a_1q\lambda^2+64\,bk^2a_{-1}a_1p\lambda\,m\\&+64\,bk^2a_{-1}a_1q\lambda\,m+64\,bk^2a_{-1}a_1pm^2\\&+64\,bk^2a_{-1}a_1qm^2-40\,ak^3la_{-1}\mu-40\,ak^3pa_{-1}\mu\\&-40\,ak^3qa_{-1}\mu+16\,bk^2a_{-1}a_1l\mu+16\,bk^2a_{-1}a_1p\mu\\&+16\,bk^2a_{-1}a_1q\mu-2\,wla_{-1}-2\,wpa_{-1}\\&-2\,wqa_{-1}=0.\end{aligned} \tag{29}$$

$$\left(m+\frac{G'}{G}\right)^{-2}\left(\frac{G'}{G}\right)^{4}: \quad \begin{aligned}&-60\,ak^3l\lambda\,a_{-1}-120\,ak^3lma_{-1}-60\,ak^3\lambda\,pa_{-1}\\&-60\,ak^3\lambda\,qa_{-1}-120\,ak^3mpa_{-1}-120\,ak^3mqa_{-1}\\&+20\,bk^2l\lambda\,a_{-1}a_1+40\,bk^2lma_{-1}a_1+20\,bk^2\lambda\,pa_{-1}a_1\\&+20\,bk^2\lambda\,qa_{-1}a_1+40\,bk^2mpa_{-1}a_1\\&+40\,bk^2mqa_{-1}a_1=0.\end{aligned} \tag{30}$$

$$\left(m+\frac{G'}{G}\right)^{-2}\left(\frac{G'}{G}\right)^{5}: \quad \begin{aligned}&-24\,ak^3la_{-1}-24\,ak^3pa_{-1}-24\,ak^3qa_{-1}\\&+8\,bk^2la_{-1}a_1+8\,bk^2pa_{-1}a_1+8\,bk^2qa_{-1}a_1=0.\end{aligned} \tag{31}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right)^{0}: \quad \begin{aligned}&14\,ak^3la_{-1}\lambda^2\mu^2+56\,ak^3la_{-1}\lambda\,m\mu^2+56\,ak^3la_{-1}m^2\mu^2\\&+14\,ak^3pa_{-1}\lambda^2\mu^2+14\,ak^3qa_{-1}\lambda^2\mu^2+16\,ak^3pa_{-1}\mu^3\\&+56\,ak^3qa_{-1}\lambda\,m\mu^2+56\,ak^3pa_{-1}m^2\mu^2+16\,ak^3la_{-1}\mu^3\\&+56\,ak^3qa_{-1}m^2\mu^2+56\,ak^3pa_{-1}\lambda\,m\mu^2-4\,bk^2a_1a_{-1}q\mu^3\\&-4\,bk^2a_1a_{-1}l\mu^3-4\,bk^2a_1a_{-1}p\mu^3+16\,ak^3qa_{-1}\mu^3\\&+2\,wla_{-1}\mu^2+2\,wpa_{-1}\mu^2+2\,wqa_{-1}\mu^2=0.\end{aligned} \tag{32}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right): \quad \begin{aligned}&28\,ak^3la_{-1}\lambda^3\mu+168\,ak^3la_{-1}\lambda^2m\mu+336\,ak^3la_{-1}m^2\lambda\,\mu\\&+224\,ak^3la_{-1}m^3\mu+28\,ak^3pa_{-1}\lambda^3\mu+28\,ak^3qa_{-1}\lambda^3\mu\\&+168\,ak^3pa_{-1}\lambda^2m\mu+168\,ak^3qa_{-1}\lambda^2m\mu+8\,wqa_{-1}m\mu\\&+336\,ak^3pa_{-1}m^2\lambda\,\mu+336\,ak^3qa_{-1}m^2\lambda\,\mu+8\,wpa_{-1}m\mu\\&+224\,ak^3pa_{-1}m^3\mu+224\,ak^3qa_{-1}m^3\mu+104\,ak^3la_{-1}\lambda\,\mu^2\\&+208\,ak^3la_{-1}m\mu^2+104\,ak^3pa_{-1}\lambda\,\mu^2+104\,ak^3qa_{-1}\lambda\,\mu^2\\&+208\,ak^3pa_{-1}m\mu^2+208\,ak^3qa_{-1}m\mu^2+4\,wqa_{-1}\lambda\,\mu\\&-12\,bk^2a_{-1}a_1l\lambda\,\mu^2-24\,bk^2a_{-1}a_1lm\mu^2+4\,wpa_{-1}\lambda\,\mu\\&-24\,bk^2a_{-1}a_1pm\mu^2-24\,bk^2a_{-1}a_1qm\mu^2+4\,wla_{-1}\lambda\,\mu\\&-12\,bk^2a_{-1}a_1p\lambda\,\mu^2+8\,wla_{-1}m\mu\\&-12\,bk^2a_{-1}a_1q\lambda\,\mu^2=0.\end{aligned} \tag{33}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right)^2: \quad \begin{aligned}&14\,ak^3l\lambda^4a_{-1}+112\,ak^3l\lambda^3ma_{-1}+336\,ak^3l\lambda^2m^2a_{-1}\\&+448\,ak^3l\lambda\,m^3a_{-1}+224\,ak^3lm^4a_{-1}+448\,ak^3\lambda\,m^3pa_{-1}\\&+14\,ak^3\lambda^4qa_{-1}+112\,ak^3\lambda^3mpa_{-1}+112\,ak^3\lambda^3mqa_{-1}\\&+336\,ak^3\lambda^2m^2pa_{-1}+336\,ak^3\lambda^2m^2qa_{-1}+14\,ak^3\lambda^4pa_{-1}\\&+448\,ak^3\lambda\,m^3qa_{-1}+224\,ak^3m^4pa_{-1}+224\,ak^3m^4qa_{-1}\\&+188\,ak^3l\lambda^2\mu\,a_{-1}+752\,ak^3l\lambda\,m\mu\,a_{-1}+4\,\mu\,pwa_{-1}\\&+188\,ak^3\lambda^2\mu\,pa_{-1}+188\,ak^3\lambda^2\mu\,qa_{-1}+8\,\lambda\,mqwa_{-1}\\&+752\,ak^3\lambda\,m\mu\,qa_{-1}+752\,ak^3m^2\mu\,pa_{-1}+8\,lm^2wa_{-1}\\&-12\,bk^2l\lambda^2\mu\,a_{-1}a_1-48\,bk^2l\lambda\,m\mu\,a_{-1}a_1+8\,m^2pwa_{-1}\\&-12\,bk^2\lambda^2\mu\,pa_{-1}a_1-12\,bk^2\lambda^2\mu\,qa_{-1}a_1+4\,\mu\,qwa_{-1}\\&-48\,bk^2\lambda\,m\mu\,qa_{-1}a_1-48\,bk^2m^2\mu\,pa_{-1}a_1+2\,l\lambda^2wa_{-1}\\&+104\,ak^3l\mu^2a_{-1}+104\,ak^3\mu^2pa_{-1}+104\,ak^3\mu^2qa_{-1}\\&-12\,bk^2l\mu^2a_{-1}a_1-12\,bk^2\mu^2pa_{-1}a_1-12\,bk^2\mu^2qa_{-1}a_1\\&-48\,bk^2m^2\mu\,qa_{-1}a_1-48\,bk^2\lambda\,m\mu\,pa_{-1}a_1+8\,l\lambda\,mwa_{-1}\\&+752\,ak^3m^2\mu\,qa_{-1}+2\,\lambda^2qwa_{-1}+8\,\lambda\,mpwa_{-1}\\&-48\,bk^2lm^2\mu\,a_{-1}a_1+8\,m^2qwa_{-1}+4\,l\mu\,wa_{-1}\\&+752\,ak^3\lambda\,m\mu\,pa_{-1}+2\,\lambda^2pwa_{-1}\\&+752\,ak^3lm^2\mu\,a_{-1}=0.\end{aligned} \tag{34}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right)^{3}: \quad \begin{aligned}&100\,ak^3l\lambda^3a_{-1}+600\,ak^3l\lambda^2ma_{-1}+1200\,ak^3l\lambda\,m^2a_{-1}\\&+800\,ak^3lm^3a_{-1}+100\,ak^3\lambda^3pa_{-1}+100\,ak^3\lambda^3qa_{-1}\\&+600\,ak^3\lambda^2mpa_{-1}+600\,ak^3\lambda^2mqa_{-1}+800\,ak^3m^3qa_{-1}\\&+1200\,ak^3\lambda\,m^2pa_{-1}+800\,ak^3m^3pa_{-1}-4\,bk^2\lambda^3qa_{-1}a_1\\&+1200\,ak^3\lambda\,m^2qa_{-1}-48\,bk^2l\lambda\,m^2a_{-1}a_1+4\,\lambda\,qwa_{-1}\\&-4\,bk^2l\lambda^3a_{-1}a_1-24\,bk^2l\lambda^2ma_{-1}a_1+640\,ak^3lm\mu\,a_{-1}\\&-32\,bk^2lm^3a_{-1}a_1-4\,bk^2\lambda^3pa_{-1}a_1-48\,bk^2\lambda\,m^2pa_{-1}a_1\\&-24\,bk^2\lambda^2mpa_{-1}a_1-24\,bk^2\lambda^2mqa_{-1}a_1+4\,\lambda\,pwa_{-1}\\&-48\,bk^2\lambda\,m^2qa_{-1}a_1-32\,bk^2m^3pa_{-1}a_1+8\,lmwa_{-1}\\&+320\,ak^3l\lambda\,\mu\,a_{-1}+320\,ak^3\lambda\,\mu\,pa_{-1}+640\,ak^3m\mu\,qa_{-1}\\&+320\,ak^3\lambda\,\mu\,qa_{-1}+640\,ak^3m\mu\,pa_{-1}-48\,bk^2m\mu\,qa_{-1}a_1\\&-24\,bk^2l\lambda\,\mu\,a_{-1}a_1-48\,bk^2lm\mu\,a_{-1}a_1+8\,mqwa_{-1}\\&-24\,bk^2\lambda\,\mu\,qa_{-1}a_1-48\,bk^2m\mu\,pa_{-1}a_1+8\,mpwa_{-1}\\&-32\,bk^2m^3qa_{-1}a_1-24\,bk^2\lambda\,\mu\,pa_{-1}a_1+4\,l\lambda\,wa_{-1}=0.\end{aligned} \tag{35}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right)^{4}: \quad \begin{aligned}&230\,ak^3l\lambda^2a_{-1}+920\,ak^3l\lambda\,ma_{-1}+920\,ak^3lm^2a_{-1}\\&+230\,ak^3\lambda^2pa_{-1}+230\,ak^3\lambda^2qa_{-1}+920\,ak^3\lambda\,mpa_{-1}\\&+920\,ak^3\lambda\,mqa_{-1}+920\,ak^3m^2pa_{-1}+920\,ak^3m^2qa_{-1}\\&-12\,bk^2l\lambda^2a_{-1}a_1-48\,bk^2l\lambda\,ma_{-1}a_1-48\,bk^2lm^2a_{-1}a_1\\&-12\,bk^2\lambda^2pa_{-1}a_1-12\,bk^2\lambda^2qa_{-1}a_1+2\,qwa_{-1}\\&-48\,bk^2\lambda\,mqa_{-1}a_1-48\,bk^2m^2pa_{-1}a_1+2\,pwa_{-1}\\&+160\,ak^3l\mu\,a_{-1}+160\,ak^3\mu\,pa_{-1}+160\,ak^3\mu\,qa_{-1}\\&-12\,bk^2l\mu\,a_{-1}a_1-12\,bk^2\mu\,pa_{-1}a_1-12\,bk^2\mu\,qa_{-1}a_1\\&-48\,bk^2\lambda\,mpa_{-1}a_1-48\,bk^2m^2qa_{-1}a_1+2\,lwa_{-1}=0.\end{aligned} \tag{36}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right)^{5}: \quad \begin{aligned}&216\,ak^3l\lambda\,a_{-1}+432\,ak^3lma_{-1}+216\,ak^3\lambda\,pa_{-1}\\&+216\,ak^3\lambda\,qa_{-1}+432\,ak^3mpa_{-1}+432\,ak^3mqa_{-1}\\&-12\,bk^2l\lambda\,a_{-1}a_1-24\,bk^2lma_{-1}a_1-12\,bk^2\lambda\,pa_{-1}a_1\\&-12\,bk^2\lambda\,qa_{-1}a_1-24\,bk^2mpa_{-1}a_1-24\,bk^2mqa_{-1}a_1=0.\end{aligned} \tag{37}$$

$$\left(m+\frac{G'}{G}\right)^{-3}\left(\frac{G'}{G}\right)^{6}: \quad \begin{aligned}&72\,ak^3la_{-1}+72\,ak^3pa_{-1}+72\,ak^3qa_{-1}\\&-4\,bk^2la_{-1}a_1-4\,bk^2pa_{-1}a_1-4\,bk^2qa_{-1}a_1=0.\end{aligned} \tag{38}$$

$$\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{0}: \quad \begin{aligned}&-36\,ak^3la_{-1}\mu^3\lambda-72\,ak^3la_{-1}\mu^3m-36\,ak^3pa_{-1}\mu^3\lambda\\&-36\,ak^3qa_{-1}\mu^3\lambda-72\,ak^3pa_{-1}\mu^3m-72\,ak^3qa_{-1}\mu^3m\\&-2\,bk^2{a_{-1}}^2l\lambda\,\mu^2-4\,bk^2{a_{-1}}^2lm\mu^2-2\,bk^2{a_{-1}}^2p\lambda\,\mu^2\\&-2\,bk^2{a_{-1}}^2q\lambda\,\mu^2-4\,bk^2{a_{-1}}^2pm\mu^2-4\,bk^2{a_{-1}}^2qm\mu^2=0.\end{aligned} \quad (39)$$

$$\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right): \quad \begin{aligned}&-108\,ak^3la_{-1}\lambda^2\mu^2-432\,ak^3la_{-1}\lambda\,m\mu^2-72\,ak^3qa_{-1}\mu^3\\&-108\,ak^3pa_{-1}\lambda^2\mu^2-108\,ak^3qa_{-1}\lambda^2\mu^2-72\,ak^3pa_{-1}\mu^3\\&-432\,ak^3qa_{-1}\lambda\,m\mu^2-432\,ak^3pa_{-1}m^2\mu^2-4\,bk^2\mu^2q{a_{-1}}^2\\&-432\,ak^3pa_{-1}\lambda\,m\mu^2-432\,ak^3la_{-1}m^2\mu^2-72\,ak^3la_{-1}\mu^3\\&-4\,bk^2l\lambda^2\mu\,{a_{-1}}^2-16\,bk^2l\lambda\,m\mu\,{a_{-1}}^2-16\,bk^2lm^2\mu\,{a_{-1}}^2\\&-432\,ak^3qa_{-1}m^2\mu^2-4\,bk^2\lambda^2\mu\,p{a_{-1}}^2-4\,bk^2\lambda^2\mu\,q{a_{-1}}^2\\&-16\,bk^2\lambda\,m\mu\,q{a_{-1}}^2-16\,bk^2m^2\mu\,p{a_{-1}}^2-4\,bk^2\mu^2p{a_{-1}}^2\\&-16\,bk^2\lambda\,m\mu\,p{a_{-1}}^2-16\,bk^2m^2\mu\,q{a_{-1}}^2-4\,bk^2l\mu^2{a_{-1}}^2=0\end{aligned} \quad (40)$$

$$\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{2}: \quad \begin{aligned}&-108\,ak^3la_{-1}\lambda^3\mu-648\,ak^3la_{-1}\lambda^2m\mu-108\,ak^3qa_{-1}\lambda^3\mu\\&-1296\,ak^3la_{-1}m^2\lambda\,\mu-2\,bk^2\lambda^3p{a_{-1}}^2-108\,ak^3pa_{-1}\lambda^3\mu\\&-1296\,ak^3pa_{-1}m^2\lambda\,\mu-864\,ak^3la_{-1}m^3\mu-2\,bk^2\lambda^3q{a_{-1}}^2\\&-648\,ak^3pa_{-1}\lambda^2m\mu-648\,ak^3qa_{-1}\lambda^2m\mu-16\,bk^2m^3q{a_{-1}}^2\\&-1296\,ak^3qa_{-1}m^2\lambda\,\mu-12\,bk^2\lambda\,\mu\,q{a_{-1}}^2-16\,bk^2m^3p{a_{-1}}^2\\&-864\,ak^3qa_{-1}m^3\mu-24\,bk^2m\mu\,p{a_{-1}}^2-24\,bk^2m\mu\,q{a_{-1}}^2\\&-324\,ak^3la_{-1}\lambda\,\mu^2-648\,ak^3la_{-1}m\mu^2-324\,ak^3pa_{-1}\lambda\,\mu^2\\&-324\,ak^3qa_{-1}\lambda\,\mu^2-648\,ak^3pa_{-1}m\mu^2-648\,ak^3qa_{-1}m\mu^2\\&-2\,bk^2l\lambda^3{a_{-1}}^2-12\,bk^2l\lambda^2m{a_{-1}}^2-24\,bk^2l\lambda\,m^2{a_{-1}}^2\\&-16\,bk^2lm^3{a_{-1}}^2-864\,ak^3pa_{-1}m^3\mu-24\,bk^2\lambda\,m^2q{a_{-1}}^2\\&-12\,bk^2\lambda^2mp{a_{-1}}^2-12\,bk^2\lambda^2mq{a_{-1}}^2-24\,bk^2\lambda\,m^2p{a_{-1}}^2\\&-12\,bk^2l\lambda\,\mu\,{a_{-1}}^2-24\,bk^2lm\mu\,{a_{-1}}^2-12\,bk^2\lambda\,\mu\,p{a_{-1}}^2=0.\end{aligned} \quad (41)$$

$$\begin{aligned}
\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{3}: \quad & -36\,ak^3l\lambda^4a_{-1}-288\,ak^3l\lambda^3ma_{-1}-864\,ak^3l\lambda^2m^2a_{-1}\\
&-1152\,ak^3l\lambda\,m^3a_{-1}-576\,ak^3lm^4a_{-1}-36\,ak^3\lambda^4pa_{-1}\\
&-36\,ak^3\lambda^4qa_{-1}-288\,ak^3\lambda^3mpa_{-1}-288\,ak^3\lambda^3mqa_{-1}\\
&-864\,ak^3\lambda^2m^2pa_{-1}-864\,ak^3\lambda^2m^2qa_{-1}-1152\,ak^3\lambda\,m^3pa_{-1}\\
&-1152\,ak^3\lambda\,m^3qa_{-1}-576\,ak^3m^4pa_{-1}-576\,ak^3m^4qa_{-1}\\
&-432\,ak^3l\lambda^2\mu\,a_{-1}-1728\,ak^3l\lambda\,m\mu\,a_{-1}-1728\,ak^3lm^2\mu\,a_{-1}\\
&-432\,ak^3\lambda^2\mu\,pa_{-1}-432\,ak^3\lambda^2\mu\,qa_{-1}-1728\,ak^3\lambda\,m\mu\,pa_{-1}\\
&-1728\,ak^3\lambda\,m\mu\,qa_{-1}-1728\,ak^3m^2\mu\,pa_{-1}-1728\,ak^3m^2\mu\,qa_{-1}\\
&-216\,ak^3l\mu^2a_{-1}-216\,ak^3\mu^2pa_{-1}-216\,ak^3\mu^2qa_{-1}\\
&-8\,bk^2l\lambda^2{a_{-1}}^2-32\,bk^2l\lambda\,m{a_{-1}}^2-32\,bk^2lm^2{a_{-1}}^2\\
&-8\,bk^2\lambda^2p{a_{-1}}^2-8\,bk^2\lambda^2q{a_{-1}}^2-32\,bk^2\lambda\,mp{a_{-1}}^2\\
&-32\,bk^2\lambda\,mq{a_{-1}}^2-32\,bk^2m^2p{a_{-1}}^2-32\,bk^2m^2q{a_{-1}}^2\\
&-8\,bk^2l\mu\,{a_{-1}}^2-8\,bk^2\mu\,p{a_{-1}}^2-8\,bk^2\mu\,q{a_{-1}}^2=0. \qquad (42)
\end{aligned}$$

$$\begin{aligned}
\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{4}: \quad & -180\,ak^3l\lambda^3a_{-1}-1080\,ak^3l\lambda^2ma_{-1}-2160\,ak^3l\lambda\,m^2a_{-1}\\
&-1440\,ak^3lm^3a_{-1}-180\,ak^3\lambda^3pa_{-1}-180\,ak^3\lambda^3qa_{-1}\\
&-1080\,ak^3\lambda^2mpa_{-1}-1080\,ak^3\lambda^2mqa_{-1}-2160\,ak^3\lambda\,m^2pa_{-1}\\
&-2160\,ak^3\lambda\,m^2qa_{-1}-1440\,ak^3m^3pa_{-1}-1440\,ak^3m^3qa_{-1}\\
&-540\,ak^3l\lambda\,\mu\,a_{-1}-1080\,ak^3lm\mu\,a_{-1}-540\,ak^3\lambda\,\mu\,pa_{-1}\\
&-540\,ak^3\lambda\,\mu\,qa_{-1}-1080\,ak^3m\mu\,pa_{-1}-1080\,ak^3m\mu\,qa_{-1}\\
&-10\,bk^2{a_{-1}}^2l\lambda-20\,bk^2{a_{-1}}^2lm-10\,bk^2{a_{-1}}^2p\lambda\\
&-10\,bk^2{a_{-1}}^2q\lambda-20\,bk^2{a_{-1}}^2pm-20\,bk^2{a_{-1}}^2qm=0. \qquad (43)
\end{aligned}$$

$$\begin{aligned}
\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{5}: \quad & -324\,ak^3l\lambda^2a_{-1}-1296\,ak^3l\lambda\,ma_{-1}-1296\,ak^3lm^2a_{-1}\\
&-324\,ak^3\lambda^2pa_{-1}-324\,ak^3\lambda^2qa_{-1}-1296\,ak^3\lambda\,mpa_{-1}\\
&-1296\,ak^3\lambda\,mqa_{-1}-1296\,ak^3m^2pa_{-1}-1296\,ak^3m^2qa_{-1}\\
&-216\,ak^3l\mu\,a_{-1}-216\,ak^3\mu\,pa_{-1}-216\,ak^3\mu\,qa_{-1}\\
&-4\,bk^2{a_{-1}}^2l-4\,bk^2{a_{-1}}^2p-4\,bk^2{a_{-1}}^2q=0. \qquad (44)
\end{aligned}$$

$$\begin{aligned}
\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{6}: \quad & -252\,ak^3l\lambda\,a_{-1}-504\,ak^3lma_{-1}-252\,ak^3\lambda\,pa_{-1}\\
&-252\,ak^3\lambda\,qa_{-1}-504\,ak^3mpa_{-1}-504\,ak^3mqa_{-1}=0. \qquad (45)
\end{aligned}$$

$$\left(m+\frac{G'}{G}\right)^{-4}\left(\frac{G'}{G}\right)^{7}: \quad -72\,ak^3la_{-1}-72\,ak^3pa_{-1}-72\,ak^3qa_{-1}=0. \tag{46}$$

$$\begin{aligned}\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{0}: \quad & 24\,ak^3la_{-1}\mu^4+24\,ak^3pa_{-1}\mu^4+24\,ak^3qa_{-1}\mu^4\\ & +4\,bk^2{a_{-1}}^2l\mu^3+4\,bk^2{a_{-1}}^2p\mu^3+4\,bk^2{a_{-1}}^2q\mu^3=0.\end{aligned} \tag{47}$$

$$\begin{aligned}\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right): \quad & 96\,ak^3la_{-1}\mu^3\lambda+192\,ak^3la_{-1}\mu^3m+96\,ak^3pa_{-1}\mu^3\lambda\\ & +96\,ak^3qa_{-1}\mu^3\lambda+192\,ak^3pa_{-1}\mu^3m+192\,ak^3qa_{-1}\mu^3m\\ & +12\,bk^2{a_{-1}}^2l\lambda\,\mu^2+24\,bk^2{a_{-1}}^2lm\mu^2+12\,bk^2{a_{-1}}^2p\lambda\,\mu^2\\ & +12\,bk^2{a_{-1}}^2q\lambda\,\mu^2+24\,bk^2{a_{-1}}^2pm\mu^2+24\,bk^2{a_{-1}}^2qm\mu^2=0.\end{aligned} \tag{48}$$

$$\begin{aligned}\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{2}: \quad & 144\,ak^3la_{-1}\lambda^2\mu^2+576\,ak^3la_{-1}\lambda\,m\mu^2+576\,ak^3la_{-1}m^2\mu^2\\ & +144\,ak^3pa_{-1}\lambda^2\mu^2+144\,ak^3qa_{-1}\lambda^2\mu^2+576\,ak^3pa_{-1}\lambda\,m\mu^2\\ & +576\,ak^3qa_{-1}\lambda\,m\mu^2+576\,ak^3pa_{-1}m^2\mu^2+576\,ak^3qa_{-1}m^2\mu^2\\ & +96\,ak^3la_{-1}\mu^3+96\,ak^3pa_{-1}\mu^3+96\,ak^3qa_{-1}\mu^3\\ & +12\,bk^2l\lambda^2\mu\,{a_{-1}}^2+48\,bk^2l\lambda\,m\mu\,{a_{-1}}^2+48\,bk^2lm^2\mu\,{a_{-1}}^2\\ & +12\,bk^2\lambda^2\mu\,p{a_{-1}}^2+12\,bk^2\lambda^2\mu\,q{a_{-1}}^2+48\,bk^2\lambda\,m\mu\,p{a_{-1}}^2\\ & +48\,bk^2\lambda\,m\mu\,q{a_{-1}}^2+48\,bk^2m^2\mu\,p{a_{-1}}^2+48\,bk^2m^2\mu\,q{a_{-1}}^2\\ & +12\,bk^2l\mu^2{a_{-1}}^2+12\,bk^2\mu^2p{a_{-1}}^2+12\,bk^2\mu^2q{a_{-1}}^2=0.\end{aligned} \tag{49}$$

$$\begin{aligned}\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{3}: \quad & 96\,ak^3la_{-1}\lambda^3\mu+576\,ak^3la_{-1}\lambda^2m\mu+1152\,ak^3la_{-1}m^2\lambda\,\mu\\ & +768\,ak^3la_{-1}m^3\mu+96\,ak^3pa_{-1}\lambda^3\mu+96\,ak^3qa_{-1}\lambda^3\mu\\ & +576\,ak^3pa_{-1}\lambda^2m\mu+576\,ak^3qa_{-1}\lambda^2m\mu+1152\,ak^3pa_{-1}m^2\lambda\,\mu\\ & +1152\,ak^3qa_{-1}m^2\lambda\,\mu+768\,ak^3pa_{-1}m^3\mu+768\,ak^3qa_{-1}m^3\mu\\ & +288\,ak^3la_{-1}\lambda\,\mu^2+576\,ak^3la_{-1}m\mu^2+288\,ak^3pa_{-1}\lambda\,\mu^2\\ & +288\,ak^3qa_{-1}\lambda\,\mu^2+576\,ak^3pa_{-1}m\mu^2+576\,ak^3qa_{-1}m\mu^2\\ & +4\,bk^2l\lambda^3{a_{-1}}^2+24\,bk^2l\lambda^2m{a_{-1}}^2+48\,bk^2l\lambda\,m^2{a_{-1}}^2\\ & +32\,bk^2lm^3{a_{-1}}^2+4\,bk^2\lambda^3p{a_{-1}}^2+4\,bk^2\lambda^3q{a_{-1}}^2\\ & +24\,bk^2\lambda^2mp{a_{-1}}^2+24\,bk^2\lambda^2mq{a_{-1}}^2+48\,bk^2\lambda\,m^2p{a_{-1}}^2\\ & +48\,bk^2\lambda\,m^2q{a_{-1}}^2+32\,bk^2m^3p{a_{-1}}^2+32\,bk^2m^3q{a_{-1}}^2\\ & +24\,bk^2l\lambda\,\mu\,{a_{-1}}^2+48\,bk^2lm\mu\,{a_{-1}}^2+24\,bk^2\lambda\,\mu\,p{a_{-1}}^2\\ & +24\,bk^2\lambda\,\mu\,q{a_{-1}}^2+48\,bk^2m\mu\,p{a_{-1}}^2+48\,bk^2m\mu\,q{a_{-1}}^2=0.\end{aligned} \tag{50}$$

$$\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{4}: \quad \begin{aligned}&24\,ak^3l\lambda^4a_{-1}+192\,ak^3l\lambda^3ma_{-1}+576\,ak^3l\lambda^2m^2a_{-1}\\&+768\,ak^3l\lambda\,m^3a_{-1}+384\,ak^3lm^4a_{-1}+24\,ak^3\lambda^4pa_{-1}\\&+24\,ak^3\lambda^4qa_{-1}+192\,ak^3\lambda^3mpa_{-1}+192\,ak^3\lambda^3mqa_{-1}\\&+576\,ak^3\lambda^2m^2pa_{-1}+576\,ak^3\lambda^2m^2qa_{-1}+768\,ak^3\lambda\,m^3pa_{-1}\\&+768\,ak^3\lambda\,m^3qa_{-1}+384\,ak^3m^4pa_{-1}+384\,ak^3m^4qa_{-1}\\&+288\,ak^3l\lambda^2\mu\,a_{-1}+1152\,ak^3l\lambda\,m\mu\,a_{-1}+1152\,ak^3lm^2\mu\,a_{-1}\\&+288\,ak^3\lambda^2\mu\,pa_{-1}+288\,ak^3\lambda^2\mu\,qa_{-1}+1152\,ak^3\lambda\,m\mu\,pa_{-1}\\&+1152\,ak^3\lambda\,m\mu\,qa_{-1}+1152\,ak^3m^2\mu\,pa_{-1}+1152\,ak^3m^2\mu\,qa_{-1}\\&+144\,ak^3l\mu^2a_{-1}+144\,ak^3\mu^2pa_{-1}+144\,ak^3\mu^2qa_{-1}\\&+12\,bk^2l\lambda^2{a_{-1}}^2+48\,bk^2l\lambda\,m{a_{-1}}^2+48\,bk^2lm^2{a_{-1}}^2\\&+12\,bk^2\lambda^2p{a_{-1}}^2+12\,bk^2\lambda^2q{a_{-1}}^2+48\,bk^2\lambda\,mp{a_{-1}}^2\\&+48\,bk^2\lambda\,mq{a_{-1}}^2+48\,bk^2m^2p{a_{-1}}^2+48\,bk^2m^2q{a_{-1}}^2\\&+12\,bk^2l\mu\,{a_{-1}}^2+12\,bk^2\mu\,p{a_{-1}}^2+12\,bk^2\mu\,q{a_{-1}}^2=0.\end{aligned} \tag{51}$$

$$\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{5}: \quad \begin{aligned}&96\,ak^3la_{-1}\lambda^3+576\,ak^3la_{-1}\lambda^2m+1152\,ak^3la_{-1}m^2\lambda\\&+768\,ak^3la_{-1}m^3+96\,ak^3pa_{-1}\lambda^3+96\,ak^3qa_{-1}\lambda^3\\&+576\,ak^3pa_{-1}\lambda^2m+576\,ak^3qa_{-1}\lambda^2m+1152\,ak^3pa_{-1}m^2\lambda\\&+1152\,ak^3qa_{-1}m^2\lambda+768\,ak^3pa_{-1}m^3+768\,ak^3qa_{-1}m^3\\&+288\,ak^3l\lambda\,\mu\,a_{-1}+576\,ak^3lm\mu\,a_{-1}+288\,ak^3\lambda\,\mu\,pa_{-1}\\&+288\,ak^3\lambda\,\mu\,qa_{-1}+576\,ak^3m\mu\,pa_{-1}+576\,ak^3m\mu\,qa_{-1}\\&+12\,bk^2l\lambda\,{a_{-1}}^2+24\,bk^2lm{a_{-1}}^2+12\,bk^2\lambda\,p{a_{-1}}^2\\&+12\,bk^2\lambda\,q{a_{-1}}^2+24\,bk^2mp{a_{-1}}^2+24\,bk^2mq{a_{-1}}^2=0.\end{aligned} \tag{52}$$

$$\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{6}: \quad \begin{aligned}&144\,ak^3la_{-1}\lambda^2+576\,ak^3la_{-1}\lambda\,m+576\,ak^3la_{-1}m^2\\&+144\,ak^3pa_{-1}\lambda^2+144\,ak^3qa_{-1}\lambda^2+576\,ak^3pa_{-1}\lambda\,m\\&+576\,ak^3qa_{-1}\lambda\,m+576\,ak^3pa_{-1}m^2+576\,ak^3qa_{-1}m^2\\&+96\,ak^3l\mu\,a_{-1}+96\,ak^3\mu\,pa_{-1}+96\,ak^3\mu\,qa_{-1}\\&+4\,bk^2l{a_{-1}}^2+4\,bk^2p{a_{-1}}^2+4\,bk^2q{a_{-1}}^2=0.\end{aligned} \tag{53}$$

$$\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{7}: \quad \begin{aligned}&96\,ak^3la_{-1}\lambda+192\,ak^3la_{-1}m+96\,ak^3pa_{-1}\lambda\\&+96\,ak^3qa_{-1}\lambda+192\,ak^3pa_{-1}m+192\,ak^3qa_{-1}m=0.\end{aligned} \tag{54}$$

$$\left(m+\frac{G'}{G}\right)^{-5}\left(\frac{G'}{G}\right)^{8}: \quad 24\,ak^3la_{-1}+24\,ak^3pa_{-1}+24\,ak^3qa_{-1}=0. \tag{55}$$

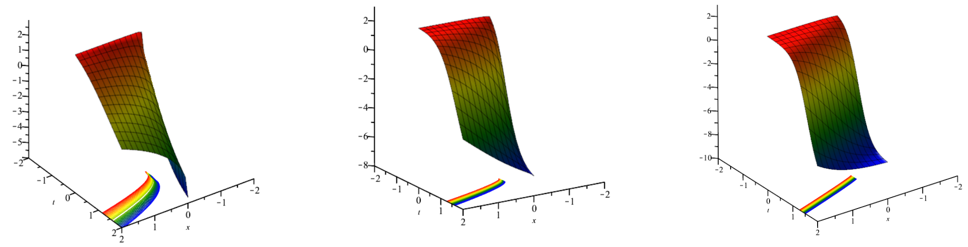
By solving the above algebraic system of Eq.(20) - Eq.(55), the following set of coefficient for the exact solution of Eq.(3.1) can be obtained, which is given as

First set : $a_{-1} = 0, a_0 = a_0, a_1 = \frac{6ak}{b}, \omega = -ak^3\lambda^2 - 4ak^3\lambda m - 4ak^3m^2 + 4ak^3\mu$, l, p, q and k are constant. The solution can be presented as the following expression

$$u_1(x,y,z,s,t) = a_0 + \frac{3ak\sqrt{(\lambda+2\,m)^2-4\,\mu}}{b}$$
$$-\frac{6akA_1\sqrt{(\lambda+2\,m)^2-4\,\mu}}{b\left(A_1+A_2\mathrm{e}^{\sqrt{(\lambda+2\,m)^2-4\,\mu}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{ly^\alpha}{\Gamma(1+\alpha)}+\frac{pz^\alpha}{\Gamma(1+\alpha)}+\frac{qs^\alpha}{\Gamma(1+\alpha)}+\frac{(-ak^3\lambda^2-4\,ak^3\lambda\,m-4\,ak^3m^2+4\,ak^3\mu)t^\alpha}{\Gamma(1+\alpha)}\right)}\right)} - \lambda. \tag{56}$$

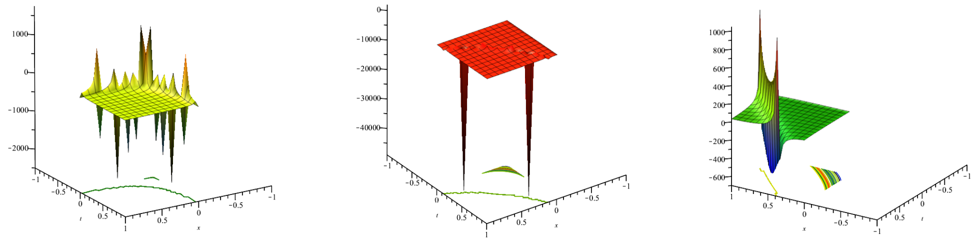
Second set: $l = -p-q, a_{-1}, a_0, a_1, \omega, p, q$ and $k \in R$. The solution can be presented as the following expression

$$u_2(x,y,z,s,t) =$$
$$\frac{a_{-1}}{\left(1-\frac{2A_1}{A_1+A_2\mathrm{e}^{\sqrt{(\lambda+2\,m)^2-4\,\mu}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{(-p-q)y^\alpha}{\Gamma(1+\alpha)}+\frac{pz^\alpha}{\Gamma(1+\alpha)}+\frac{qs^\alpha}{\Gamma(1+\alpha)}+\frac{\omega t^\alpha}{\Gamma(1+\alpha)}\right)}}\right)\sqrt{(\lambda+2\,m)^2-4\,\mu}-\frac{\lambda}{2}} + a_0$$
$$+a_1\left(\left(1-\frac{2A_1}{A_1+A_2\mathrm{e}^{\sqrt{(\lambda+2\,m)^2-4\,\mu}\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\frac{(-p-q)y^\alpha}{\Gamma(1+\alpha)}+\frac{pz^\alpha}{\Gamma(1+\alpha)}+\frac{qs^\alpha}{\Gamma(1+\alpha)}+\frac{\omega t^\alpha}{\Gamma(1+\alpha)}\right)}}\right)\sqrt{(\lambda+2\,m)^2-4\,\mu}-\frac{\lambda}{2}\right). \tag{57}$$



$[a]$ $[b]$ $[c]$

Fig.1 The solutions $u_1(x, y, z, s, t)$, [a], [b] and [c] show $\alpha = 0.3, 0.6$ and 0.9 respectively, where $a_0 = 1, a = 1, k = 1, l = 1, p = 1, q = 1, y = 1, z = 1, s = 1, A_1 = 3, A_2 = 1, b = 1, m = 0.8, \lambda = 1, \mu = 1.$



$[a]$ $[b]$ $[c]$

Fig.2 The solutions $u_2(x, y, z, s, t)$, [a], [b] and [c] show $\alpha = 0.3, 0.6$ and 0.9 respectively , where,$a_0 = 10, a_{-1} = 10, a_1 = 1, k = 1, p = 1, q = 1, y = 1, z = 1, s = 1, A_1 = 10, A_2 = 1, b = 1, m = 1, \lambda = 1, \mu = 1, \omega = 1.$

5. CONCLUSIONS

In summary, we have proposed the (4+1)-dimensional fractional Boiti–Leon–Manna–Pempinelli (BLMP) equation. the $\left(m + \frac{G'}{G}\right)$ expansion method have been implemented for obtaining the new exact solutions of The (4+1)-dimensional fractional Boiti–Leon–Manna–Pempinelli equation. The performance of this method is reliable and effective. Thus, we deduce that the proposed method can be extended to solve many systems of nonlinear fractional partial differential equations.

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