

Simulation of Incompressible Cylindrical Duct Flow with Electrically Conducting Fluid Using Finite Difference Method

K. B. Patel

*Department of Mathematics,
Veer Narmad South Gujarat University, Surat Gujarat, India.
E-mail: patel_kaushal1@yahoo.com*

Abstract

In this paper we deal with the steady state flow of incompressible and electrically conducting fluid through a cylindrical channel. In this we consider the effect of magnetic field or Hartmann number on velocity profile. An external uniform magnetic field is applied to the fluid which is directed in a co-circular cylinder perpendicular to the flow direction. The governing partial differential equations of cylindrical co-ordinate system are transform in Cartesian system and then solve numerically using finite difference method. And finally we discuss the effects of magnetic and Hartmann parameter on the velocity.

Keywords: Cylindrical duct, Electro conducting fluid, Hartmann number, Navier - Stokes equation, Energy equations.

1. INTRODUCTION

The increasing number of technical utilization of magmatic fluid in industries, since the last century due to its substantial applications. For example, magneto-hydro dynamics (MHD) steam plants and MHD generators are used in the modern power plants. The basic concept of the MHD generator is to generate electrical energy from the motion of conductive fluid that is crossing a perpendicular magnetic field. Carnot efficiency is improved by the presence of MHD unit. Another example is the MHD pumps and flow meters. In this type of pumps, the electrical energy is converted

directly to a force which is applied on the working fluid. MHD separation in metal casting with superconducting coils is another important application.

A very useful proposed application which involves electromagnetic fluid is the lithium cooling blanket in a nuclear fusion reactor. The high-temperature plasma is maintained in the reactor by means of magnetic field. The liquid-lithium circulation loops, which will be located between the plasma and magnetic windings, are called lithium blankets. The lithium performs two functions: it absorbs the thermal energy released by the reaction and it participates in nuclear reactions in which tritium is produced. The lithium blanket is thus a very important reactor component. On other hand, the blanket will be acted upon by an extremely strong magnetic field. Consequently, to calculate the flow of liquid metal in channels or pipes situated at different angles to the magnetic field, and to determine the required pressure drop, heat transfer, etc., knowledge of the appropriate MHD relationships will be necessary.

Williams published results of experiments with electrolytes flowing in insulated tubes. The tubes were placed between the poles of a magnet, and the potential difference across the flow was measured using wires passed through the walls. Hartmann and Lazarus made some very comprehensive theoretical and experimental studies of this subject. They performed their experiments with mercury which has an electrical conductivity 100,000 times greater than that of an electrolyte. This made it possible to observe a wider range of phenomena than in the experiments by Williams. They examine the change in drag and the suppression of turbulence caused by magnetic field. Hartmann obtained the exact solution of the flow between two parallel, non-conducting walls with the applied magnetic field normal to the walls. Shercliff [1, 2], in 1956, has solved the problem of rectangular duct, from which he noticed that for high Hartmann numbers M the velocity distribution consists of a uniform core with a boundary layer near the walls. In 1962, Gold and Lykoudis [3] obtained an analytical solution for the MFM flow in a circular tube with zero wall conductivity while in 1968, Gardner and Lykoudis [4, 5, 6, and 7] have acquired experimentally some results for circular tube with and without heat transfer. The MFM flow is also examined numerically by Al-Khawaja et al. for the case of circular tube with heat transfer and for the case of uniform wall heat flux with and without free convection. The solution for MFM square duct flow is obtained using spectral method by Al-Khawaja and Selmi [8] for the case of uniform wall temperature. This result enabled to solve the problem for a circular pipe in an approximate manner for large M assuming walls of zero conductivity and, subsequently, walls with small conductivity.

2. PROBLEM STATEMENT

The forced convection in a horizontal co-circular pipe of radius 'a' and 'b' in a uniform transverse magnetic field B_0 as shown in figure 1. A homogeneous, incompressible, viscous, electrically-conducting fluid flows through a horizontal cylindrical pipe and is subjected to a uniform surface temperature and a uniform surface heat flux. In conjunction with defining this problem, the following assumptions are made:

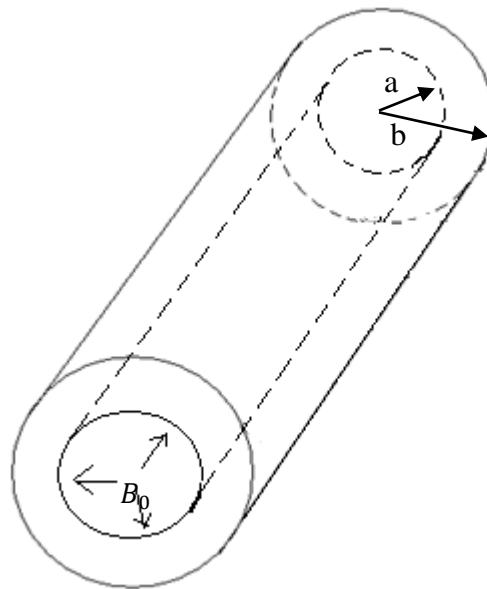


Figure 1: MFM circular duct flow

- a) Assume to be the fluid is incompressible and change of temperature of fluid does not affect the properties of fluid.
- b) The pipe is sufficiently long that it can be assumed the flow and heat transfer are fully developed and entrance or exit effects can be neglected.
- c) Only pressure and temperature vary linearly with axial direction.
- c) The contributions of viscous and Joulean dissipation in the energy equation are small and can be neglected. This assumption has been shown to be applicable to a similar problem when no external electric field is imposed on the flow.
- d) The induced magnetic field produced as a result of interaction of applied field, B_0 , with either main or secondary flow, will be assumed negligibly small compared to B_0 .

This assumption follows from the fact that the magnetic Reynolds number based on the flow is much smaller than unity under conditions found in typical applications.

The motion of an electrically conducting fluid in the presence of a magnetic field obeys the well-known equations of magnetohydrodynamics. The fluid is treated as a continuum, and the classical results of fluid dynamics and electro dynamics are combined to express the phenomenon. For the steady flow of a viscous, incompressible fluid with constant properties, the full magnetohydrodynamic system can be reduced to two equations involving the velocity, pressure, and magnetic field, i.e. the modified Navier-Stokes equation and the induction equation, along with the solenoidal conditions on the two vector quantities:

Here we consider an incompressible, Newtonian flow with a velocity field $\vec{V} = (u_r, u_\theta, u_z)$ and magnetic field $\vec{H} = (h_r, h_\theta, h_z)0$. For incompressible Newtonian liquid metal fluid and steady-state conditions, the modified Navier-Stokes equations under the effect of magnetic field body force including induction and energy equations in cylindrical coordinate system of vector forms are, respectively,

$$\frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{1}{r} \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0 \quad (1)$$

r-component:

$$\rho \left(u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) = - \frac{\partial \left(p + \frac{\mu |H|^2}{2} \right)}{\partial r} + \mu \left(h_r \frac{\partial h_r}{\partial r} + \frac{h_\theta}{r} \frac{\partial h_r}{\partial \theta} - \frac{h_\theta^2}{r} + h_z \frac{\partial h_r}{\partial z} \right) + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \quad (2)$$

θ –Component:

$$\rho \left(u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_\theta}{\partial z} \right) = - \frac{\partial \left(p + \frac{\mu |H|^2}{2} \right)}{\partial \theta} + \mu \left(h_r \frac{\partial h_\theta}{\partial r} + \frac{h_\theta}{r} \frac{\partial h_\theta}{\partial \theta} - \frac{h_r h_\theta}{r} + h_z \frac{\partial h_\theta}{\partial z} \right) + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \quad (3)$$

z - Component:

$$\rho \left(u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) = - \frac{\partial \left(p + \frac{\mu |H|^2}{2} \right)}{\partial z} + \mu \left(h_r \frac{\partial h_z}{\partial r} + \frac{h_\theta}{r} \frac{\partial h_z}{\partial \theta} - \frac{h_r h_z}{r} + h_z \frac{\partial h_z}{\partial z} \right) + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \quad (4)$$

The energy equation in cylindrical coordinates allows for non-constant physical properties, energy generation, and conversion of mechanical to internal energy using viscous dissipation, which is expressed in terms of unspecified viscous stress-tensor components τ_{ij} :

$$\begin{aligned} \rho c_v u_r \frac{\partial H}{\partial r} + \rho c_v \frac{u_\theta}{r} \frac{\partial H}{\partial \theta} + \rho c_v u_z \frac{\partial H}{\partial z} &= \frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial H}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(k \frac{\partial H}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial H}{\partial z} \right) - \\ H \frac{\partial P}{\partial H} \Big|_\rho \left[\frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \right] &- \tau_{rr} \frac{\partial u_r}{\partial r} - \tau_{\theta\theta} \frac{1}{r} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right) - \tau_{zz} \frac{\partial u_z}{\partial z} - \\ \tau_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] &- \tau_{rz} \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) - \tau_{\theta z} \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) + \mu_f \Phi + \frac{|J|^2}{\sigma} \end{aligned} \quad (5)$$

$$\begin{aligned} \rho c_p u_r \frac{\partial T}{\partial r} + \rho c_p \frac{u_\theta}{r} \frac{\partial T}{\partial \theta} + \rho c_p u_z \frac{\partial T}{\partial z} \\ = \frac{k}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{k}{r^2} \left(\frac{\partial^2 T}{\partial \theta^2} \right) + K \left(\frac{\partial^2 T}{\partial z^2} \right) + 2\mu \left(\frac{\partial u_r}{\partial r} \right)^2 + \frac{2\mu}{r^2} \left(\frac{\partial u_\theta}{\partial \theta} + u_r \right)^2 + 2\mu \left(\frac{\partial u_z}{\partial z} \right)^2 + \\ \mu \left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right)^2 + \mu \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right)^2 + \mu \left[\frac{1}{r} \frac{\partial u_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) \right]^2 + \mu_f \Phi + \frac{|J|^2}{\sigma} \end{aligned} \quad (6)$$

Where C_v and C_p is the heat capacity at constant volume and pressure respectively, k the thermal conductivity, and The last two terms in the right hand of the energy equation, Equation 5, represent the viscous and Joulean dissipations, respectively. It can be applied to non-Newtonian fluids if the appropriate constitutive equation relating the viscous stress to the rate of strain is known.

In addition to solenoidal conditions on the two vectors

$$\frac{1}{r} \frac{\partial (r u_r)}{\partial r} + \frac{1}{r} \frac{\partial (u_\theta)}{\partial \theta} + \frac{\partial (u_z)}{\partial z} = 0 \quad \text{And} \quad \frac{1}{r} \frac{\partial (r h_r)}{\partial r} + \frac{1}{r} \frac{\partial (h_\theta)}{\partial \theta} + \frac{\partial (h_z)}{\partial z} = 0$$

Now we transform cylindrical systems to Cartesian systems. We convert the cylindrical system to Cartesian System by taking the transformation

$$x(r, \theta) = r \cos \theta$$

$$y(r, \theta) = r \sin \theta$$

By the simplification system of equations is

$$\rho(V \cdot \nabla)V + \nabla \left(p + \mu \frac{|H|^2}{2} \right) = \mu_f \nabla^2 V + \mu(H \cdot \nabla)H \quad (7)$$

$$\nabla^2 H + \mu\sigma[(H \cdot \nabla)V - (V \cdot \nabla)H] = 0 \quad (8)$$

$$\rho c(V \cdot \nabla)T = k\nabla^2 T + \mu_f \Phi + \frac{|J|^2}{\sigma} \quad (9)$$

And solenoidal conditions on the two vectors

$$\nabla \cdot V = 0 \quad \text{And} \quad \nabla \cdot H = 0 \quad (10)$$

For very small magnetic Reynolds number R_M (i.e the induced magnetic field produced as a result of interaction of applied field, B_0 , will be assumed negligibly small compared to B_0), The induced equation, equation 8, can be derived from Maxwell's equation along with the two solenoidal conditions, equation 10. The last two terms in the right hand of the energy equation, Equation 9, represent the viscous and Joule dissipations, respectively. That term can be neglected compared to the other ones in the equation.

After simplifications by assuming fully developed flow, that is two dimensional flows given in Figure 2, and since the flow is laminar due to the variations of turbulence in the presence of magnetic field, the dimensionless governing equations for the flow become

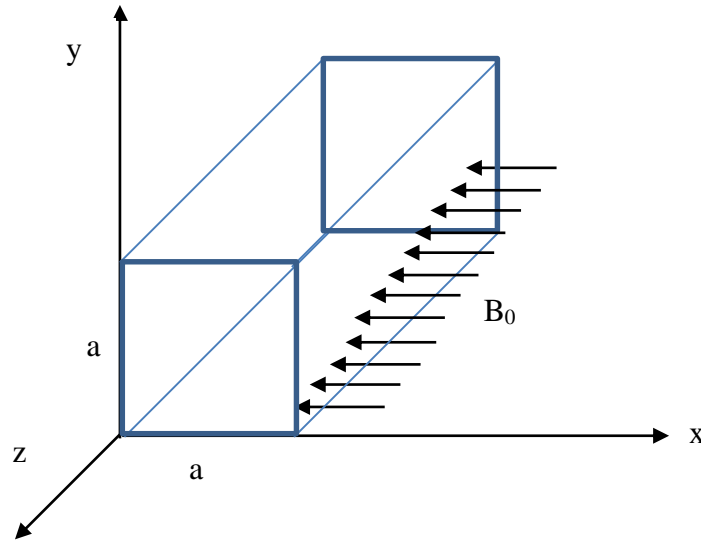


Fig.2: MFM square duct flow

$$\nabla^2 V - M \frac{\partial H}{\partial x^*} = 1 \quad (11)$$

$$\nabla^2 H - M \frac{\partial V}{\partial x^*} = 1 \quad (12)$$

$$\nabla^2 T + 4Nu VT = 0 \quad (13)$$

And

$$\nabla^2 T - 4V = 0 \quad (14)$$

Where the negative dimensionless pressure gradient γ is related to V by

$$\gamma = \frac{1}{\int_0^1 \int_0^1 v dx^* dy^*} \quad (15)$$

From the force and energy balances one can show, respectively, that $fRe = -2\gamma$ and $Nu = -1/T_m$. Where the mean dimensionless temperature is given by

$$T_m = \frac{\int_0^1 \int_0^1 TV dx^* dy^*}{\int_0^1 \int_0^1 v dx^* dy^*} \quad (16)$$

The boundary conditions are $w^* = 0$ (from no-slip condition), $H^* = 0$ (from electrically insulated surface), and $T = 0$ (for isothermal surface and constant surface heat flux).

3. NUMERICAL DIFFERENTIATIONS:

In this paper, the MFM problem for two heat transfer limits; constant temperature and constant heat flux boundary conditions, is investigated numerically for square duct (Figure 2). The modified dimensionless Navier-Stokes equations with uniform-temperature-condition having energy equation (Equation 7), and uniform-heat-flux-condition having energy equation (Equation 8) are transferred into finite-difference equations (using the central-difference scheme).

$$v_{i-1,j}^* + v_{i+1,j}^* + v_{i,j-1}^* + v_{i,j+1}^* - 4v_{i,j}^* - \frac{1}{2}M\Delta x^*(H_{i+1,j}^* - H_{i-1,j}^*) = (\Delta x^*)^2 \quad (17)$$

$$H_{i-1,j}^* + H_{i+1,j}^* + H_{i,j-1}^* + H_{i,j+1}^* - 4H_{i,j}^* - \frac{1}{2}M\Delta x^*(v_{i+1,j}^* - v_{i-1,j}^*) = 0 \quad (18)$$

$$T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j} + 4(\Delta x^*)^2 Nu v_{i,j} T_{i,j} = 0 \quad (19)$$

$$\text{And } T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j} - 4(\Delta x^*)^2 v_{i,j} = 0 \quad (20)$$

With the following definitions of dimensionless pressure gradient and mean dimensionless temperature given, respectively, as

$$\gamma = \frac{1}{\sum_{i=0}^I \sum_{j=0}^J v_{i,j}^* (\Delta x^*)^2} \quad (21)$$

$$\text{And } T_m = \frac{\sum_{i=0}^I \sum_{j=0}^J T_{i,j} v_{i,j} (\Delta x^*)^2}{\sum_{i=0}^I \sum_{j=0}^J v_{i,j} (\Delta x^*)^2} \quad (22)$$

4. RESULTS:

Some noticeable heat transfer results are obtained for the MFM circular duct flow with uniform temperature and heat flux boundary conditions. The flow (velocity and pressure) was studied so extensively in reference for the same flow conditions. For more details, we should refer to reference to notice, in the provided figures, the flattening of the axial velocity due to the presence of the magnetic field and the increase of the friction factor with the field. The negative dimensionless temperature distributions at the mid-plane either along or normal to the magnetic field always decrease as the Hartmann number increases for both boundary condition limits. This is because the temperature distributions are more homogenous as the magnetic field is turned on. This can be seen from the results presented in and is due to the fact that velocity profile becomes more flattened as M increases, particularly along the direction of the magnetic field. Also we notice that the temperature distributions along and normal to the field are almost identical for any Hartmann number. The uniformity of the temperature across the duct is greater for the former case but it is constant during the applied magnetics process.

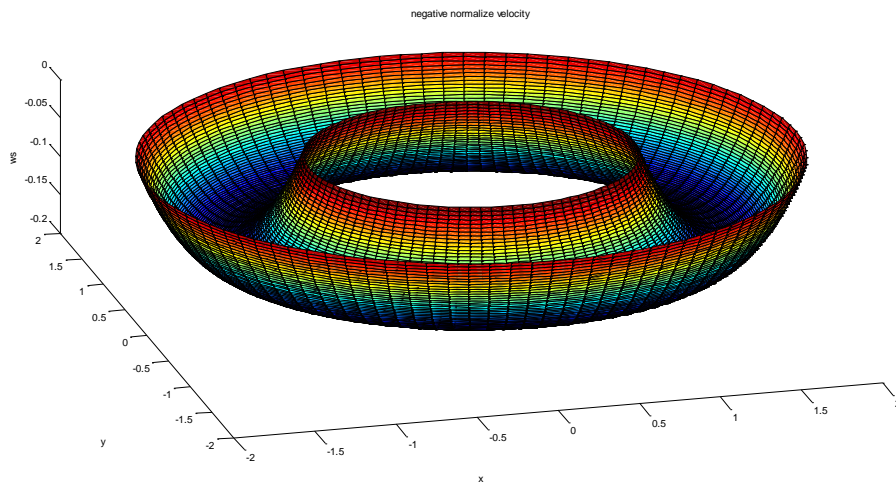


Fig.3: Negative normalized axial velocity

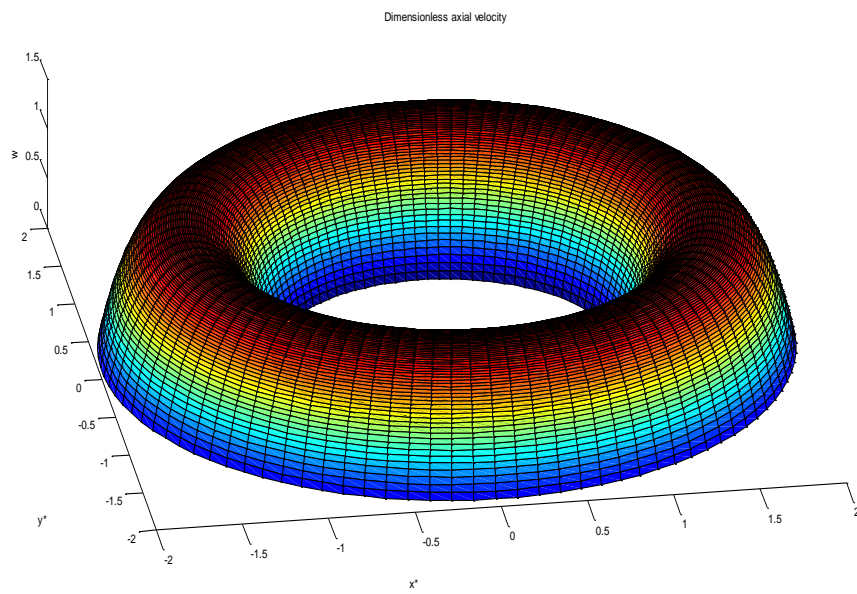


Fig.4: Dimensionless axial velocity

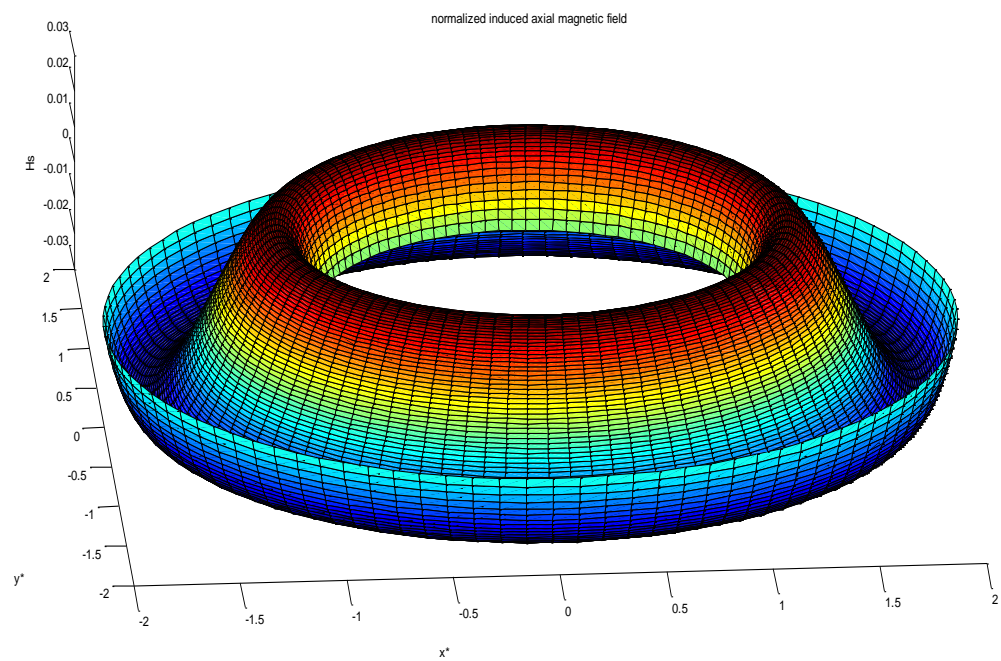


Fig.5: Normalized induced axial magnetic field

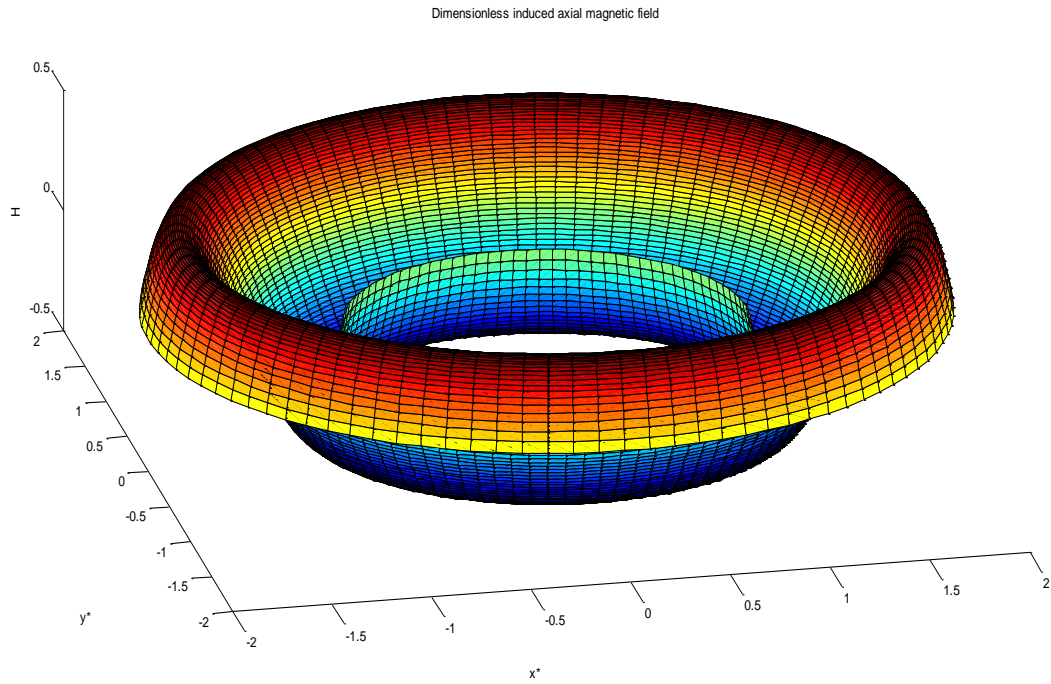


Fig.6: Dimensional less induced axial magnetic field

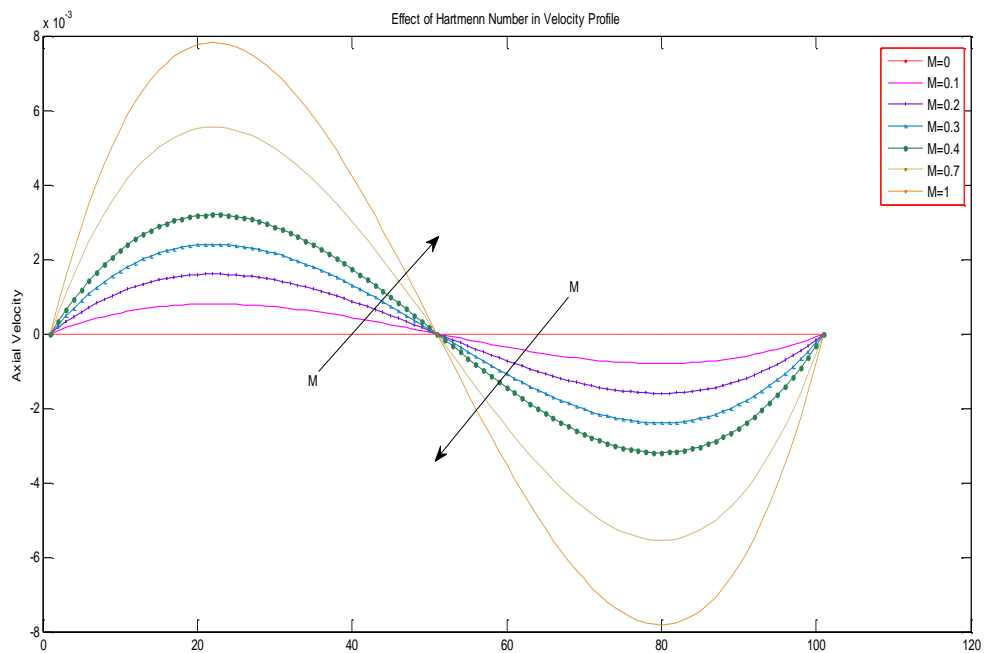


Fig 7: Effect of Hartmann Number on velocity profile

5. CONCLUSION:

The Study of co – Circular Cylindrical duct flow with electrically conducting fluid and with two heat transfer limits has been studied. The problem is analyzed numerically when a uniform transverse magnetic field is applied to the duct. The assumption of laminar flow is mostly valid in MFM flows since the turbulences will be damped out due the opposing force induced in the flow. The fluid mechanic part of this problem was considered extensively and the results were shown using the spectral method. Also, the heat transfer results for only uniform temperature boundary condition were shown. In the present work, we consider two heat transfer limits (uniform heat flux and temperature boundary conditions) numerically using iterative Gauss-Seidel method, and the software package MatLab is utilized to achieve this approach. The results obtained for the case of constant temperature condition agree very well with reference.

REFERENCE

- [1] Shercliff, J. A., The Flow of Conducting Fluids in Circular Pipes under Transverse Magnetic Field. *J. Fluid Mech.*, Vol. 1, 1956, p. 644.
- [2] Shercilff, J. A., Magnetohydrodynamic Pipe Flow, part 2 High Hartmann Number. *Journal of Fluid Mech.*, Vol. 13, 1962, p. 513.
- [3] Gold, R., Magnetohydrodynamic Pipe Flow. *Journal of Fluid Mech.*, Part 1, Vol. 13, 1962, p. 505.
- [4] Gardner, R. A., Laminar Pipe Flow in a Transverse Magnetic Field with Heat Transfer. *International Journal of Heat Mass Transfer*", Vol. 11, 1968, pp. 1076-1081.
- [5] Gardner, R. A. and Lykoudis, P. S., Magneto-Fluid-Mechanic Pipe Flow in a Transverse Magnetic Field Part Two. Heat Transfer. *Journal of Fluid Mech.*, Vol. 48, Part 1, 1971, pp. 129-141.
- [6] Al-Khawaja M. J., Gardner R. A., Agarwal R., Numerical Study of magneto-fluidmechanics Forced convection Pipe Flow. *Engineering Journal of Qatar University*, Vol.7, 1994, pp. 115-134.
- [7] Al-Khawaja M. J., Agarwal R. K., Gardner R. A., Numerical study of magneto – fluid mechanic combined free-and-forced convection Heat Transfer. *Int. J. Heat Mass Transfer*, Vol. 42, 1999, pp. 467-475.
- [8] Al-Khawaja M. J. and Selmi M., Highly Accurate Solution of a Laminar Square Duct Flow in a Transverse Magnetic Field With Heat Transfer Using Spectral Method”, *Journal of Heat Transfer*, Vol. 128, 2006, pp. 413-417.

- [9] Ozizik, N. M., Heat Transfer: A Basic Approach, McGraw-Hill, ISBN 0-07-047982-8, North Carolina State University, pp. 289-291, Chap. 7.
- [10] Thompson J. F., Warsi Z.U.A, C., Mastin W., Numerical Grid generation: Foundations and Applications, North Holland, January 1985.
- [11] John D. Anderson. J. R., Computational Fluid Dynamics: The basics with Applications, McGraw-Hill, New York, 1995.
- [12] Thompson J. F., Soni B. K., Weatherill N. P., Hand book of Grid Generation, CRC Press, Boca Raton London, New York, Washington, 1998.