Analysis of the Iridium 33 and Cosmos 2251 Collision using Velocity Perturbations of the Fragments

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Abstract

The accidental collision of Iridium 33 and Cosmos 2251 satellites in February 2009 produced the second largest space debris in the history of satellite fragmentations. Fragment products of the collision were continuously found and cataloged for a long time. This study examines the collision by analyzing the paths of the colliding satellites prior to the collision and the velocity perturbations of 1307 fragments cataloged through 168 days from the aftermath of the collision. The collision produced two distinct clouds of debris and no center-of-mass debris, consistent with a phenomenology learnt from the Delta 180 collision experiment of 1986. The angle of encounter of 101.71° meant that no high energy ricochet fragments like those observed in the Solwind ASAT experiment, were produced. The velocity perturbations of the fragments of collision were calculated in the three orthogonal (radial, down-range and cross-range) directions in the parent satellite’s frame of references. They all exhibited Gaussian patterns belonging to two distinct debris clouds. For the fragments of both satellites, the Gaussian patterns in both down-range and cross-range directions were shifted in the direction of the oncoming satellite as a result of impact. In the radial direction, slightly more fragments of Iridium 33 received velocity perturbations downwards, while the opposite was true for the fragments of Cosmos 2251, perhaps suggesting that the upper part of the former satellite impacted the lower part of the latter. Interestingly, the far more numerous Cosmos fragments received greater velocity changes on average than their less numerous Iridium counterparts. The largest remnants of both debris which inherited the designations of their parent satellites, both received small velocity perturbations in the retrograde and downward directions.
Introduction
On 10 February 2009, the first ever accidental collision between two large orbiting satellites took place over northern Siberia [1]. Iridium 33, a U.S. operational communications satellite (International Designator 1997-051C; U.S. Satellite Number 24946) and Cosmos 2251, a Russian decommissioned military communications satellite (International Designator 1993-036A; U.S. Satellite Number 22675) collided at 1656 GMT at a latitude of 72.51°N, longitude 97.88°E, and altitude of 789 km [1, 2]. The two satellites were in nearly circular orbits having eccentricities of .0002251 and .0016015 for Iridium 33 and Cosmos 2251, respectively [3]. Iridium 33 was northbound with an inclination of 86.3989° and Cosmos 2251 was southbound having an inclination 74.0357° [3].

The collision of Iridium 33 and Cosmos 2251 satellites produced the second largest space debris in the history of satellite fragmentations, and the largest since the Fengyun ASAT experiment two years earlier [4]. Normally, most of the trackable debris are discovered and cataloged within 150 days after a breakup [4]. In the case of the Cosmos-Iridium collision, additional debris were found and cataloged beyond that date. The U.S. Space Surveillance Network catalogs the orbital elements of each trackable object in orbit. They can be accessed from the www.space-track.org website of the Department of Defense [3]. In this study, we have considered 1307 fragments of the Cosmos-Iridium collision, which were cataloged through Day 209 of 2009 and calculated their ejection velocities or velocity perturbations from the orbital elements sets. We have used the method prescribed by Badhwar, et al. [5], which was successfully utilized to analyze the Solwind ASAT experiment [6], Delta 180 collision experiment [7] and the Spot 1 Ariane rocket fragmentation [8].

The Colliding Satellites
Cosmos 2251 and Iridium 33 were two moderately large satellites having dry masses of 900 kg and 560 kg, respectively [1]. The Cosmos satellite was roughly cylindrical in shape having a length of 3 m and diameter 2 m with a 5 m long gravity gradient boom [9]. The Iridium satellite was of triangular cylindrical shape having a length of 2 m and diameter 1 m [9]. It had two 3.5 m x 0.6 m large solar panels and three 1.2 m x 0.6 m communication panels attached to its sides [9]. As calculated below, the satellites collided at an angle of 101.71° at a relative speed of 11.57 km/s.

Fig.1. Geometry of the Cosmos 2251 and Iridium 33 encounter and the angle of encounter.
Figure 1 shows the geometry of the Cosmos 2251 – Iridium 33 encounter. Iridium 33 was northbound while Cosmos 2251 was southbound prior to the collision which occurred at the point A or C. The angle of encounter is thus:

$$\theta = \pi - A - C$$  \hspace{1cm} (1)

The angles A and C can be found from the right spherical triangles ABE and CDE. By the law of sines, we have

$$\sin A = \frac{\cos B}{\cos b}$$  \hspace{1cm} (2)

and

$$\sin C = \frac{\cos D}{\cos d}$$  \hspace{1cm} (3)

where B and D are the inclinations of Iridium 33 and Cosmos 2251, respectively, prior to the collision: \( B = i_1 = 86.3989^\circ \) and \( D = i_C = 74.0357^\circ \). Also, \( b = d = \lambda = 72.51^\circ \), the latitude of the location of impact A or C. Equations (2) and (3) furnish: \( A = 12.063^\circ \) and \( C = 66.227^\circ \), whence the angle of encounter is, from Eq. (1):

$$\theta = 101.71^\circ .$$

From the point of view of either Iridium 33 or Cosmos 2251, the other satellite arrived from a roughly transverse direction with a significant head-on component equal to \( \cos(\pi - \theta) = 0.203 \) or 20.3\% of the latter’s velocity. To use the convention of wind direction, in a frame of reference of Iridium 33 facing north, Cosmos 2251 came from the WNW direction. Likewise, in the frame of reference of Cosmos 2251 facing north, Iridium 33 arrived from the ENE direction.

In a vertical plane, the velocity of a satellite \( \vec{v} \) consists of a down-range component \( v_d \) and a vertical component \( v_r \). In terms of the gravitational parameter \( \mu \), the semi major axis \( a \), eccentricity \( e \) and the radial distance from the center of the Earth \( r \), one has:

$$v = \sqrt{\frac{\mu}{r} \left( \frac{2}{r} - \frac{1}{a} \right)}$$  \hspace{1cm} (4)

$$v_d = \frac{1}{r} \sqrt{\mu a (1 - e^2)}$$  \hspace{1cm} (5)

And

$$v_r = \pm \frac{1}{a} \sqrt{\mu e^2 - \frac{\mu}{a} (r - a)^2}$$  \hspace{1cm} (6)

In Eq. (6), the + sign corresponds to the ascending node of the satellite (true anomaly \( \nu > \pi \)), whereas the – sign corresponds to the descending node (\( \nu < \pi \)). The true anomaly \( \nu \) is obtained from the mean anomaly \( M \) via the eccentric anomaly \( \omega \):

$$\nu = 2 \tan^{-1} \left( \frac{1 + e}{1 - e} \tan \frac{\omega}{2} \right)$$  \hspace{1cm} (7)
and
\[ \omega \approx M + e \sin M + \frac{1}{2} e^2 \sin 2M + \frac{1}{8} e^3 (3 \sin 3M - \sin M) \quad (8) \]

The semi major axis of the satellite is calculated from the mean motion \( n \):
\[ a = \frac{\mu}{n^2} \quad (9) \]

The slope angle of the satellite \( \alpha \) is obtained from \( v_r \) and \( v_d \):
\[ \alpha = \tan^{-1} \frac{v_r}{v_d} \quad (10) \]

Both of the colliding satellites had nearly circular orbits \( e = 0.002253 \) for Iridium 33 and \( e = 0.016015 \) for Cosmos 2251) and both were descending \( (\nu = 270.5^\circ \) for Iridium 33 and \( \nu = 264.3^\circ \) for Cosmos 2251) prior to the collision. The calculated slope angles were: \( \alpha \approx 0^\circ \) for Iridium 33 and \( \alpha \approx -0.94^\circ \) for Cosmos 2251. Thus both satellites were travelling nearly horizontally prior to the collision with Cosmos 2251 having a slight downward slope of less than \( 1^\circ \).

The relative velocity of impact between the two satellites is estimated from the angle of encounter \( \theta \) and the individual velocities of the two satellites \( v_i \) and \( v_C \):
\[ v_{ic} = \sqrt{v_i^2 + v_C^2 - 2v_i v_C \cos \theta} \quad (11) \]
giving \( v_{ic} = 11.57 \text{ km/s} \). Owing to the slight head-on component, this was the highest velocity of impact of any collision event in orbit thus far.

The accidental collision between Iridium 33 and Cosmos 2251 bears great resemblance with the planned Delta 180 collision experiment conducted in September 1986, when the 930 kg payload of a Delta launch vehicle was made to collide with its 1370 kg second stage rocket. In the latter event, two distinct debris clouds were produced, and no center-of-mass clouds of any significance were created contrary to expectations \([10, 11]\). This is a salient feature of hypervelocity collision phenomenology in space first learnt from that event \([10, 11]\). The absence of any center-of-mass debris cloud in the Iridium 33 – Cosmos 2251 collision confirms that phenomenology.

There is another aspect of hypervelocity collision phenomenology in space, whose presence or absence is even more intriguing. A few fragments called “anomalous fragments” were found in high apogee orbits in the Solwind ASAT experiment \([12]\) and in the Delta 180 collision experiment in space \([10, 11]\). These high energy fragments have been identified as being “ricochet fragments” in hypervelocity impact at oblique incidence and the necessary condition for their formation has been established which is that the angle between the projectile and the ricochet must be greater than \( 124^\circ \) \([6, 13]\). Since the angle of encounter between Cosmos 2251 and Iridium 33 was \( 101.71^\circ \), no ricochet fragments were to be expected.
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according to the above criterion, and indeed none was observed. This further serves to validate the condition for creation of ricochet fragments in orbital space.

The Gabbard diagram is one of the earliest tools used to study the fragments of a satellite breakup. It plots the heights of apogee and perigee of the fragments against the period of revolution. Figure 2 shows the Gabbard diagrams for 383 fragments of Iridium 33 and 924 fragments of Cosmos 2251 satellites cataloged through day 208 of 2009. Certainly there were no fragments with unusually high periods or apogee to be classified in the category of ricochet fragments.

Fig. 2. Gabbard diagrams of the fragments of Iridium 33 and Cosmos 2251 satellites.

Figure 2 shows that the Cosmos 2251 fragments were scattered over far greater ranges in periods and apogee/perigee heights than the Iridium 33 fragments. The Iridium 33 and Cosmos 2251 satellites had essentially the same periods of 100.40 min and 100.62 min, respectively, prior to the collision. Whereas the periods of the Iridium fragments ranged from 96.66 min to 105.59 min, those of the Cosmos fragments ranges from 91.87 min to 109.98 min, or twice the range of the former. This means that the more numerous Cosmos fragments were also ejected with greater velocity changes than their less numerous Iridium counterparts from the same collision. Fragments inside the arms of the “X”, especially those in the lower ends of period or apogee, mostly belonging to Cosmos 2251, showed significant drag effects.

Velocity Perturbations of the Fragments

Exact solutions for the velocity perturbations of a fragment were obtained by Badhwar et al. [5]. Upon fragmentation, the velocity of a fragment has the components \( v_r + dv_r \), \( v_q + dv_q \) and \( dv_x \), where the velocity perturbation
components of the fragment are in the three orthogonal directions (radial, down-range and cross-range) are given by [5]:

\[ dv_r = \pm \sqrt{\mu \left( \frac{2}{r^2} - \frac{1}{r^2} \right) - \frac{\mu a^i}{r^2} (1 - e^2)} - v_r \] (12)

\[ dv_d = \frac{\cos \zeta}{r} \sqrt{\mu a (1 - e^2) - v_d} \] (13)

And

\[ dv_x = \frac{\sin \zeta}{r} \sqrt{\mu a (1 - e^2)} \] (14)

In the above equations, \( a \) is the semi major axis and \( e' \) the eccentricity of the fragment’s orbit, and \( \zeta \) is the plane change angle of the fragment’s orbit from the parent’s orbit. In Eq. (12), the + sign corresponds to the ascending node of the fragment (true anomaly \( \nu < \pi \)), whereas the - sign corresponds to the descending node (\( \nu > \pi \)).

The plane change angle \( \zeta \) is calculated from the inclinations \( i \) and \( i' \) of the parent’s and fragment’s orbits, respectively, and the latitude of the breakup point \( \lambda \) as:

\[ \zeta = \pm \cos^{-1} \frac{\cos i \cos i' + \sqrt{(\cos^2 \lambda - \cos^2 i)(\cos^2 \lambda - \cos^2 i')}}{\cos^2 \lambda} \] (15)

Here the + sign corresponds to \( i' > i \) and the – sign corresponds to \( i' < i \) on the northbound orbits with the opposite sense on the southbound orbits.

The true anomaly \( \nu' \) of the fragment at the time of the breakup, which dictates the sign of \( v_r + dv_r \) in Eq. (12), is determined from the argument of latitude \( u' \) and the argument of perigee \( \omega' \) at the time of fragmentation as

\[ \nu' = u' - \omega' \] (16)

The argument of latitude \( u' \) is given by

\[ u' = \sin^{-1} \left( \frac{\sin \lambda}{\sin i'} \right) \] (17)

for northbound motion of the fragment at the time of fragmentation, or by

\[ u' = \pi - \sin^{-1} \left( \frac{\sin \lambda}{\sin i'} \right) \] (18)

for southbound motion. The argument of perigee of the fragment \( \omega' \) at the time of breakup \( t \) is determined from its value \( \omega'_0 \) at the time of observation \( t_0 \) by [14]:

\[ \omega' = \omega'_0 - \frac{4.98(5 \cos^2 \lambda - 1)(t_0 - t)}{(a/r_\oplus)^{3/2}(1-e^2)^2} \] (19)

where \( r_\oplus \) is the reference radius of the Earth, \( \omega \) and \( \omega'_0 \) are expressed in degrees, and \( t \) and \( t_0 \) are expressed in days.

In this study, we have calculated the velocity perturbations of 383 fragments of Iridium 33 and 924 fragments of Cosmos 2251 which were cataloged through Day 209 of 2009, i.e., 168 days after the collision, which took place on Day 41 of 2009.
The orbital elements of the fragments are taken from www.space-track.org website of the Department of Defense [2]. The list of fragments does not include the largest remnants of the two colliding satellites, which assumed their parent’s designation and are studied separately. Fragments continued to be cataloged after the cut-off date. However, the cut-off period is of sufficient duration during which most of the larger fragments were accounted for. Moreover, the accuracy of the data is expected to deteriorate because of perturbations of various kinds, especially atmospheric drag.

**Fig. 3.** Distributions of the velocity perturbations components of the fragments of Iridium 33 in the radial, down-range and cross-range directions. The numbers of fragments with positive and negative velocity perturbations are marked.

Figure 3 shows the histograms of the velocity perturbations components of the fragments of Iridium 33 in the three orthogonal directions in the parent satellite’s frame of reference. All three distributions resembled Gaussian patterns. The directions
of the oncoming Cosmos satellite are also shown on the down-range and cross-range distribution plots. Clearly, the Gaussian patterns are shifted in these directions. The numbers of fragments with velocity perturbations in the positive and negative senses are also shown. Interestingly, more than half of the fragments received velocity perturbations in the negative sense in all three directions. Specifically, 55% of the fragments had velocity perturbations in the downward direction; 64% of them had velocity perturbations in the retrograde direction; and 68% of them had velocity perturbations in the negative cross-range direction. The last two results are as one would expect given the direction of the Cosmos satellite prior to the collision. More fragments were thrown in the direction of the colliding satellite in accordance with momentum conservation. The mean value of the change in speed of the 383 Iridium fragments was 55.79 m/s.

![Graphs showing velocity perturbation distributions](image)

**Fig. 4.** Distributions of the velocity perturbations components of the fragments of Cosmos 2251 in the radial, down-range and cross-range directions. The numbers of fragments with positive and negative velocity perturbations are marked.
Figure 4 shows the histograms of the velocity perturbations components of the fragments of Cosmos 2251 in the three orthogonal directions in the parent satellite’s frame of reference. Once again, all three distributions resembled Gaussian patterns. The directions of the oncoming Iridium satellite are also shown on the down-range and cross-range distribution plots. The numbers of fragments with velocity perturbations in the positive and negative senses are again shown. In this case, 52% of the fragments received velocity perturbations in the upward direction; 75% of them had velocity perturbations in the retrograde direction; and 54% of them had velocity perturbations in the positive cross-range direction. The last two results are, once again, consistent with the direction of the Iridium satellite prior to the collision. In this case, the mean value of the change in speed of the Cosmos fragments was 91.76 m/s, which was 64% greater than that of the Iridium fragments. This is a seemingly contradictory result that the 2.41 times more numerous fragments of Cosmos can suffer such greater velocity changes compared with the Iridium fragments. One possible explanation would be that the Cosmos fragments were on average far smaller than the Iridium fragments.

As is gleaned from Figs. 3 and 4, slightly more than half of the Iridium 33 fragments received velocity perturbations in the radially downward direction whereas slightly more than half of the Cosmos 2251 fragments received velocity perturbations in the radially upward direction. This may be construed to suggest that perhaps the upper part of Iridium 33 struck the lower part of Cosmos 2251. This scenario seems quite plausible as it has been reported that at least two of the lower antennas of Iridium 33 remained intact following the collision [15].

**Fig. 5.** Scatterplots of the velocity perturbations components of Iridium 33 satellite in the horizontal (down-range – cross-range) plane and a vertical (radial – cross-range) plane. The numbers of fragments in each quadrant are marked as well as the directions of the colliding satellites.
Figure 5 shows the scatter-plots of the velocity perturbations components of Iridium 33 satellite in the horizontal (down-range – cross-range) plane and a vertical (radial – cross-range) plane. The numbers of fragments in each quadrant are marked as well as the directions of the colliding satellites. In the horizontal plane, Cosmos 2251 arrived from the first quadrant and struck Iridium 33 at the origin. As a result, the fewest number of fragments (54) were ejected in the first quadrant, while the highest number of fragments (176) were ejected in the diametrically opposite third quadrant. In the vertical plane, Cosmos 2251 came from the positive $dv_x$ axis and consequently far more fragments (259 out of 383) were ejected in the negative $dv_x$ quadrants. Overall, more fragments (211 out of 383) went below the horizontal plane as opposed to above.

![Figure 5](image1.png)

**Fig. 5.** Scatterplots of the velocity perturbations components of Iridium 33 satellite in the horizontal (down-range – cross-range) plane and a vertical (radial – cross-range) plane. The numbers of fragments in each quadrant are marked as well as the directions of the colliding satellites.

Figure 6 shows the scatter-plots of the velocity perturbations components of Cosmos 2251 satellite in the horizontal (down-range – cross-range) plane and a vertical (radial – cross-range) plane. The numbers of fragments in each quadrant are marked as well as the directions of the colliding satellites.

![Figure 6](image2.png)

**Fig. 6.** Scatterplots of the velocity perturbations components of Cosmos 2251 satellite in the horizontal (down-range – cross-range) plane and a vertical (radial – cross-range) plane. The numbers of fragments in each quadrant are marked as well as the directions of the colliding satellites.

Figure 6 shows the scatter-plots of the velocity perturbations components of Cosmos 2251 satellite in the horizontal (down-range – cross-range) plane and a vertical (radial – cross-range) plane. The numbers of fragments in each quadrant are marked as well as the directions of the colliding satellites. In the horizontal plane, Iridium 33 arrived from the fourth quadrant and struck Cosmos 2251 at the origin. In consequence, the largest number of fragments (390) headed in the diametrically opposite second quadrant. In the vertical plane, Iridium 33 came from the negative $dv_x$ axis and consequently more fragments (503 out of 924) were ejected in the positive $dv_x$
quadrants. Overall, slightly more fragments (484 out of 924) received positive \( dv \)'s as opposed to negative.

**Fig. 7.** Three-dimensional scatter-plot of the velocity perturbations components of Iridium 33 satellite in the radial, down-range and cross-range directions. The direction of the oncoming Cosmos 2251 satellite is shown.

Figure 7 is a three-dimensional scatter-plot of the velocity perturbations components of Iridium 33 satellite in the radial, down-range and cross-range directions, giving a bird’s eye-view perspective. Prior to the collision, the direction of Iridium 33 was approximately along positive \( dv_r \) direction. The direction of the oncoming Cosmos 2251 is as shown in the figure. Overall, most of the fragments were bundled in the middle with relatively small velocity spreads with a handful of fragments in the peripheral space. Conspicuously, a small region near the origin appears devoid of fragments indicating perhaps that most of the Iridium fragments received finite velocity perturbations.

**Fig. 8.** Three-dimensional scatter-plot of the velocity perturbations components of Cosmos 2251 satellite in the radial, down-range and cross-range directions. The direction of the oncoming Iridium 33 satellite is shown.
Figure 8 is the corresponding three-dimensional scatter-plot of the velocity perturbations components of Cosmos 2251 satellite in the radial, down-range and cross-range directions. The direction of Cosmos prior to the collision was approximately along the down-range direction, whereas the direction of the oncoming Iridium 33 satellite is shown as marked. The velocity perturbations spread of the Cosmos fragments was far greater than those of the Iridium fragments, as noted earlier. This is particularly true in the horizontal plane consistent with Fig. 6. One must be reminded that due to the very large number of fragments, many fragments are hidden from the view.

Following the collision, the fragment having the largest radar cross-section (RCS) assumes the title and satellite number of the parent. The velocity perturbations received by the largest remnants of Iridium 33 and Cosmos 2251 were calculated. The results for the Iridium remnant were the following: $dv_r = -5.48$ m/s; $dv_d = -8.86$ m/s; $dv_x = -3.51$ m/s; and $dv = 11.00$ m/s; whereas those for the Cosmos remnant were: $dv_r = -6.15$ m/s; $dv_d = -1.60$ m/s; $dv_x = -4.33$ m/s; and $dv = 7.69$ m/s. Recalling that the average velocity changes of the Iridium and Cosmos fragments were $dv = 55.79$ m/s and $dv = 91.76$ m/s, respectively, the velocity changes of the largest remnants were far smaller. It has been suggested that much of Iridium 33 was left intact following the collision [15]. The same can also be inferred for Cosmos 2251. According to one study, the largest remnant of Iridium 33 may well contain more than half of the mass of the original satellite [4].

The largest remnants of both Iridium 33 and Cosmos 2251 both suffered negative velocity perturbations in the vertical and down-range directions. This can be explained by the fact that the two satellites were travelling nearly horizontally and both had encountered a head-on component of the other’s momentum at collision. In the cross-range direction, the main remnant of Iridium 33 suffered velocity change along the oncoming satellite’s direction, where the majority of its fragments also headed. However, for the main remnant of Cosmos 2251, the deflection in the cross-range direction was opposite that of the direction of the oncoming satellite. This does not violate the momentum conservation law, however, since the vast majority of the Cosmos fragments were deflected in the opposite direction (i.e., in the direction of Iridium 33) with large velocity changes.

Discussion
The hypervelocity collision between Iridium 33 and Cosmos 2251 satellites in 2009 was the first accidental collision of two large satellites in orbit. It differed from the planned Delta 180 experiment in 1986 in that the impact occurred at an obtuse angle of 103.71° as opposed to an acute glancing angle of 19.1° in the latter case [7]. Consequently, the relative velocity of impact was far greater in the Iridium 33 – Cosmos 2251 collision and the fragment production was far more prolific (over 1,700 trackable fragments as opposed to 381 in the Delta 180 experiment) [10, 11]. The difference in the angles of impact also meant that no high energy ricochet fragments were produced in the Iridium 33 – Cosmos 2251 collision event as were in the
Solwind ASAT experiment [12] and the Delta 180 collision experiment [10, 11]. This also validates the necessary condition for ricochet formation [13] which may be applied in future collision events in space.

References
