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On soft $\beta-$open sets

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Abstract

Soft β-open sets and soft β-closed sets in a soft topological space are studied. Properties and characerizations of soft β-closed sets and soft β-open sets are derived. Its relation with other sets in the soft topological space are also studied.

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1. Introduction

To solve several practical problems in economics, engineering, social science and medical science, Molodtsov [4] introduced the theory of soft sets as a new mathematical application for handling few vagueness which the traditional mathematical tool could not resolve. Shabir and Naz [15] studied the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Also, the authors introduced various soft separations axioms and studied their properties.

Recently, several research papers were published on soft set theory and their applications [14]. Maji et al. [10] elaborated on the operations on soft sets, and some basic properties of these operations have also been revealed. Several of these research papers namely [1–3, 5–13, 16, 17] forms the basis for the theoretical aspects to apply topology on soft sets and help the development of information system and engineering. Few other authors defined over an initial universe set with a fixed set of parameters. Our treatment here is different from that of in paper[15].

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2. Preliminaries

The following definitions are essential for the development of the paper.

Definition 2.1. [4] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U. The pair (F, E) or simply F_E, is called a *soft set* over U, where F is a mapping given by $F : E \longrightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in U$, $F(e)$ may be considered as the set of e - approximate elements of the soft set F. The collection of all soft sets over U and E is denoted by $S(U)$. If $A \subset E$, then the pair (F, A) or simply F_A, is called a *soft set* over U, where F is a mapping $F : A \longrightarrow P(U)$. Note that for $e \notin A$, $F(e) = \emptyset$.

Definition 2.2. [10] The *union of two soft sets* of F_B and G_C over the common universe U is the soft set H_D, where B and C are subsets of the parameter set E, $D = B \cup C$ and for all $e \in D$, $H(e) = F(e)$ if $e \in B - C$, $H(e) = G(e)$ if $e \in C - B$ and $H(e) = F(e) \cup G(e)$ if $e \in B \cap C$, we write $F_B \widetilde{\cup} G_C = H_D$.

Definition 2.3. [10] The *intersecion of two soft sets* of F_B and G_C over the common universe U is the soft set H_D, where $D = B \cap C$ and for all $e \in D$, $H(e) = F(e) \cap G(e)$ if $D = B \cap C$. We write $F_B \widetilde{\cap} G_C = H_D$.

Definition 2.4. [10] Let F_B and G_C be soft sets over a common universe set U and $B, C \subseteq E$. Then F_B is a *soft subset* of G_C, denoted by $F_B \widetilde{\subset} G_C$, if (i) $B \subset C$ and (ii) for all $e \in B$, $F(e) = G(e)$. Also, G_C, is called the soft super set of F_B and is denoted by $F_B \widetilde{\supset} G_C$.

Definition 2.5. [10] The soft sets F_B and G_C over a common universe set U are said to be *soft equal*, if $F_B \widetilde{\subset} G_C$, and $F_B \widetilde{\supset} G_C$. Then we write $F_B = G_C$.

Definition 2.6. [10] A soft set F_B over U is called a *null soft set* denoted by F_ϕ, if for all $e \in B$, $F(e) = \emptyset$.

Definition 2.7. [9] The *relative complement* of a soft set F_A, denoted by F_A^c, is defined by the approximate function $f_{A^c}(e) = f_A^c(e)$, where $f_A^c(e)$ is the soft complement of the soft set $f_A(e)$, that is $f_A^c(e) = U - f_A(e)$ for all $e \in E$. It is easy to see that $(F_A^c)^c = F_A$, $F_\phi^c = F_E$ and $F_E^c = F_\emptyset$.

Definition 2.8. [10] Let U be an initial universe and E be a set of parameters. If $B \subset E$, the soft set F_B over U is called an *absolute soft set* , if for all $e \in B$, $F(e) = U$.

Definition 2.9. [6] Let U be an initial universe and E be a set of parameters. Let τ be a subcollection of $S(U)$, the collection of soft sets defined on U. Then τ is a *soft topology* if it satisfies the following conditions.

(i) $F_\emptyset, F_E \in \tau$.

(ii) The union of any number of soft sets in τ belongs to τ.

(iii) The intersection of any two soft sets in τ belongs to τ.

If this definition is considered for further development, then the results are similar to that of results in topological spaces. Therefore, throughout the paper the following definition of soft topology is used.

Definition 2.10. [9] Let U be an initial universe and E be a set of parameters. Let $F_A \in S(U)$. A *soft topology* on F_A, denoted by $\widetilde{\tau}$, is a collection of soft subsets of F_A having the following properties:

1. $F_A, F_\phi \in \widetilde{\tau}$
2. $\{F_{A_i} \widetilde{\subseteq} F_A : i \in I\} \widetilde{\subseteq} \widetilde{\tau} \Rightarrow \widetilde{\bigcup}_{i \in I} F_{A_i} \in \widetilde{\tau}$
3. $\{F_{A_i} \widetilde{\subseteq} F_A : 1 \leq i \leq n, n \in \mathbf{N}\} \widetilde{\subseteq} \widetilde{\tau} \Rightarrow \widetilde{\bigcap}_{i=1}^{n} F_{A_i} \in \widetilde{\tau}$.

The pair $(F_A, \widetilde{\tau})$ is called a soft topological space.

The definition of soft closed sets is taken as in [9], then Example 2.1 in [17] gives a soft topological space in $(F_A, \widetilde{\tau})$ which there is no soft closed set. Therefore, we follow the following proper perfect definition of soft closed in soft topological spaces $(F_A, \widetilde{\tau})$.

Definition 2.11. [8] Let $(F_A, \widetilde{\tau})$ be a soft topological space in F_A. Elements of $\widetilde{\tau}$ are called *soft open* sets. A soft setF_B in F_A is said to be a *soft closed* set in F_A, if its relative complement, $F_B^c \cap F_A$ belongs to $\widetilde{\tau}$.

Definition 2.12. [16] Let $(F_A, \widetilde{\tau})$ be a soft topological space and F_B be a soft set in F_A.

(i) The *soft interior* of F_B is the soft set int(F_B)=$\widetilde{\cup}\{F_C$: F_C is soft open set and $F_C \widetilde{\subseteq} F_B\}$.

(ii) The *soft closure* of F_B is the soft set cl(F_B)=$\widetilde{\cap}\{F_C$: F_C is soft closed set and $F_B \widetilde{\subseteq} F_C\}$.

Its clearly int(F_B) is the largest soft open set contained in F_B and cl(F_B) is the smallest soft closed set containing F_B.

Lemma 2.13. [16] Let $(F_A, \widetilde{\tau})$ be a soft topological space and F_B and F_C be a soft subset of F_A. Then the following hold.

(i) int(int(F_B))=int(F_B),

(ii) $F_B \widetilde{\subseteq} F_C$ implies int(F_B)$\widetilde{\subseteq}$int(F_C).

(iii) int(F_B)$\widetilde{\cap}$int(F_C)=int($F_B \widetilde{\cap} F_C$).

(iv) int(F_B)$\widetilde{\cup}$int(F_C)$\widetilde{\subseteq}$int($F_B \widetilde{\cup} F_C$).

(v) F_B is soft open set if and only if F_B=int(F_B).

Lemma 2.14. [16] Let $(F_A, \widetilde{\tau})$ be a soft topological space and F_B and F_C be a soft subset of F_A. Then the following hold.

(i) cl(cl(F_B))=cl(F_B).

(ii) $F_B \widetilde{\subseteq} F_C$ implies cl$(F_B)\widetilde{\subseteq}cl(F_C)$.

(iii) cl$(F_B)\widetilde{\cap}$cl$(F_C)\widetilde{\supseteq}cl(F_B\widetilde{\cap}F_C)$.

(iv) cl$(F_B)\widetilde{\cup}$cl(F_C)=cl$(F_B\widetilde{\cup}F_C)$.

(v) F_B is soft closed set if and only if F_B=cl(F_B).

Lemma 2.15. [8] Arbitrary union of soft open sets is soft open and finite intersection of soft closed sets is soft closed.

Proposition 2.16. [16] Let $(F_A, \widetilde{\tau})$ be a soft topological space over F_A. Then the following hold.

(i) $F_\emptyset$, F_E are soft closed sets in F_A

(ii) The union of any two soft closed sets is a soft closed set in F_A.

(iii) The intersection of any two soft closed sets is a soft closed set in F_A.

Proposition 2.17. [16] If $\{F_{B_\alpha} | \alpha \in I\}$ is a collection of soft sets, then the following hold.

(i) $\widetilde{\cup}$int$(F_{B_\alpha})\widetilde{\subseteq}$ int$(\widetilde{\cup}F_{B_\alpha})$.

(ii) $\widetilde{\cup}$cl$(F_{B_\alpha})\widetilde{\subseteq}$ cl$(\widetilde{\cup}F_{B_\alpha})$.

Lemma 2.18. [6]

(i) For every soft open set F_B in a soft topological space $(F_A, \widetilde{\tau})$ and every soft set F_C, we have cl$(F_C)\widetilde{\cap}F_B\widetilde{\subseteq}cl(F_C\widetilde{\cap}F_B)$.

(ii) For every soft closed set F_B in a soft topological space $(F_A, \widetilde{\tau})$ and every soft set F_C, we have int$(F_B\widetilde{\cup}F_C)\widetilde{\subseteq}int(F_B)\widetilde{\cup}F_C$.

Definition 2.19. [6] Let F_B be a soft subset of F_A of a soft topological space $(F_A, \widetilde{\tau})$. F_B is a soft α-open set, is $F_B\widetilde{\subseteq}$int(cl(int(F_B))) and a soft α-closed set, if cl(int(cl(F_B)))$\widetilde{\subseteq}F_B$.

Definition 2.20. [7] Let F_B be a soft subset of F_A of a soft topological space $(F_A, \widetilde{\tau})$. F_B is a soft semi open set, if $F_B\widetilde{\subseteq}$cl(int(F_B)) and a soft semi closed set, if int(cl(F_B))$\widetilde{\subseteq}F_B$.

Definition 2.21. [8] Let F_B be a soft subset of F_A of a soft topological space $(F_A, \widetilde{\tau})$. F_B is a soft pre-open set, if $F_B\widetilde{\subseteq}$ int(cl(F_B)) and a soft pre-closed set, if cl(int(F_B))$\widetilde{\subseteq}F_B$.

3. Soft β-open set and Soft β-closed set

Definition 3.1. Let F_B be a soft subset of F_A of a soft topological space $(F_A, \widetilde{\tau})$. F_B is a soft β-open set (or) a soft semi-pre open set, if $F_B \widetilde{\subseteq}$cl(int(cl($F_B$))). The complement of a soft β-open set is a soft β-closed set.

Theorem 3.2. Let F_B be a soft subset of F_A be a soft topological space of $(F_A, \widetilde{\tau})$. Then F_B is a soft $\beta-$closed set if only if int(cl(int(F_B)))$\widetilde{\subseteq} F_B$.

Proof. Let F_B be a soft β-closed set. Then F_B^c is a soft $\beta-$open set and so $F_B^c \widetilde{\subseteq}$ cl(int(cl(F_B^c))). Therefore, $F_B^c \widetilde{\subseteq}$ [int(cl(int(F_B)))]^c and so $F_B \widetilde{\supseteq}$ int(cl(int(F_B))). Conversely, let int(cl(int(F_B)))$\widetilde{\subseteq} F_B$. Then [int(cl(int($F_B$)))]$^c \widetilde{\supseteq} F_B^c$ and so [cl(int(cl(F_B^c)))] $\widetilde{\supseteq} F_B^c$. Therefore, F_B is a soft β-closed set. ■

Example 3.3. Let $U = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \widetilde{\subseteq} E$.
$F_A = \{(e_1, \{a, b, c\}), (e_1, \{a, b, c\})\}$, $F_1 = \{(e_1, \{a\}) (e_2, \{b, c\})\}$,
$F_2 = \{(e_1, \{b\}) (e_2, \{a, c\})\}$, $F_3 = \{(e_2, \{c\})\}$, $F_4 = \{(e_1, \{a, b\}) (e_2, \{a, b, c\})\}$,
$F_5 = \{(e_1, \{a, c\}) (e_2, \{b\})\}$, $F_6 = \{(e_1, \{a\}) (e_2, \{b\})\}$,
$F_7 = \{(e_1, \{a, c\}), (e_2, \{b, c\})\}$, $F_8 = \{(e_1, \{b, c\}) (e_2, \{a, b, c\})\}$,
$F_9 = \{(e_1, \{c\}) (e_2, \{a, b\})\}$, $F_{10} = \{(e_1, \{b, c\}) (e_2, \{a\})\}$,
$F_{11} = \{(e_1, \{a, c\}) (e_2, \{b\})\}$, $F_{12} = \{(e_1, \{a, b, c\}) (e_2, \{a, b\})\}$, $F_{13} = \{(e_1, \{c\})\}$,
$F_{14} = \{(e_1, \{b\}), (e_2, \{a, c\})\}$, $F_{15} = \{(e_1, \{b, c\}), (e_2, \{a, c\})\}$,
$F_{16} = \{(e_1, \{b\}), (e_2, \{a\})\}$, $F_{17} = \{(e_1, \{a\})\}$, $F_{18} = \{(e_1, \{a, b\}), (e_2, \{c\})\}$, $F_{19} = F_A$ and $F_{20} = F_\phi$. Let $\widetilde{\tau} = \left\{F_\phi, F_A, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9\right\}$. Then $(F_A, \widetilde{\tau})$ is a soft topological space. Soft closed sets are $\{F_A, F_\phi, F_{10}, F_{11}, F_{12}, F_{13},$
$F_{14}, F_{15}, F_{16}, F_{17}, F_{18}\}$.

(i) Let us take $F_B = \{(e_1, \{b\}), (e_2, \{c\})\}$. Then int(F_B)=F_3, cl(int(F_B))=F_{18}, and int(cl(int(F_B)))=F_3, and so $F_B \widetilde{\subseteq}$cl(int($F_B$)). Hence F_B is a soft semi-open set but not a soft α-open set.

(ii) Let us take $F_B = \{(e_1, \{a\}), (e_2, \{a, c\})\}$. Then cl(F_B)=F_A, int(cl(F_B))= F_A, and cl(int(cl(F_B))) = F_A, and so $F_B \widetilde{\subseteq}$int(cl($F_B$)). Hence F_B is a soft pre-open set but not a soft α-open set.

(iii) Let us take $F_B = \{(e_1, \{c\}), (e_2, \{b, c\})\}$. Then cl(F_B)=F_A, int(cl(F_B)) = F_A, and int(F_B)=F_ϕ, cl(int(F_B)) = F_ϕ, and so $F_B \widetilde{\subseteq}$int(cl($F_B$)). Hence F_B is a soft pre-open set but not a soft semi-open set.

(iv) Let us take $F_B = \{(e_1, \{a, b\}), (e_2, \{c\})\}$. Then cl(F_B)=F_{12}, int(cl(F_B)) = F_3, and int(F_B)=F_3, cl(int(F_B)) = F_{12}, and so $F_B \widetilde{\subseteq}$cl(int($F_B$)). Hence F_B is a soft semi-open set but not a soft pre-open set.

(v) Let us take $F_B = \{(e_1, \{a, b\}), (e_2, \{a\})\}$. Then cl(F_B)=F_{12}, int(cl(F_B))=F_9, and cl(int(cl(F_B))) = F_{12}, and so $F_B \widetilde{\subseteq}$cl(int(cl($F_B$))). Hence F_B is a soft β-open set but it is neither a soft semi-open set nor a soft pre-open set.

4. Properties of soft β-open set and Soft β-closed set

Theorem 4.1. In a soft topological space $(F_A, \tilde{\tau})$, we have the following conditions.

(i) F_ϕ, F_A are soft β-open sets.

(ii) The union of any number of soft β-open subsets is a soft β-open set.

Proof.

(i) F_ϕ and F_A are soft open and so are soft β-open sets.

(ii) Let F_α belongs to soft β-open set in F_A for $\forall\alpha \in I$, where I is an index set. Then for all $\alpha \in I$, $F_\alpha \tilde{\subseteq}$ cl(int(cl(F_α))) $\Rightarrow \tilde{\bigcup}_{\alpha\in I} F_\alpha \tilde{\subseteq} \tilde{\bigcup}_{\alpha\in I}$cl(int(cl($F_\alpha$))) $\tilde{\subseteq}$ cl($\tilde{\bigcup}_{\alpha\in I}$(int(cl($F_\alpha$))))$\tilde{\subseteq}$cl(int($\tilde{\bigcup}_{\alpha\in I}$(cl($F_\alpha$))))$\tilde{\subseteq}$ cl(int(cl($\tilde{\bigcup}_{\alpha\in I} F_\alpha$))). Therefore, union of any number of soft β-open subsets is a soft β-open set. ■

The intersection of two soft β-open sets need not be soft β-open set as is illustrated by the following example.

Remark 4.2. Let $(F_A, \tilde{\tau})$ be the soft topological space of Example 3.3. We take $F_B = \{(e_1, \{b\}), (e_2, \{b\})\}$, then cl($F_B$)=$F_{12}$, int(cl($F_B$))=$F_9$, and cl(int(cl($F_B$))) = F_{12}, and so $F_B\tilde{\subseteq}$cl(int(cl(F_B))). Hence F_B is a soft β-open set and $F_C = \{(e_1, \{b\}), (e_2, \{a, c\})\}$, then cl($F_C$)=$F_{14}$, int(cl($F_C$))=$F_2$, and cl(int(cl($F_C$))) = F_{14}, and so $F_C\tilde{\subseteq}$cl(int(cl(F_C))). Hence F_C is a soft β-open set.Now, let $F_B \tilde{\cap} F_C = \{(e_1, \{b\}), (e_2, \{b\})\} \tilde{\cap}$ $\{(e_1, \{b\}), (e_2, \{a, c\})\} = \{(e_1, \{b\})\}$. If $F_D = \{(e_1, \{b\})\}$, then cl(F_D)=F_{16}, int(cl(F_D))=F_ϕ, and cl(int(cl(F_D))) = F_ϕ, and so $F_D \tilde{\nsubseteq}$cl(int(cl(F_D))). Hence F_D is not a soft β-open set.

Theorem 4.3. Any soft topological space $(F_A, \tilde{\tau})$, arbitrary intersection of soft β-closed sets is a soft β-closed set.

Proof. Let F_α be a soft β-closed set in F_A for every $\forall\alpha \in I$, where I is an index set. Then for every $\forall\alpha \in I$, $F_\alpha \tilde{\supseteq}$int(cl(int($F_\alpha$))) $\Rightarrow \tilde{\bigcap}_{\alpha\in I} F_\alpha \tilde{\supseteq} \tilde{\bigcap}_{\alpha\in I}$int(cl(int($F_\alpha$))). $\tilde{\supseteq}$ int($\tilde{\bigcap}_{\alpha\in I}$(cl(int($F_\alpha$))))$\tilde{\supseteq}$int(cl($\tilde{\bigcap}_{\alpha\in I}$(int($F_\alpha$))))$\tilde{\supseteq}$ int(cl(int($\tilde{\bigcap}_{\alpha\in I} F_\alpha$))). ■

The union of two soft β-closed sets need not be soft β-closed set as is illustrated by the following example.

Remark 4.4. Let $(F_A, \tilde{\tau})$ be the soft topological space of Example 3.3. If F_B , F_C are any two soft β-closed sets in $(F_A, \tilde{\tau})$, then the following example shows that $F_B \tilde{\cup} F_C$ need not be a soft β-closed set.

We take $F_B = \{(e_1, \{a, b, c\}), (e_2, \{b\})\}$, then int($F_B$)=$F_5$, cl(int($F_B$))=$F_{11}$, and int(cl(int($F_B$))) = F_5, and so cl(int(cl(F_B)))$\tilde{\subseteq} F_B$. Hence F_B is a soft β-closed set, and $F_C = \{(e_1, \{a, b, c\}), (e_2, \{c\})\}$, then int($F_C$)=$F_3$, cl(int($F_C$))=$F_{18}$, and int(cl(int($F_C$))) = F_3, and so cl(int(cl(F_C)))$\tilde{\subseteq} F_C$. Hence F_C is a soft β-closed set. F_B and F_C are soft

β-closed sets but $F_B\widetilde{\cup}F_C$=$\{(e_1,\{a,b,c\}),(e_2,\{b\})\}\widetilde{\cup}\{(e_1,\{a,b,c\}),(e_2,\{c\})\}$ $=\{(e_1,\{a,b,c\}),(e_2,\{b,c\})\}$. If F_D=$\{(e_1,\{a,b,c\}),(e_2,\{b,c\})\}$, then int$(F_D)$ = F_1, cl(int(F_D))=F_A, and int(cl(int(F_D))) = F_A, and so cl(int(cl(F_D)))$\widetilde{\nsubseteq}F_D$. Hence F_D is not a soft β-closed set.

Theorem 4.5. If F_B is a soft β-open set in a soft topological space $(F_A,\widetilde{\tau})$ which is also a soft semi-closed set, then F_B is a soft semi-open set.

Proof. If F_B is soft β-open set, then $F_B\widetilde{\subseteq}$cl(int(cl(F_B))) and F_B is soft semi-closed set, then int(cl(F_B))$\widetilde{\subseteq}F_B$ and so int(cl(F_B))$\widetilde{\subseteq}F_B\widetilde{\subseteq}$ cl(int(cl(F_B))). If int(cl(F_B))=F_C, then $F_C\widetilde{\subseteq}F_B\widetilde{\subseteq}$ cl(F_C). Since F_C is a soft open set, F_B is a soft semi-open set. ■

Corollary 4.6. If F_B is a soft β-closed set in a soft topological space $(F_A,\widetilde{\tau})$ which is also a soft semi-open set, then F_B is a soft semi-closed set.

Theorem 4.7. In an indiscrete soft topological space $(F_A,\widetilde{\tau})$, each soft β-open set is a soft pre-open set.

Proof. If $F_B = F_\phi$, then F_B is a soft β-open set as well as a soft pre-open set. Let $F_B \neq F_\phi$. If F_B is soft β-open, then $F_B\widetilde{\subseteq}$ cl(int(cl(F_B)) = F_A = int(cl(F_B)), since F_ϕ and F_A are the only soft open sets. Hence F_B is a soft pre-open set. ■

Theorem 4.8. A soft subset F_B in a soft topological space $(F_A,\widetilde{\tau})$, is soft β-closed set if and only if cl(F_A-cl(int(F_B))) $-$ (F_A-cl(F_B))$\widetilde{\supseteq}$cl$(F_B) - F_B$.

Proof. cl(F_A-cl(int(F_B))) $-$ (F_A- cl(F_B))$\widetilde{\supseteq}$cl$(F_B) - F_B$
$\Leftrightarrow$ cl(F_A- int(cl(int(F_B)))) $-$ (F_A-cl(F_B))$\widetilde{\supseteq}$cl$(F_B) - F_B$
$\Leftrightarrow$ (F_A- int(cl(int(F_B))))$\widetilde{\cap}$ cl(F_B))$\widetilde{\supseteq}$ cl$(F_B) - F_B$
$\Leftrightarrow$ ($F_A\widetilde{\cap}$ cl(F_B))$-$ [int(cl(int(F_B)))$\widetilde{\cap}$ cl(F_B))]$\widetilde{\supseteq}$ cl$(F_B) - F_B$
$\Leftrightarrow$ cl$(F_B)-$ int(cl(int(F_B)))$\widetilde{\supseteq}$ cl$(F_B) - F_B$
$\Leftrightarrow$ $F_B\widetilde{\subseteq}$ int(cl(int(F_B))) $\Leftrightarrow$ F_B is a soft β-closed set. ■

Theorem 4.9. If F_B is a soft β-open set in a soft topological space $(F_A,\widetilde{\tau})$ which is also a soft α-closed set, then F_B is a soft closed set.

Proof. Let F_B be a soft β-open set in F_A. Then $F_B\widetilde{\subseteq}$cl(int(cl(F_B))). Since F_B is soft α-closed set, then cl(int(cl(F_B)))$\widetilde{\subseteq}F_B$, and so cl(int(cl(F_B)))$\widetilde{\subseteq}F_B\widetilde{\subseteq}$ cl(int(cl(F_B))). Hence F_B=cl(int(cl(F_B))) which is soft closed. ■

Definition 4.10. Let $(F_A,\widetilde{\tau})$ be a soft topological space and let F_B be a soft subset of F_A.

(i) The soft β-interior of F_B is the soft set β-int$(F_B) = \widetilde{\cup}\{F_C : F_C$ is soft β-open set and $F_C\widetilde{\subseteq}F_B\}$.

(ii) The soft β-closure of F_B is the soft set β-cl$(F_B) = \widetilde{\cap}\{F_C : F_C$ is soft β-closed set and $F_B\widetilde{\subseteq}F_C\}$.

Clearly, β-cl(F_B) is the smallest soft β-closed set in F_A which contains F_B and β-int(F_B) is the largest soft β-open set in F_A which is contained in F_B.

Theorem 4.11. Let $(F_A, \widetilde{\tau})$ be a soft topological space and let F_B be a soft subset of F_A. Then the following hold.

(i) F_B is soft β-closed if and only if F_B=β-cl(F_B).

(ii) F_B is soft β-open set if and only if F_B=β-int(F_B).

Proof.

(i) Let F_B= β-cl$(F_B) = \widetilde{\cap}\{F_C : F_C$ is soft β-closed set and $F_B \widetilde{\subseteq} F_C\}$. This shows that $F_B \in \{F_C : F_C$ is a soft β-closed set and $F_B \widetilde{\subseteq} F_C\}$. Hence F_B is a soft β-closed set. Conversely, let F_B be a soft β-closed set. Since $F_B \widetilde{\subseteq} F_B$ and F_B is a soft β-closed set, $F_B \in \{F_C : F_C$ is a soft β-closed set and $F_B \widetilde{\subseteq} F_C\}$. $F_B = \widetilde{\cap}\{F_C : F_C$ is soft β-closed set and $F_B \widetilde{\subseteq} F_C\}$.

(ii) Let F_B= β-int$(F_B) = \widetilde{\cup}\{F_C : F_C$ is soft β-open set and $F_C \widetilde{\subseteq} F_B\}$. This shows that $F_B \in \{F_C : F_C$ is a soft β-open set and $F_C \widetilde{\subseteq} F_B\}$. Hence F_B is a soft β-open set. Conversely, let F_B be a soft β-open set. Since $F_B \widetilde{\subseteq} F_B$ and F_B is a soft β-open set, $F_B \in \{F_C : F_C$ is a soft β-open set and $F_C \widetilde{\subseteq} F_B\}$. Further, $F_B \widetilde{\subseteq} F_C$ for all such that F_B. $F_B = \widetilde{\cap}\{F_C : F_C$ is soft β-open set and $F_C \widetilde{\subseteq} F_B\}$. ■

Theorem 4.12. Let $(F_A, \widetilde{\tau})$ be a soft topological space and let F_B and F_C be two soft subsets of F_A. Then the following hold.

(i) β-cl(F_ϕ)=F_ϕ and β-cl(F_A)=F_A.

(ii) β-cl(β-cl(F_B))=β-cl(F_B).

(iii) $F_B \widetilde{\subseteq} F_C \Rightarrow \beta$-cl$(F_B) \widetilde{\subseteq} \beta$-cl$(F_C)$.

(iv) β-cl$(F_B \widetilde{\cup} F_C)$=$\beta$-cl$(F_B) \widetilde{\cup} \beta$-cl$(F_C)$.

(v) β-cl$(F_B \widetilde{\cap} F_C) \widetilde{\subseteq} \beta$-cl$(F_B) \widetilde{\cap} \beta$-cl$(F_C)$.

(vi) $(\beta$-cl$(F_B))^c$=β-int(F_B^c).

Proof. Let $(F_A, \widetilde{\tau})$ be a soft topological space and let F_B be a soft subset of F_A.

(i) Since F_ϕ and F_A are soft β-closed sets, by Theorem 3.14 (i), β-cl(F_ϕ)=F_ϕ and β-cl(F_A)=F_A.

(ii) β-cl(F_B) belongs to soft β-closed set in F_A by definition of soft closure and [Theorem 3.5], β-cl(β-cl(F_B))=β-cl(F_B) by Theorem 3.14 (i).

(iii) By definition of soft β-closure, β-cl$(F_B) = \widetilde{\cap}\{F_D : F_D$ is soft β-open set and $F_D \widetilde{\subseteq} F_B\}$. β-cl$(F_C) = \widetilde{\cap}\{F_E : F_E$ is soft β-open set and $F_E \widetilde{\subseteq} F_C\}$. Since $F_B \widetilde{\subseteq} \beta$-cl$(F_B)$ and $F_C \widetilde{\subseteq} \beta$-cl$(F_C)$. Now, $F_B \widetilde{\subseteq} F_C \widetilde{\subseteq} \beta$-cl$(F_C)$, which implies that $F_B \widetilde{\subseteq} \beta$-cl$(F_C)$. But β-cl(F_B) is the smallest soft β-closed set containing in F_B. Therefore, β-cl$(F_B) \widetilde{\subseteq} \beta$-cl$(F_C)$.

(iv) Since $F_B \widetilde{\subseteq} (F_B \widetilde{\cup} F_C)$ and $F_C \widetilde{\subseteq} (F_B \widetilde{\cup} F_C)$. Since β-cl$(F_B) \widetilde{\subseteq} \beta$-cl$(F_B \widetilde{\cup} F_C)$ and β-cl$(F_C) \widetilde{\subseteq} \beta$-cl$(F_B \widetilde{\cup} F_C)$ by (ii), which implies that β-cl$(F_B) \widetilde{\cup} \beta$-cl$(F_C) \widetilde{\subseteq} \beta$-cl$(F_B \widetilde{\cup} F_C) \longrightarrow (1)$. Since β-cl(F_B) and β-cl(F_C) are soft β-closed sets in $(F_A, \widetilde{\tau})$, β-cl$(F_B) \widetilde{\cup} \beta$-cl$(F_C)$ is a soft β-closed set in $(F_A, \widetilde{\tau})$. Then $F_B \widetilde{\subseteq} \beta$-cl$(F_B)$ and $F_C \widetilde{\subseteq} \beta$-cl$(F_C)$, $\Rightarrow (F_B \widetilde{\cup} F_C) \widetilde{\subseteq} \beta$-cl$(F_B)$ $\widetilde{\cup} \beta$-cl(F_C). That is β-cl$(F_B) \widetilde{\cup} \beta$-cl$(F_C)$ is a soft β-closed set containing $F_B \widetilde{\cup} F_C$. But, β-cl$(F_B \widetilde{\cup} F_C)$ is the smallest soft β-closed set containing $F_B \widetilde{\cup} F_C$. Hence β-cl$(F_B \widetilde{\cup} F_C) \widetilde{\subseteq} \beta$-cl$(F_B) \widetilde{\cup} \beta$-cl$(F_C) \longrightarrow (2)$. From (1) and (2), β-cl$(F_B \widetilde{\cup} F_C)$=$\beta$-cl$(F_B) \widetilde{\cup} \beta$-cl$(F_C)$.

(v) Since $F_B \widetilde{\cap} F_C \widetilde{\subseteq} F_B$ and $F_B \widetilde{\cap} F_C \widetilde{\subseteq} F_C$, which implies that β-cl$(F_B \widetilde{\cap} F_C) \widetilde{\subseteq} \beta$-cl$(F_B)$ and β-cl$(F_B \widetilde{\cap} F_C) \widetilde{\subseteq} \beta$-cl$(F_C)$. Therefore, β-cl$(F_B \widetilde{\cap} F_C) \widetilde{\subseteq} \beta$-cl$(F_B) \widetilde{\cap} \beta$- cl$(F_C)$.

(vi) $(\beta$-cl$(F_B))^c$=$[\widetilde{\cap}\{F_C : F_B \widetilde{\subseteq} F_C$ and F_C belongs to soft β-closed set in $F_A\}]^c$ =$\widetilde{\cup}\{(F_C)^c : (F_B)^c \widetilde{\supseteq} (F_C)^c$ and $(F_C)^c$ belongs to soft β-open set in $F_A\}$ =$\widetilde{\cup}\{F_C^c :$ $F_C^c \widetilde{\subseteq} F_B^c$ and F_C^c belongs to soft β-open set in $F_A\}$
=β-int(F_B^c). ■

Theorem 4.13. Let $(F_A, \widetilde{\tau})$ be a soft topological space and F_B be a soft subset of F_A. Then the following hold.

(i) β-int(F_ϕ)=F_ϕ and β-int(F_A)=F_A.

(ii) β-int$(\beta$-int$(F_B))$=β-int(F_B).

(iii) $F_B \widetilde{\subseteq} F_C \Rightarrow \beta$-int$(F_B) \widetilde{\subseteq} \beta$-int$(F_C)$.

(iv) β-int$(F_B \widetilde{\cap} F_C)$=$\beta$-int$(F_B) \widetilde{\cap} \beta$-int$(F_C)$.

(v) β-int$(F_B \widetilde{\cup} F_C) \widetilde{\supseteq} \beta$-int$(F_B)$ $\widetilde{\cup} \beta$-int(F_C).

(vi) $(\beta$-int$(F_B))^c$=β-cl(F_B^c).

Proof. Let $(F_A, \widetilde{\tau})$ be a soft topological space and let F_B and F_C be two soft subsets of F_A.

(i) Since F_ϕ and F_A are soft β-open sets, by Theorem 3.14 (ii).
β-int(F_ϕ)=F_ϕ and β-int(F_A)=F_A.

(ii) β-int(F_B) is a soft β-open set in F_A by definition of soft interior and [Theorem 3.4] and so β-int$(\beta$-int$(F_B))$=β-int(F_B) by Theorem 3.14 (ii).

(iii) By definition of soft β-interior, β-int$(F_B) = \widetilde{\bigcup}\{F_D : F_D$ is soft β-open set and $F_D \widetilde{\subseteq} F_B\}$ and β-int$(F_C) = \widetilde{\bigcup}\{F_E : F_E$ is soft β-open set and $F_E \widetilde{\subseteq} F_C\}$. Now, β-int$(F_B) \widetilde{\subseteq} F_B \widetilde{\subseteq} F_C \Rightarrow \beta$-int$(F_B) \widetilde{\subseteq} F_C$. Since β-int(F_C) is the largest soft β-open set contained in F_C. Therefore, β-int$(F_B) \widetilde{\subseteq} \beta$-int$(F_C)$.

(iv) Since $F_B \widetilde{\supseteq} (F_B \widetilde{\cap} F_C)$ and $F_C \widetilde{\supseteq} (F_B \widetilde{\cap} F_C)$, since β-int$(F_B) \widetilde{\supseteq} \beta$-int$(F_B \widetilde{\cap} F_C)$ and β-int$(F_C) \widetilde{\supseteq} \beta$-int$(F_B \widetilde{\cap} F_C)$ by (i), which implies that β-int$(F_B) \widetilde{\cap} \beta$-int$(F_C) \widetilde{\supseteq} \beta$-int$(F_B \widetilde{\cap} F_C) \longrightarrow (3)$. Since β-int(F_B) and β-int(F_C) are soft β-open set in $(F_A, \widetilde{\tau})$, β-int$(F_B) \widetilde{\cap} \beta$-int$(F_C)$ is a soft β-open set in $(F_A, \widetilde{\tau})$. Then $F_B \widetilde{\supseteq} \beta$-int$(F_B)$ and $F_C \widetilde{\supseteq} \beta$-int$(F_C)$, which implies that $(F_B \widetilde{\cap} F_C) \widetilde{\supseteq} \beta$-int$(F_B) \widetilde{\cap} \beta$-int$(F_C)$. That is β-int$(F_B) \widetilde{\cap} \beta$-int$(F_C)$ is a soft β-open set contained in $F_B \widetilde{\cap} F_C$. But, β-int$(F_B \widetilde{\cap} F_C)$ is the largest soft β-open set contained in $F_B \widetilde{\cap} F_C$. Hence β-int$(F_B \widetilde{\cap} F_C) \widetilde{\supseteq} \beta$-int$(F_B) \widetilde{\cap} \beta$-int$(F_C) \longrightarrow (4)$. From (3) and (4), β-int$(F_B \widetilde{\cap} F_C)$=$\beta$-int$(F_B) \widetilde{\cap} \beta$-int$(F_C)$.

(v) Since, $F_B \widetilde{\cup} F_C \widetilde{\supseteq} F_B$ and $F_B \widetilde{\cup} F_C \widetilde{\supseteq} F_C$, which implies that β-int$(F_B \widetilde{\cup} F_C) \widetilde{\supseteq} \beta$- int$(F_B)$ and β-int$(F_B \widetilde{\cup} F_C) \widetilde{\supseteq} \beta$- int$(F_C)$. Therefore,$\beta$-int$(F_B \widetilde{\cup} F_C) \widetilde{\supseteq} \beta$-int$(F_B) \widetilde{\cup} \beta$- int$(F_C)$.

(vi) $(\beta$-int$(F_B))^c$=$[\widetilde{\bigcup}\{F_C : F_C \widetilde{\subseteq} F_B$ and F_C belongs to soft β-open set in $F_A\}]^c$ = $\widetilde{\bigcap}\{(F_C)^c : (F_C)^c \widetilde{\supseteq} (F_B)^c$ and $(F_C)^c$ belongs to soft β-closed set in $F_A\}$ = $\widetilde{\bigcap}\{F_C^c : F_B^c \widetilde{\subseteq} F_C^c$ and F_C^c belongs to soft β-closd set in $F_A\}$
= β-cl(F_B^c). ■

Theorem 4.14. Let $(F_A, \widetilde{\tau})$ be a soft tolopogical space. Then the following hold.

(i) $\widetilde{\tau} \widetilde{\subset}$ soft β-open set in $(F_A, \widetilde{\tau})$.

(ii) If F_B is a soft subset of F_A and F_C is a soft pre-open set of F_A such that $F_C \widetilde{\subseteq} F_B \widetilde{\subseteq}$ cl(int(F_C)), then F_B is a soft β-open set.

Proof.

(i) It is obvious.

(ii) Since F_C is a soft pre-open set, we have $F_C \widetilde{\subseteq}$int(cl$(F_C)$). Then $F_B \widetilde{\subseteq}$cl(int(int(cl$(F_C)$))))= cl(int(cl$(F_C)$))$\widetilde{\subseteq}$cl(int(cl$(F_B)$))), so F_B is a soft β-open set. ■

Theorem 4.15. Let F_B be a soft subset of F_A in a soft topological space $(F_A, \widetilde{\tau})$. Then the following hold.

(i) α-cl$(F_B) = F_B \widetilde{\cup}$cl(int(cl$(F_B)$))).

(ii) α-int$(F_B) = F_B \widetilde{\cap}$int(cl(int$(F_B)$))).

Proof.

(i) cl(int(cl(F_B)$\widetilde{\cup}$ cl(int(F_B))))$\widetilde{\subseteq}$ cl (int(cl(F_B)$\widetilde{\cup}$ cl(int(F_B)) = cl(int(cl(F_B)))$\widetilde{\subseteq}$$F_B$$\widetilde{\cup}$ cl(int(cl(F_B))) which implies that $F_B\widetilde{\cup}$ cl(int(cl(F_B))) is a soft α-closed set which implies that α-cl(F_B)$\widetilde{\subseteq}$$F_B$$\widetilde{\cup}$ cl(int(cl(F_B))). Conversely, α-cl(F_B) is a soft α-closed set, which implies that cl(int(cl(F_B)))$\widetilde{\subseteq}$ cl(int(cl(α-cl(F_B)))$\widetilde{\subseteq}$$\alpha$-cl($F_B$) which implies that $F_B\widetilde{\cup}$ cl(int(cl(F_B)))$\widetilde{\subseteq}$
α-cl(F_B) which implies that α-cl(F_B) = $F_B\widetilde{\cup}$cl(int(cl(F_B))).

(ii) int(cl(int(F_B)$\widetilde{\cap}$ int(cl(F_B))))$\widetilde{\supseteq}$ int(cl(int(F_B)$\widetilde{\cap}$ int(cl(F_B)))) = int(cl(int(F_B)))$\widetilde{\supseteq}$$F_B$$\widetilde{\cap}$ int(cl(int(F_B))) which implies that $F_B\widetilde{\cap}$int(cl(int(F_B))) is a soft α-closed, which implies that α-int(F_B)$\widetilde{\supseteq}$$F_B$$\widetilde{\cap}$int(cl(int($F_B$))). Conversely, α-int(F_B) is a soft α-open set, which implies that int(cl(int(F_B)))$\widetilde{\supseteq}$ int(cl(int($\alpha-$int(F_B)))$\widetilde{\supseteq}$$\alpha-$int($F_B$) which implies that $F_B\widetilde{\cap}$int(cl(int(F_B)))$\widetilde{\supseteq}$
$\alpha-$int(F_B) which implies that α-int(F_B) = $F_B\widetilde{\cap}$int(cl(int(F_B))). ■

5. Conclusion

We studied in this paper about Soft β-open sets and soft β-closed sets over the soft topological space. We also defined some properties of soft β-closed sets and soft β-open sets and examples where it can be applied were also described. Examples with details and several applications related to soft β-open sets and soft β-closed sets were dealt in detail.

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