Analysis of the Nimbus-6 Rocket Fragmentation in Orbit using Theory and Computations

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Abstract
The Nimbus-6 second stage Delta rocket was the eight among several such rocket bodies to have exploded following the successful mission in placing its payload in orbit. In this study, we examine the Nimbus-6 rocket body explosion using theory and computations. The Gabbard diagram of the fragments departed significantly from the inclined ‘X’ shape for those of circular orbits with small eccentricity, the reason for which is assessed to be explosion of the propellant tank in the ‘Clam’ model. The scatterplots of the velocity perturbation components in three mutually perpendicular planes indicated that the majority of the fragments headed in one octant of space in the forward, leftward and downward directions in the fragmenting rocket’s frame of reference. The diametrically opposite octant containing the rupture location was largely devoid of fragments. The angular distribution of the fragments in a cylindrical projection map showing the locations of fragments having the largest and smallest velocity perturbations substantiates this finding. The average values of the velocity perturbations and specific kinetic energy enhancements of the fragments indicate that the fragmentation of the Nimbus-6 rocket body in orbit was far more energetic than those of the NOAA-3 and Landsat-1 rocket bodies.

INTRODUCTION
Since 1973 several Delta second stage rocket bodies (RB) had exploded in low-Earth orbit at various intervals following the successful performance of their missions. The Nimbus-6 RB (International Designator 1975-52B; U.S. Satellite Number 7946) was the eighth of these events which occurred nearly 191 months after the successful
deployment of the Nimbus-6 payload [1, 2]. The cause of the explosion is believed to be the ignition of the residual propellant left in the RB in Sun-synchronous orbit [1, 2]. The event took place at 0856 GMT on 1 May 1991 at 66°N latitude and 38°W longitude over Greenland [1, 2]. In this paper, we study the Nimbus-6 RB explosion by (1) examining the Gabbard diagram of the orbits of the fragments produced; and (2) calculating and analyzing the magnitudes and directions of the fragment velocities. The method of Badhwar, et al. [3] is used for the latter purpose. The event location and relevant data are taken from *History of on-orbit Satellite Fragmentations* [2]. The data for the orbital elements of the fragments are taken from *Space-track.org* [4]. The results are compared with the fragmentations of NOAA-3 and Landsat-1 RBs [5, 6] to shed additional light on the low-Earth-orbit upper stage fragmentation phenomenology.

**GABBARD DIAGRAM OF THE FRAGMENTS**

The Gabbard diagram is one of the earliest tools to investigate a satellite fragmentation in orbit [7]. It is a plot of the apogee and perigee heights, \( h_A \) and \( h_P \), respectively, of the fragments as functions of their periods, \( P \). For a satellite having semi-major axis \( a \) and eccentricity \( e \), the apsidal heights are given by

\[
\begin{align*}
    h_A &= a(1 + e) - r_\oplus \\
    h_P &= a(1 - e) - r_\oplus
\end{align*}
\]

where \( r_\oplus \) is the reference radius of the Earth, and

\[
a = \frac{3GM}{n^2}
\]

is the semi-major axis, \( GM \) the gravitational parameter of the Earth and \( n \) the mean motion of the satellite. The period, by definition is, \( P = 2\pi/n \).

**VELOCITY PERTURBATIONS OF THE FRAGMENTS**

The second and more important tool to analyze a satellite fragmentation event consists of calculating the velocity perturbations of the fragments from their orbital elements. Exact solutions of the three orthogonal components of the velocity perturbation of a fragment in the radial, down-range and cross-range directions of the parent satellite denoted by \( dv_r \), \( dv_d \) and \( dv_x \), respectively, were obtained [3] as follows:

\[
\begin{align*}
    dv_r &= \pm \sqrt{\mu \left( \frac{2}{r} - \frac{1}{a'} \right) - \frac{\mu a'}{r^2} (1 - e'^2)} - v_r \\
    dv_d &= \frac{\cos \zeta}{r} \sqrt{\mu a'(1 - e'^2)} - v_d
\end{align*}
\]
and

\[ dv_x = \frac{\sin \zeta}{r} \sqrt{\mu a'(1 - e'^2)} \]  

(6)

where \( v_r \) and \( v_d \) are respectively the radial and down-range components of the velocity of the parent satellite at the point of fragmentation; \( a' \) and \( e' \) are respectively the semi-major axis and eccentricity of the fragment’s orbit; and \( r \) is the radial distance of the fragmentation point from the centre of the Earth. The plane-change angle of the fragment’s orbit is

\[ \zeta = \pm \cos^{-1} \frac{\cos i' \sqrt{(\cos^2 \lambda - \cos^2 i)(\cos^2 \lambda - \cos^2 i')}}{\cos^2 \lambda} \]  

(7)

where \( i \) and \( i' \) are the inclinations of the parent and fragment’s orbits, respectively and \( \lambda \) is the latitude of the fragmentation point. In Eq. (4), the + and – signs correspond to the ascending and descending modes of the fragment, respectively; and in Eq. (7), the + sign corresponds to \( i' > i \) and the – sign corresponds to \( i' < i \) on the northbound orbits with the opposite sense on the southbound orbits [3].

RESULTS

The fragmentation of Nimbus-6 RB took place on 1 May (Day 121) 1991 at 08:56 GMT above Greenland at 66°N latitude and 38°W longitude [1, 2]. The event occurred 191 months after successful deployment of the Nimbus-6 payload [1, 2]. The pre-event orbital elements data of the fragmenting RB were of Day 112 of 1991 [2]. In accordance with the latter, the satellite had inclination of \( i = 99.5801^\circ \); eccentricity \( e = 0.006217 \); and mean motion \( n = 13.43007146 \) rev/day. The data for the satellite yield: \( P = 107.2221 \) min; \( a = 7,476.238 \) km; \( h_a = 1102.741 \) km; \( h_p = 1093.445 \) km; \( v_d = 7,299.899955 \) km/s; and \( v_r = -4.13299 \) m/s. The data for the orbital elements of the fragments are taken from Space-track.org [4]. Altogether 151 fragments including the largest remnant and catalogued through Day 43 of 1992 were taken into consideration.

Fig. 1 is the Gabbard diagram of the 151 fragments of the Nimbus-6 RB. Since the orbit of the rocket body prior to fragmentation was nearly circular (\( e = 0.006217 \)) and the fragmentation altitude was relatively high (\( h = 1090 \) km), one would expect a classical ‘X’ form with nearly equal number of fragments on either side of the inclined ‘X’ for an isotropic fragmentation in the ‘Octant’ Model [8]. In contrast, there were fewer fragments on the left hand side, with most of them within the ‘forbidden zone’. This automatically raises the possibility that the fragmentation might not have taken place in the Octant Model, but may well have taken place in the ‘Clam’ or the ‘Half-segment’ model [8]. The answer to this question would become clear from the following analyses.
The velocity perturbations of the Landsat-1 RB fragments were calculated using Eqs. (4) – (7). The frequency distributions of $dv_d$, $dv_x$, $dv_r$ and $dv$ of the fragments are shown in Fig. 2, together with their fitted curves. The distributions of $dv_d$, $dv_x$ and $dv_r$ are, by and large Gaussian, with those of $dv_d$ and $dv_x$ shifted in the forward direction with that of $dv_r$ in the downward direction. This indicates strong directionalities in the spread of fragments in this event. The frequency distribution of $dv$’s in the Nimbus-6 RB fragmentation (Fig. 2) could be fitted with a ‘Truncated Normal Distribution’ curve in contrast to those of the NOAA-3 and Landsat-1 RB’s, which were exponential [5, 6].
Fig. 2. Frequency distributions of the velocity perturbations of the fragments of Nimbus-6 Rocket Body.
A cursory look at the velocity perturbations in Fig. 2 indicates that their magnitudes were quite large. The average values of the velocity perturbations $\langle dv \rangle$ were calculated for the NOAA-3, Landsat-1 and Nimbus-6 RB fragments for comparison (Table I). Also calculated were the average specific kinetic energy enhancements $\langle \frac{1}{2} dv^2 \rangle$ for the fragments. (Table I). The values indicate a marked difference between the fragments of NOAA-3 and Landsat-1 on one hand and those of Nimbus-6 on the other. The average velocity perturbation $\langle dv \rangle$ of the Nimbus-6 fragments was 2.79 and 3.51 times larger than those of the NOAA-3 and Landsat-1 fragments, respectively. Likewise, the average specific kinetic energy enhancement $\langle \frac{1}{2} dv^2 \rangle$ of the Nimbus-6 fragments was 6.92 and 8.86 times greater than those of NOAA-3 and Landsat-1 RB fragments, respectively. The inference is made that the Nimbus-6 RB explosion was far more energetic than the NOAA-3 and Landsat-1 RB explosions.

<table>
<thead>
<tr>
<th>Fragmenting RB</th>
<th>$\langle dv \rangle$, m/s</th>
<th>$\langle \frac{1}{2} dv^2 \rangle$, m$^2$/s$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOAA-3</td>
<td>107.77</td>
<td>9,611.79</td>
</tr>
<tr>
<td>Landsat-1</td>
<td>85.56</td>
<td>7,501.45</td>
</tr>
<tr>
<td>Nimbus-6</td>
<td>300.69</td>
<td>66,468.20</td>
</tr>
</tbody>
</table>

The scatterplots of velocity perturbation components of the Nimbus-6 RB fragments are shown in three mutually perpendicular planes in Fig. 3: (1) In the horizontal plane, viewed from vertically above; (2) In a vertical plane containing the momentum of the parent; and (3) In another vertical plane containing the angular momentum vector of the parent. All three plots indicate that the fragment spread was far from isotropic, with significant concentration of fragments in a single quadrant. This implies that in the three dimensional space, a large concentration of fragments is found in a single octant, which in the orthogonal coordinate system ($dv_d, dv_v, dv_r$), is the fifth octant. This would further suggest that the explosive fragmentation of the Nimbus-6 RB occurred in the ‘Clam’ model and not in the ‘Half-segment’ model [8], since in the latter case, the fragment concentration will be in two diametrically opposite directions [8].
Fig. 3. Scatterplots of the velocity perturbation components of Nimbus-6 RB fragments.
At this juncture, it will be appropriate to briefly review the Clam Model of an exploding spherical tank according to the PISCES Code [8, 9]. A schematic diagram is shown in Fig. 4. In this model, the rupture to the tank occurs at a single location, through which burnt gases (represented by blue dotted arrows in Fig. 4) escape into the hemisphere facing the rupture. In this figure, the rupture occurs at at the 12 O’clock position. The distribution of resulting fragments is highly non-uniform in magnitude and anisotropic in direction. In Fig. 4, the red arrows represent the velocity perturbation vectors of the fragments emanating from the 3, 6, 9 and 12 O’clock positions. Fragments from the rupture location have the greatest velocity perturbations whereas fragments from its anti-podal location have the least velocity changes. Very importantly, fragments from the hemisphere opposite the rupture location head out in a narrow cone of angle 54°, which could easily be contained in a single octant of space while the diametrically opposite octant would be largely devoid of fragments.

![Fig. 4. Schematic diagram of exploding spherical tank in the Clam Model. Blue arrows represent escaping gases whereas red arrows represent velocities of fragments from the 3, 6, 9 and 12 O’clock positions.](image-url)
The three-dimensional scatterplot of the velocity perturbation components of Nimbus-6 RB fragments (Fig. 5) lends support to our contention that the explosion of the RB happened in the ‘Clam’ model of exploding propellant tanks. In that figure, \((dv_d, dv_x, dv_r)\) define a right-handed system of coordinates in the local frame of the fragmenting RB. A large concentration of fragments in a cone in the forward, downward and leftward directions given by positive \(dv_d\) and \(dv_x\) and negative \(dv_r\) values, respectively, is clearly discernible. This suggests that the rupture location of the fragmenting RB was on the diametrically opposite side. Also evident is a lack of fragments in the opposite hemisphere of space. Furthermore, fragments with the largest displacements seem to lie in the border regions between the two hemispheres in accordance with the ‘Clam’ model of Fig. 4.

The explosion of the Nimbus-6 RB in the ‘Clam’ model will become further evident in the angular distribution of its fragments. In the local coordinated system, the latitude \(\lambda\) and longitude \(\phi\) of a fragment are given by:
$$\lambda = \sin^{-1} \frac{dv_x}{dv}$$  \hspace{1cm} (8)$$

$$\phi = \tan^{-1} \frac{dv_x}{dv_d} + n\pi$$  \hspace{1cm} (9)

where $n = 0$ if $dv_d > 0$; $n = 1$ if $dv_d < 0$ and $dv_x > 0$; and $n = -1$ if $dv_d < 0$ and $dv_x < 0$.

The angular coordinates of the fragments are calculated in accordance with Eqs. (8) and (9) and plotted on a Lambert’s equidistant cylindrical projection map (Fig. 6). In that map, the latitudes and longitudes are equally spaced and the octants of space are marked. Very tellingly, 81 of the 151 fragments, representing 53.64% of the total, are found in the single Octant V. The remaining 46.36% of the fragments are distributed among the remaining 7 octants of space. Significantly too, Octant III, diametrically opposite to Octant V, is largely devoid of fragments. This almost certainly suggests that the rupture location was facing Octant III. Finally, the locations of the fragments with the greatest and smallest velocity perturbations are shown in Fig. 6. The 3 fragments with $dv < 50$ m/s were all found in Octant III whereas the 3 fragments with $dv > 800$ m/s were located in Octants I and VIII (both adjacent to Octant V), which further supports the view that the Nimbus-6 RB had fragmented in the ‘Clam’ model.

![Angular distribution of fragments of Nimbus-6 RB in local frame of reference of the fragmenting parent.](image-url)
SUMMARY & DISCUSSION

The explosive fragmentation of the Nimbus-6 second stage Delta RB is now analyzed using exact solutions of the fragments’ velocity perturbations. The results are compared with those of the NOAA-3 and Landsat-1 RB fragmentations. The fragmentation of Nimbus-6 RB was far more energetic than those of NOAA-3 and Landsat-1 RB’s. The distribution of fragment velocities in the Nimbus-6 event was a ‘Truncate Normal’ distribution as opposed to those of the NOAA-3 and Landsat-1 event, which were ‘exponential’. Velocity perturbations components and the angular distribution of the fragments indicate that the Nimbus-6 RB exploded in the ‘Clam’ model of exploding propellant tanks. This is totally different from those of the NOAA-3 and Landsat-1 RB fragmentations, which exploded in the ‘Octant’ model. It will be interesting to see if any of the other upper stage rocket fragmentations explode in the ‘Half-segment’ model.

REFERENCES


