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On Intuitionistic Fuzzy n–Normed I_λ–Convergence of Sequence spaces Defined by Orlicz Function

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Abstract

Recently, the notion of I_λ–convergence in an intuitionistic fuzzy n–normed spaces was introduced by Konwar et al. [N. Konwar and P. Debnath. I_λ–convergence in intuitionistic fuzzy n-normed linear space. 07 2016]. In this article with the help of the notion I_λ–convergence , we introduce some new Orlicz sequence spaces. Further, we examine some topological properties on these spaces.

Keywords: Intuitionistic fuzzy n–Normed Spaces, I_λ–convergence, Orlicz function.

1. INTRODUCTION

Let $\mathbb{N}$ and $\mathbb{R}$, denote the sets of all natural and real numbers, respectively. Let X be a nonempty set, a family I of subsets of X is said to be ideal in X if and only if (i) $\emptyset \in I$, (ii) for each $A, B \in I$ we have $A \cup B \in I$, (iii) for each $A \in I$ and $B \subset A$ we have $B \in I$ and I is called an admissible in X if and only if $I \neq X$ and it contains all singletons. A filter on X is a non–empty family $\mathcal{F}$ of subsets of X satisfying (i) $\emptyset \notin \mathcal{F}$, (ii) for each $A, B \in \mathcal{F}$ we have $A \cap B \in \mathcal{F}$, (iii) for each $A \in \mathcal{F}$ and $B \supset A$ we have $B \in \mathcal{F}$. For each ideal I there is a filter $\mathcal{F}(I)$ corresponding to I, that is,

$\mathcal{F}(I) = \{K \subseteq X : K^c \in I\}$. Depends on the structure of ideals of subsets of $\mathbb{N}$, Kostyrko et al. [11, 12] defined the notion of I–convergence as a generalization of statistical convergence introduced by Fast [3] and Steinhaus [21]. Later, the notion of I–convergence was further investigated and generalized by using different operators, for instance [5, 7, 8, 16, 20, 23]. Let $\lambda = (\lambda_n)$ be a non–decreasing sequence of positive numbers tending to infinity such that $\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 1$. The generalized laVallée–Poussin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in J_n} (x_k). \tag{1.1}$$

Where $J_n = [n - \lambda_n + 1, n]$, (see, [15]). One of the generalization of our interest in this paper is the notion of I_λ–convergence in an Intuitionistic fuzzy n–normed space provided by Konwar et al. [10].

Recall in [13] that an Orlicz function is a function $M : [0, \infty) \longrightarrow [0, \infty)$, which is continuous, convex and non–decreasing with $M(0) = 0, M(x) > 0$ and $M(x) \to \infty$ as $x \to \infty$. If the convexity of an Orlicz function is replaced by $M(x+y) \leq M(x)+M(y)$ then this function is called Modulus function, which was introduced by Nakano [18] and it was further investigated with applications to sequences by Maddox [22], Musielak [17], Tripathy [24]. It is well known if M is a convex function and $M(0) = 0$, then $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 < \lambda < 1$. Lindenstrauss and Tzafriri [14] used the idea of Orlicz function to define the sequence space

$$l_M = \left\{ x = (x_n) \in \omega : \sum_{n=1}^{\infty} M\left(\frac{|x_n|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}$$

which is called an Orlicz sequence space. This space is a Banach space with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{n=1}^{\infty} M\left(\frac{|x_n|}{\rho}\right) \leq 1 \right\}$$

For $M(t) = t^p$ for $1 \leq p < \infty$, the space l_M coincides with the classical sequence space l_p. An Orlicz function M is said to satisfy $\triangle_2$–condition for all values of u if there exists a constant $K > 0$ such that $M(Lu) \leq KLM(u)$ for all values $L > 1$. Subsequently, Orlicz function was used to defined sequence spaces by Parashar and Choudhary [19], Khan et al. [25]. Later on, with the help of the notion of I–convergence, Tripathy and Hazarika [24] introduced some new sequence spaces defined by Orlicz function and further studied by Khan et al. [4, 6, 9] and many others.

In this paper, we define some new intuitionistic fuzzy n–normed I_λ–convergent sequence spaces by using Orlicz function and study some of the topological and algebraic properties on these spaces. Further, we present some inclusion relations concerning this new resulting.

Now, we recall some of the definitions that will be used throughout the paper.

Definition 1.1. [1, 26] Let X be a linear space over $\mathbb{R}$ of dimension $d \geq n$, $*$ is a continuous t–norm, $\diamond$ is a continuous t–conorm. An intuitionisitic fuzzy subset $(\mu, \nu)_n$ of $X^n \times \mathbb{R}$ is called an intuitionisitic fuzzy n–norm on X if and only if

(1) for all $t \in \mathbb{R}$ with $t \leq 0$, $\mu(x_1, x_2, \ldots, x_n, t) = 0$,

(2) for all $t \in \mathbb{R}$ with $t > 0$, $\mu(x_1, x_2, \ldots, x_n, t) = 1$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent,

(3) $\mu(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$,

(4) for all $c \in \mathbb{R}$ with $c \neq 0$, $\mu(f_1, f_2, \ldots, cf_n, t) = \mu(x_1, x_2, \ldots, x_n, t/|c|)$,

(5) for all $s, t \in \mathbb{R}$,

$$\mu(x_1, x_2, \ldots, x_n + x_n^{'}, s + t) \geq \mu(x_1, x_2, \ldots, x_n, s) * \mu(x_1, x_2, \ldots, x_n^{'}, t)\},$$

(6) $\lim_{t \to \infty} \mu(x_1, x_2, \ldots, x_n, t) = 1$,

(7) for all $t \in \mathbb{R}$ with $t \leq 0$, $\nu(x_1, x_2, \ldots, x_n, t) = 1$,

(8) for all $t \in \mathbb{R}$ with $t > 0$, $\nu(x_1, x_2, \ldots, x_n, t) = 0$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent,

(9) $\nu(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$,

(10) for all $c \in \mathbb{R}$ with $c \neq 0$, $\nu(f_1, f_2, \ldots, cf_n, t) = \nu(x_1, x_2, \ldots, x_n, t/|c|)$,

(11) for all $s, t \in \mathbb{R}$,

$$\nu(x_1, x_2, \ldots, x_n + x_n^{'}, s + t) \leq \nu(x_1, x_2, \ldots, x_n, s) \diamond \nu(x_1, x_2, \ldots, x_n^{'}, t)\},$$

(12) $\lim_{t \to \infty} \nu(x_1, x_2, \ldots, x_n, t) = 0$.

Then the five–tuple $(X, \mu, \nu, *, \diamond)$ is called intuitionisitic fuzzy n–normed linear space (for short IFnNS).

Definition 1.2. [26] Let $(X, \mu, \nu, *, \diamond)$ be an IFnNS. A sequence $x = (x_k)$ in X is convergent to $L \in X$ if for each $y_1, y_2, \ldots, y_{n-1} \in X$

$$\lim_{k\to\infty} \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) = 1 \text{ and } \lim_{k\to\infty} \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) = 0 \, \forall \, t > 0.$$

Definition 1.3. [26] Let $(X, \mu, \nu, *, \diamond)$ be an IFnNS. A sequence $x = (x_k)$ in X is said to be Cauchy sequence if for each $y_1, y_2, \ldots, y_{n-1} \in X$

$$\lim_{k\to\infty} \mu(y_1, y_2, \ldots, y_{n-1}, x_{k+p} - x_k, t) = 1 \text{ and } \lim_{k\to\infty} \nu(y_1, y_2, \ldots, y_{n-1}, x_{k+p} - x_k, t) = 0 \, \forall \, t > 0$$

and it is uniformly on $p = 1, 2, 3, \ldots$

Definition 1.4. [2] Let $(X, \mu, \nu, *, \diamond)$ be an IFnNS. For $t > 0$ we define an open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ and $y_1, y_2, \ldots, y_{n-1} \in X$ as

$$B(x, r, t) = \{\mu(y_1, y_2, \ldots, y_{n-1}, y - x, t) > 1 - r \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, y - x, t) < r\}.$$

Definition 1.5. [10] Let $I \subset 2^{\mathbb{N}}$ and let $(X, \mu, \nu, *, \diamond)$ be an IFnNS. A sequence $x = (x_k)$ in X is said to be I_λ–convergent to $L \in X$ with respect to the intuitionistic fuzzy n–norm $(\mu, \nu)_n$ if, for every $\epsilon > 0$, $t > 0$ and $y_1, y_2 \ldots, y_{n-1} \in X$, we have

$$\Bigg\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \leq 1 - \epsilon$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \geq \epsilon \Bigg\} \in I.$$

In this case, L is called the I_λ–limit of the sequence (x_k) and we write $I_\lambda^{(\mu,\nu)_n}$–$\lim x = L$.

2. MAIN RESULTS

In this section, we define some new intuitionistic fuzzy n–normed I_λ–convergent sequence spaces by using Orlicz function M and study some topological and algebraic properties of these spaces. Further, we present some inclusion relations concerning this new resulting.

$$c_{(\mu,\nu)_n}^{I_\lambda}(M) = \Bigg\{ x = (x_k) \in X : \Big\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t)}{\rho}\right) \leq 1 - \epsilon$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t)}{\rho}\right) \geq \epsilon \Big\} \in I \Bigg\}. \tag{2.1}$$

$$c^{I_\lambda}_{0(\mu,\nu)_n}(M) = \Big\{x = (x_k) \in X : \{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k, t)}{\rho}\right) \leq 1-\epsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k, t)}{\rho}\right) \geq \epsilon\} \in I\Big\}. \tag{2.2}$$

We define an open ball with center $x \in X$ and radius r with respect to t as follows:

$$B(x, r, t)(M) = \Big\{(z_k) \in X : \{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - z_k, t)}{\rho}\right) > 1-r \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - z_k, t)}{\rho}\right) < r\}\Big\}. \tag{2.3}$$

Theorem 2.1. The spaces $c^{I_\lambda}_{(\mu,\nu)_n}(M)$ and $c^{I_\lambda}_{0(\mu,\nu)_n}(M)$ are linear spaces over $\mathbb{R}$.

Proof. We will prove the result for $c^{I_\lambda}_{(\mu,\nu)_n}(M)$. The proof for the other space will follow similarly. Let $x = (x_k), z = (z_k) \in c^{I_\lambda}_{(\mu,\nu)_n}(M)$ and α, β are scalars. Chose $\epsilon \in (0, 1)$ and ρ_i for $i = 1, 2, 3, \ldots$ such that for any Orlicz function M and a non–decreasing sequence of positive numbers $\lambda = (\lambda_n)$, we have $\frac{1}{\lambda_n}\sum_{k\in J_n} M(\frac{1-\epsilon}{\rho_3}) > 1-\epsilon$ and $\frac{1}{\lambda_n}\sum_{k\in J_n} M(\frac{\epsilon}{\rho_3}) < \epsilon$. Therefore,

$$\Big\{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2})}{\rho_1}\right) \leq 1-\epsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2})}{\rho_1}\right) \geq \epsilon\Big\} \in I$$

and

$$\Big\{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, z_k - L_2, \frac{t}{2})}{\rho_2}\right) \leq 1-\epsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, z_k - L_2, \frac{t}{2})}{\rho_2}\right) \geq \epsilon\Big\} \in I.$$

Let $\rho_3 = \max\{\rho_1, \rho_2\}$, for some $L_1, L_2 \in \mathbb{R}$. Let

$$A_1 = \Big\{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2|\alpha|})}{\rho_1}\right) > 1-\epsilon \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2|\alpha|})}{\rho_1}\right) < \epsilon\Big\} \in \mathcal{F}(I)$$

and

$$A_2 = \Big\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, z_k - L_2, \frac{t}{2|\beta|})}{\rho_2} \right) > 1 - \epsilon$$
$$\text{and } \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, z_k - L_2, \frac{t}{2|\beta|})}{\rho_2} \right) < \epsilon \Big\} \in \mathcal{F}(I).$$

Since M is non–decreasing and convex function, we have

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k + \beta z_k - (\alpha L_1 + \beta L_2), t)}{\rho_3} \right)$$
$$\geq \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k - \alpha L_1, \frac{t}{2}) * \mu(y_1, y_2, \ldots, y_{n-1}, \beta z_k - \beta L_2, \frac{t}{2})}{\rho_3} \right)$$
$$= \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2|\alpha|}) * \mu(y_1, y_2, \ldots, y_{n-1}, z_k - \beta L_2, \frac{t}{2|\beta|})}{\rho_3} \right)$$
$$\geq \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{(1-\epsilon) * (1-\epsilon)}{\rho_3} \right) > \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{1-\epsilon}{\rho_3} \right) > 1 - \epsilon.$$

and

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k + \beta z_k - (\alpha L_1 + \beta L_2), t)}{\rho_3} \right)$$
$$\leq \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k - \alpha L_1, \frac{t}{2}) * \nu(y_1, y_2, \ldots, y_{n-1}, \beta z_k - \beta L_2, \frac{t}{2})}{\rho_3} \right)$$
$$= \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2|\alpha|}) * \nu(y_1, y_2, \ldots, y_{n-1}, z_k - \beta L_2, \frac{t}{2|\beta|})}{\rho_3} \right)$$
$$\leq \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\epsilon \diamond \epsilon}{\rho_3} \right) < \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\epsilon}{\rho_3} \right) < \epsilon.$$

Therefore,

$$\Big\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k + \beta z_k - (\alpha L_1 + \beta L_2), t)}{\rho_3} \right) \leq 1 - \epsilon$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M\left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k + \beta(z_k) - (\alpha L_1 + \beta L_2), t)}{\rho_3} \right) \geq \epsilon \Big\} \supseteq A \cap A_2.$$

Therefore, $(\alpha x + \beta z) \in c^{I_\lambda}_{(\mu,\nu)_n}$. Hence, $c^{I_\lambda}_{(\mu,\nu)_n}(M)$ is a linear space. ■

Theorem 2.2. Every open ball $B(x,r,t)(M)$ is an open set in $c^{I_\lambda}_{(\mu,\nu)_n}(M)$.

Proof. Let $B(x,r,t)(M)$ be an open ball with center x and radius r with respect to t, that is

$$B(x,r,t)(M)=\Big\{z=(z_k)\in X:\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t)}{\rho}\right)>1-r \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t)}{\rho}\right)<r\}\Big\}.$$

Let $z\in B(x,r,t)(M)$. Then

$$\frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t)}{\rho}\right)>1-r$$
$$\text{and } \frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t)}{\rho}\right)<r.$$

Then, there exists $t_0\in(0,t)$ such that

$$\frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t_0)}{\rho}\right)>1-r$$
$$\text{and } \frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t_0)}{\rho}\right)<r.$$

Putting $r_0=\frac{1}{\lambda_n}\sum_{k\in J_n}M\Big(\frac{\mu(y_1,y_2,\dots,y_{n-1},(x_k)-(z_k),t_0)}{\rho}\Big)$, we have $r_0>1-r$, there exists
$s\in(0,1)$ such that $r_0>1-s>1-r$. For $r_0>1-s$, we have $r_1,r_2\in(0,1)$ such that $r_0*r_1>1-s$ and $(1-r_0)\diamond(1-r_2)\le s$. Putting $r_3=\max\{r_1,r_2\}$. Consider the ball $B(z,1-r_3,t-t_0)(M)$. We prove that $B(z,1-r_3,t-t_0)(M)\subset B(x,r,t)(M)$. Let $q=(q_k)\in B(z,1-r_3,t-t_0)(M)$, then

$$\frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},z_k-q_k,t-t_0)}{\rho}\right)>r_3$$

and

$$\frac{1}{\lambda_n}\sum_{k\in J_n}M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},z_k-q_k,t-t_0)}{\rho}\right)<1-r_3.$$

Therefore,

$$\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x_k-q_k,t)}{\rho}\right)$$
$$\geq \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t_0)*\mu(y_1,y_2,\dots,y_{n-1},z_k-q_k,t-t_0)}{\rho}\right)$$
$$\geq r_0*r_3\geq r_0*r_3>1-s>1-r$$

and

$$\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x_k-q_k,t)}{\rho}\right)$$
$$\leq \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x_k-z_k,t_0)\diamond\nu(y_1,y_2,\dots,y_{n-1},z_k-q_k,t-t_0)}{\rho}\right)$$
$$\leq (1-r_0)\diamond(1-r_3)\leq(1-r_0)\diamond(1-r_3)\leq s<r.$$

Thus, $q\in B(x,r,t)(M)$ and hence $B(z,1-r_3,t-t_0)(M)\subset B(x,r,t)(M)$. ■

Definition 2.1. Let $(X,\mu,\nu,*,\diamond)$ be an IfnNS. Define

$$\tau^{I_\lambda}_{(\mu,\nu)_n}(M)=\{A\subset c^{I_\lambda}_{(\mu,\nu)_n}(M):\ \text{for each } x\in A \text{ there exists } t>0$$
$$r\in(0,1) \text{ such that } B(x,r,t)(M)\subset A\}.$$

Then, $\tau^{I_\lambda}_{(\mu,\nu)_n}(M)$ is a topology on $c^{I_\lambda}_{(\mu,\nu)_n}(M)$.

Theorem 2.3. The sequence spaces $c^{I_\lambda}_{(\mu,\nu)_n}(M)$ and $c^{I_\lambda}_{0(\mu,\nu)_n}(M)$ are Hausdorff spaces.

Proof. We prove the result for $c^{I_\lambda}_{(\mu,\nu)_n}(M)$. The result for $c^{I_\lambda}_{0(\mu,\nu)_n}(M)$ can be established Similarly. Let $x,z\in c^{I_\lambda}_{(\mu,\nu)_n}(M)$ such that $x\neq z$. Then

$$0<\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x-z,t)}{\rho}\right)<1$$
$$\text{and } 0<\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x-z,t)}{\rho}\right)<1.$$

Putting $r_1=\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\dots,y_{n-1},x-z,t)}{\rho}\right)$ and $r_2=\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\dots,y_{n-1},x-z,t)}{\rho}\right)$ and $r=\max\{r_1,1-r_2\}$. For each $r_0\in$

$(r, 1)$, there exist r_3 and r_4 such that $r_3 * r_3 \geq r_0$ and $(1 - r_4) \diamond (1 - r_4) \leq (1 - r_0)$. Putting $r_5 = \max\{r_3, 1 - r_4\}$ and consider the open balls $B(x, 1 - r_5, \frac{t}{2})$ and $B(z, 1 - r_5, \frac{t}{2})$. Then clearly $B(x, 1 - r_5, \frac{t}{2}) \cap B(z, 1 - r_5, \frac{t}{2}) = \emptyset$. For if there exists $q \in B(x, 1 - r_5, \frac{t}{2}) \cap B(z, 1 - r_5, \frac{t}{2})$, we have

$$\begin{aligned} r_1 =& \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x - z, t)}{\rho} \right) \\ \geq& \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x - q, \frac{t}{2}) * \mu(y_1, y_2, \ldots, y_{n-1}, q - z, \frac{t}{2})}{\rho} \right) \\ \geq& r_5 * r_5 \geq r_3 * r_3 \geq r_0 > r_1 \end{aligned}$$

and

$$\begin{aligned} r_2 =& \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x - z, t)}{\rho} \right) \\ \leq& \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x - q, \frac{t}{2}) \diamond \nu(y_1, y_2, \ldots, y_{n-1}, q - z, \frac{t}{2})}{\rho} \right) \\ \leq& (1 - r_5) \diamond (1 - r_5) \leq (1 - r_4) \diamond (1 - r_4) \leq (1 - r_0) < r_2 \end{aligned}$$

which is a contradiction. Hence, $c^{I_\lambda}_{(\mu,\nu)_n}(M)$ is Hausdorff space. ■

Theorem 2.4. Let $c^{I_\lambda}_{(\mu,\nu)_n}(M)$ be an IFnNS and $\tau^{I_\lambda}_{(\mu,\nu)_n}(M)$ is a topology on $c^{I_\lambda}_{(\mu,\nu)_n}(M)$. Then a sequence $(x_k) \in c^{I_\lambda}_{(\mu,\nu)_n}(M)$, $x_k \to x$ if and only if

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M \Big(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x, t)}{\rho} \Big) \to 1$$
$$\text{and } \frac{1}{\lambda_n} \sum_{k \in J_n} M \Big(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x, t)}{\rho} \Big) \to 0 \text{ as } k \to \infty.$$

Proof. Fix $t > 0$. Suppose $x_k \to x$. Then for $r \in (0, 1)$, there exists $n_0 \in \mathbb{N}$ such that $(x_k) \in B(x, r, t)(M)$ for all $k \geq n_0$. Therefore,

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x, t)}{\rho} \right) < r$$
$$\text{and } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x, t)}{\rho} \right) < r,$$

Hence,

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x, t)}{\rho} \right) \to 1$$
$$\text{and } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x, t)}{\rho} \right) \to 0$$

as $k \to \infty$. Conversely, if for each $t > 0$,

$$\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_k-x,t)}{\rho}\right) \to 1$$
$$\text{and } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1}x_k-x,t)}{\rho}\right) \to 0 \text{ as } k \to \infty,$$

then for $r \in (0,1)$, there exists $n_0 \in \mathbb{N}$ such that

$$1-\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_k-x,t)}{\rho}\right) < r$$
$$\text{and } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_k-x,t)}{\rho}\right) < r$$

for all $k \geq n_0$. It follows that

$$\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_k-x,t)}{\rho}\right) > 1-r$$
$$\text{and } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_k-x,t)}{\rho}\right) < r$$

for all $k \geq n_0$. Thus $(x_k) \in B(x,r,t)(M)$ for all $k \geq n_0$ and hence $x_k \to x$. ∎

Theorem 2.5. A sequence $x = (x_k) \in c^{I_\lambda}_{(\mu,\nu)_n}(M)$ is I–convergent if and only if for every $\epsilon > 0$ and $t > 0$ there exists a number $N = N(x,\epsilon,t)$ such that

$$\Big\{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_k-x_N,\frac{t}{2})}{\rho}\right) > 1-\epsilon$$
$$\text{and } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_k-x_N,\frac{t}{2})}{\rho}\right) < \epsilon\Big\} \in \mathcal{F}(I).$$

Proof. Suppose that $I^{(\mu,\nu)_n}_\lambda - \lim x = L$ and let $\epsilon > 0, t > 0$. For a given $\epsilon > 0$, choose $s > 0$ such that $(1-\epsilon)*(1-\epsilon) > 1-s$ and $\epsilon \diamond \epsilon < s$. Then for each $x \in c^{I_\lambda}_{(\mu,\nu)_n}(M)$,

$$A = \Big\{n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_k-x_N,\frac{t}{2})}{\rho}\right) \leq 1-\epsilon$$
$$\text{or } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_k-x_N,\frac{t}{2})}{\rho}\right) \geq \epsilon\Big\} \in I.$$

which implies that

$$A^c = \Big\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, \frac{t}{2})}{\rho} \right) > 1 - \epsilon$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, \frac{t}{2})}{\rho} \right) < \epsilon \Big\} \in \mathcal{F}(I).$$

Conversely let us choose $N \in A$. Then,

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, \frac{t}{2})}{\rho} \right) \leq 1 - \epsilon$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, \frac{t}{2})}{\rho} \right) \geq \epsilon.$$

Now, we show that there exists a number $N = N(x, \epsilon, t)$ such that

$$\Big\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t)}{\rho} \right) \leq 1 - s$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t)}{\rho} \right) \geq s \Big\} \in I.$$

For this, define for each $x \in c^{I_\lambda}_{(\mu,\nu)n}(M)$

$$B = \Big\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t)}{\rho} \right) \leq 1 - s$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t)}{\rho} \right) \geq s \Big\} \in I.$$

Now, we have to show that $B \subset A$. Suppose that $B \subsetneq A$. Then there exists $n \in B$ and $n \notin A$. Therefore, we have

$$\frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\mu(y_1, y_2, \ldots, y_{n-1}, x_n - x_N, t)}{\rho} \right) \leq 1 - s$$
$$\text{or } \frac{1}{\lambda_n} \sum_{k \in J_n} M \left(\frac{\nu(y_1, y_2, \ldots, y_{n-1}, x_n - x_N, t)}{\rho} \right) \geq s.$$

In particular $\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_k-L,\frac{t}{2})}{\rho}\right) > 1-s$. Therefore, we have

$$\begin{aligned}
1-s &\geq \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_n-x_N,t)}{\rho}\right)\\
&\geq \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\mu(y_1,y_2,\ldots,y_{n-1},x_n-L,\frac{t}{2}) * \mu(y_1,y_2,\ldots,y_{n-1},x_N-L,\frac{t}{2})}{\rho}\right)\\
&\geq (1-\epsilon)*(1-\epsilon) > 1-s.
\end{aligned}$$

which is not possible. On the other hand

$$\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_n-x_N,t)}{\rho}\right) \geq s$$
$$\text{or } \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_n-x_N,\frac{t}{2})}{\rho}\right) < s$$

In particular $\frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_N-L,\frac{t}{2})}{\rho}\right) < s$. Therefore, we have

$$\begin{aligned}
s &\leq \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_n-x_N,t)}{\rho}\right)\\
&\leq \frac{1}{\lambda_n}\sum_{k\in J_n} M\left(\frac{\nu(y_1,y_2,\ldots,y_{n-1},x_n-L,\frac{t}{2}) \diamond \nu(y_1,y_2,\ldots,y_{n-1},x_N-L,\frac{t}{2})}{\rho}\right)\\
&\leq \epsilon \diamond \epsilon < s,
\end{aligned}$$

which is not possible. Hence, $B\subset A$. $A\in I$ implies $B\in I$. ■

3. CONCLUSION

In this paper, we introduced some new intuitionistic fuzzy n–normed sequence spaces by using the idea of Orlicz function and the notion of I_λ–convergence, that is, $c^{I_\lambda}_{(\mu,\nu)_n}(M)$ and $c^{I_\lambda}_{0(\mu,\nu)_n}(M)$ and studied some topological and algebraic properties of these spaces. These definitions and results in this paper provide new tools to deal with the convergence problems of sequences occurring in many branches of science and engineering.

4. AUTHORS CONTRIBUTIONS

All authors of the manuscript have read and agreed to its content and are accountable for all aspects of the accuracy and integrity of the manuscript.

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6. COMPETING INTERESTS

The authors declare that they have no competing interests.

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